I. OVERVIEW

This two-year Exploration project in the Engaged Student Learning track is a systematic effort to identify and address mathematics-related learning issues in introductory physics courses; the setting is the Arizona State University (ASU) Polytechnic campus, one of four campuses of ASU. The physics courses include algebra- and calculus-based general physics. Through this project, we seek to establish the basis for design and development of new strategies for improving student engagement in introductory physics. Specifically, by focusing on the crucial mathematics skills and concepts needed in introductory physics, we hope to obtain maximum leverage by addressing arguably one of the most significant obstacles to effective student engagement. Ultimately, we expect ongoing and related work based on this project to result in evidence of improved student learning in physics and we hope that it may improve retention rates as well, increasing the probability of student success in those courses. With this short-term Exploration project, we will lay the basis for more extended work by identifying key mathematics difficulties and formulating a materials development strategy for addressing those difficulties, with preliminary testing of research-based instructional materials. More extended materials development work is planned for future projects, such as a Design and Development I project. Project findings and products are expected to contribute substantively to the body of research and of evidence-based instructional materials available for undergraduate physics courses, and should be a resource for other researchers and curriculum developers.

A. Instructional context

ASU enrolls over 70,000 students across its campuses, and awards over 13,000 undergraduate and graduate degrees each year. ASU is among the nation's leading post-secondary institutions in number of degrees awarded to Hispanic and Native American students. The Polytechnic campus, located in east Mesa, currently enrolls over 11,000 students and has a specific focus on engineering and technology. The College of Letters and Sciences (CLS) is responsible for teaching all of the mathematics and physics courses on the Polytechnic campus, primarily in service to the Ira A. Fulton Schools of Engineering (FSE); FSE represents the largest college on the campus. FSE on the Polytechnic campus offers degree programs in engineering, technology, and applied sciences, including such majors as manufacturing engineering, information technology, and environmental resource management. The popular majors require students to take one of the general physics courses, and so these courses serve as key gateways (and potential obstacles) to student success.

B. Identification of the problem

There has been a rapid expansion in physics course enrollments on the Polytechnic campus in the past several years. Large and persistent D/F/W rates (grades of D or F, or course withdrawals) in some of these courses have been traced in significant part to students' inadequate preparation in key mathematical concepts and methods. Discussions among mathematics and physics faculty suggest that these problems are not primarily due to failings in the ASU mathematics courses, but have deeper roots in students' pre-university education and/or are exacerbated by long-established (and hard to alter) sequencing mismatches between mathematics and physics courses. (That is, certain topics are addressed only in mathematics courses taken long after the students are first exposed to them in their physics course.) For example, students' difficulties with trigonometry, basic algebra, graphing, symbolic representation, and vector manipulation/analysis are among the issues that have been identified.

Although this problem exists in concentrated form at ASU Polytechnic, there is no reason to think that it is unique to ASU; rather, a reading of the literature (see Section IV below) suggests that the situation at ASU is representative of a broad-based problem that afflicts many U.S. colleges and universities. At ASU, in order to make significant progress in students' success in physics courses, the faculty has concluded that mathematics learning issues will have to be addressed much more effectively than is now the case, and that this will have to be done within the context of the physics courses *themselves*. We believe that our efforts to address this problem can serve as a model to other institutions facing similar issues.

C. Plan for addressing the problem

This project is unusual in that its Senior Personnel represent 50% of all tenured and tenure-track faculty who have been responsible for mathematics and physics instruction on the Polytechnic campus. (PI Meltzer is on the Teachers College faculty; all others are part of CLS. Numerous adjunct and temporary faculty also are involved in mathematics and physics instruction on the campus.) Together with the other three tenured/tenure-track CLS faculty who teach math and physics on the Polytechnic campus, they have identified a key problem and achieved consensus on a plan for addressing it; thus they are in an unusually strong position to make substantive, effective, and long-lasting changes in instruction that have excellent prospects for becoming institutionalized. Together, they will be able to effectuate instructional reforms that, for one or two isolated faculty, might not be possible. We believe this unified, consensus approach is perhaps the strongest single feature of the present project: It can serve as a "proof of concept" of methods for addressing similar problems in other institutional contexts where the faculty may not be as unified or circumstances do not allow immediate concerted action, as they do here at ASU.

The plan involves *systematically* identifying and addressing mathematics difficulties encountered by ASU Polytechnic students in their physics courses. The project staff includes a senior educational researcher in physics (Meltzer), along with highly experienced researchers and course instructors in mathematics (Kang) and physics (Peng). The project is deeply grounded in the extensive body of research and practice developed by the mathematics- and physics-education research communities during the past 20 years.

II. PROJECT OUTLINE

Here we briefly outline the main features of the project, and in the following sections we develop them in more detail. We propose to systematically identify specific mathematical concepts and skills that cause particular difficulty for students enrolled in our general introductory algebra- and calculus-based physics courses (PHY 111-112 and PHY 121-131). We will then develop instructional strategies and materials to address the learning issues, carry out reformed and supplemented instruction that incorporates the new materials, and assess the learning outcomes. In brief outline, we will:

i. use standardized diagnostics and develop new diagnostics to probe physics students' mathematics ideas and reasoning processes; carry out one-on-one interviews to validate the diagnostics; administer the diagnostics at the beginning of courses;

ii. develop and test instructional materials as well as review and adapt research-based materials in the literature to address issues identified by the diagnostics, revising the materials when necessary to address the physics context;

iii. designate time blocks in physics courses during both recitation and lab periods for small-group instruction on mathematics concepts using research-based materials;

iv. assess impact by using standardized and locally developed diagnostics administered post-instruction, and compare with baseline data being compiled as part of a WIDER project.

III. RESULTS FROM PRIOR NSF SUPPORT, AND QUALIFICATIONS OF PROJECT PERSONNEL

Over the past 20 years, PI David Meltzer has received NSF funding as PI or Co-PI on ten separate physics-education-related projects (12 overall), including two during the past five years. Project titles, award numbers, and key publications are listed in the biography pages. Recent work has focused on research into student learning of thermodynamics, along with development of active-learning, inquiry-based curricular materials for thermodynamics in college-level physics and chemistry courses. He and his collaborators have given more than 50 invited and contributed presentations and published more than a dozen refereed papers related to this work; two doctoral dissertations were supported by these grants. The project team has widely disseminated dozens of research-based curricular worksheets, diagnostic test questions, and other instructional materials. Strong evidence for effectiveness of the materials is contained in the references. For example, use of our research-based tutorial on the second law of thermodynamics resulted in score gains on diagnostic exams from ≈10% correct without tutorial use, up to ≈55% correct with use, with similar results reported at two different universities (Christensen, Meltzer, and Ogilvie, 2009). Most of the work produced by these projects, including papers, presentations, and curricular materials, may be viewed or downloaded directly; see Thermo PER (2014).

Meltzer's most recent NSF-sponsored project is in collaboration with Co-PI Kang: "WIDER: EAGER: Recognizing, assessing, and enhancing evidence-based instructional practices in STEM at Arizona State University, Polytechnic," DUE #1256333, $298,233, 24 months. Standard conceptual diagnostic exams have been systematically administered and analyzed for the first time in physics and mathematics courses on the Polytechnic campus of ASU. Data from these instruments will serve as a baseline for guiding and assessing progress as evidence-based instructional practices are integrated more fully into STEM education at ASU, and in particular will serve these purposes specifically for the present project. Several invited and contributed talks have been given that reported on project-related work (Meltzer and Thornton, 2013 a,b); Meltzer, 2013a,b).

Co-PI Yun Kang has been PI or co-PI on two NSF-supported projects; she is Co-PI on the WIDER: EAGER project cited above, and PI on a project that just began in September 2013: DMS #1313313, **"**Multiscale Modeling of Division of Labor in Social Insects." Kang is a mathematical biologist with research interests in dynamical systems and nonlinear population dynamics. She has five years of teaching experience at ASU, having taught undergraduate courses in calculus and differential equations multiple times. She serves as a committee member of the Association for Women in Mathematics (AWM) mentor network.

Co-PI Xihong Peng is an experienced researcher and instructor, having taught a range of undergraduate physics courses on the Polytechnic campus since arriving in 2008, including the large-enrollment PHY 111-112 specifically targeted in this proposal. Together, the three Senior Personnel named in this proposal constitute 50% of the tenured and tenure-track physics and mathematics faculty on ASU's Polytechnic campus.

IV. REVIEW OF THE LITERATURE, AND RELEVANCE TO THIS PROJECT

The literature on the relation between mathematics knowledge and performance in physics courses is extensive, and many different aspects of the problem have been studied. Here we carry out an extended review of key research themes of the recent past in order to identify their relationship to the current proposal. We emphasize **with boldface type** the specific elements of this research that will be applied in the current project.

To begin, we note that PI Meltzer (2002) reported a significant correlation between students' algebra skills and learning of physics as measured by performance on qualitative, conceptual physics questions which themselves required virtually no algebraic manipulation. However, the reasons for this relationship were not explored and remained (and still remain) somewhat obscure, despite valuable work on correlations between performance and students' reasoning ability reported by Coletta and co-workers, e.g., Coletta and Phillips (2005); Coletta, Phillips, and Steinert (2007).

Sherin (2001) has pointed out that "successful [physics] students learn to understand what equations say in a fundamental sense; they have a feel for expressions, and this guides their work." This is an excellent caution against focusing too narrowly on mathematical techniques and skills *in themselves* as presumed keys to success for introductory physics students. Sherin's work and much that followed has demonstrated that *students' understanding of the concepts underlying mathematical problem solving* are in fact what is central to success in physics. The concrete expressions of this broad idea in classroom practice have occupied numerous recent investigators who have sought to improve student learning. In the present project, **we will focus on probing students' understanding of underlying mathematical concepts, and not merely testing their technical skill in applying particular algorithms.** In part, this is done by soliciting and analyzing students' explanations of the reasoning they use when solving problems.

An early project to develop curricular materials that focused on students' mathematical thinking in introductory physics was that of Steinberg, Wittmann, and Redish (1997). Their work focused on mathematical concepts related to wave propagation, and is now embodied as a component of Vol. 1 of *Activity-Based Tutorials* (Wittmann, Steinberg, and Redish, 2004); additional work related to modern physics in included in Vol. 2. This valuable work provides a useful model for our project as we explore other areas of mathematical thinking related to introductory physics.

Dray and Manogue (1999; 2003; 2004) identified difficulties posed to physics students by the sharply divergent symbols and techniques used by the physics and mathematics communities for common procedures such as line and surface integrals. They conclude that "physicists tend to think geometrically, but lower-division mathematics classes have become increasingly algebraic"; moreover: "physics problems don't fit templates, so skill at solving template problems is not enough." They offer suggestions built on the theme that "rather than a plethora of formulas for different cases, physicists need a few key ideas that will be remembered later on." Further elaboration of their approach was focused on upper-level physics courses (e.g., in the "Paradigms" project; Manogue et al., 2006; Thompson et al., 2012), so it has limited application to our project, except conceptually. That is**, a key part of our project will focus on identifying specific examples of such "language mismatches" that may be affecting our introductory physics students. We will also strive to identify those "few key ideas" that can have a lasting impact on student thinking.**

The mathematical difficulties encountered by students are by no means all traceable to language mismatches in themselves. In their analysis of students' ideas and confusions regarding the "arrow" representation of electric field vectors, Gire and Price (2013) provide an excellent example of other aspects of students' mathematical thinking that will be addressed in our project. Students' difficulties with symbolic representations used in physics may not have *direct* sources or analogues in material encountered in their mathematics studies. Therefore, **we can't make the assumption that difficulties that are apparently "mathematical" in origin are necessarily caused by *or* addressable through actions taken in the students' mathematics courses.** Related work on this theme by Torigoe and Gladding (2007a,b; 2011) is discussed below.

PI Meltzer and his former student Nguyen built upon work by Knight (1995) and probed students' reasoning on graphical representations of vectors (Nguyen and Meltzer, 2003). They developed a diagnostic test and used it to explore students' thinking (this diagnostic is reproduced below on pp. 12-14). **The vector diagnostic—along with the accompanying analysis—will be part of the initial guidance as we begin this project.** Later, related work by Flores, Kanim, and Kautz (2004) is also relevant. Meltzer and Nguyen also contributed to a preliminary investigation by Christensen (Meltzer's former student) of students' reasoning with vector dot and cross products: see Christensen, Nguyen, and Meltzer (2004). Portions of this diagnostic are also reproduced below, and will also be employed in this project. Since this early work, a number of groups have published brief proceedings reports of students' thinking related to vector operations in physics. A review of this work along with a comprehensive report of a long-term investigation in Mexico, together with a new diagnostic exam, were published this year by Barniol and Zavala (2014); see also references cited in that work. **The valuable work by Barniol and Zavala (2014) will provide a solid foundation for our investigation of students' vector ideas in physics courses, as we assess the degree to which their diagnostic is applicable and informative in our context at ASU.**

Ambrose (2004) examined students' difficulties related to vector gradient and curl in intermediate-level mechanics. Ambrose found that even after instruction, students' had only weak grasp on the meaning of curl in a purely mathematical context. Students had even more trouble in a physics context, on problems asking them to identify which vector force fields were conservative. Ambrose and Wittmann (2014) developed "Intermediate Mechanics Tutorials" (guided problem-solving worksheets) to address these and other learning issues in intermediate mechanics, and we have used them with success in our own course. However, other relevant mathematics concepts (such as those involved in harmonic motion) have only a small research base in the physics education literature (e.g. Galle and Meredith, 2014; discussed below), although numerous relevant tutorials are incorporated in the work of Ambrose and Wittmann. PI Meltzer made use of the Intermediate Mechanics Tutorials when he taught an intermediate mechanics course in 2009, and some of those materials have potential value for the PHY 131 course (calculus-based electricity and magnetism). In addition to expanding the research base on these concepts, **we will assess the effectiveness of some of the Intermediate Mechanics Tutorials in our PHY 131 course**.

Another extensive source of relevant materials on intermediate mechanics is the “Classical Mechanics/Math Methods I” archive; see University of Colorado (CU): Mechanics (2014). This set of instructional materials includes a few that are focused explicitly on mathematical issues (e.g., line integrals) and a large compendium of math-related “Student Difficulties.” This listing, useful as it is, consists only of brief statements that identify confusions noted by instructors (e.g., “Don’t realize r-hat points a different direction for each little dA”); **the CU compendium and instructional materials provide a starting point for further research, and will help guide us in our initial work in the calculus-based PHY 121-131 sequence.**

Pepper et al. (2012) of the University of Colorado group also reported an extended investigation of mathematical difficulties within the context of intermediate electricity and magnetism. They focus on vector calculus concepts, as well as other mathematical ideas that have a more explicit physics context (i.e., Gauss’s law and scalar potential). The large number of guided tutorials they have developed for this course includes a few that focus explicitly on mathematical topics such as “Separation of Variables” and “Complex Exponentials,” although most are set in explicit physics contexts; see University of Colorado (CU): E&M (2014). **We will examine relevant CU tutorials and make appropriate application of their research findings in the development of our diagnostics and curricular materials for PHY 131.**

Extensive studies of mathematics difficulties encountered by upper-level thermal physics students have been reported by Thompson, Bucy, Christensen, Pollock and co-workers. (For example: Thompson, Bucy, and Mountcastle, 2006; Pollock et al., 2007; Christensen and Thompson, 2010; 2012.) They have provided diagnostic models for detecting whether, and to what extent, ostensibly “mathematical” difficulties are actually related to the use or translation of the concept *in the physics context*, and are therefore not merely a confusion whose origin lies in the students’ mathematics education in itself. Analogous work was discussed by Wagner, Manogue, and Thompson (2012). However, these workers also affirm that students’ difficulties in employing mathematics concepts in a physics context may well have their origins in weak understanding of the mathematics concepts themselves (e.g., Christensen and Pollock, 2012). Although curricular materials developed by these workers focus on upper-level thermodynamics, they provide insight into the approach we will follow in this project, that is: **we must ascertain the degree to which the physics context itself is responsible for students’ difficulties in executing certain mathematical procedures.** This is analogous to, but distinct from, the work of the University of Maryland group, discussed next. At the same time—indeed, the other side of the same coin—**we must assess the relative contribution of weak understanding of mathematics concepts to physics students’ mathematical difficulties**.

The University of Maryland group has focused attention on “student’s perception or judgment of the kind of knowledge that is appropriate to bring to bear in a particular situation.” They emphasize that “students often ‘get stuck’ using a limited group of [mathematical] skills or reasoning and fail to notice that a different set of tools (which they possess and know how to use effectively) could quickly and easily solve their problem” (Bing and Redish, 2009; Gupta and Elby, 2011). Our own investigation will explore the possibility of analogous “utilization failures” among our own students. However, our preliminary studies suggest that our students do not generally possess alternative sets of tools that they “know how to use effectively”; instead, it seems that they have never actually mastered certain key tools in the first place. Still, **our investigation will address possible utilization failures by *varying the context* in which students are asked to apply specific mathematical concepts**, thus providing multiple opportunities for students to access their knowledge and—one hopes—avoiding perceptual traps that might manifest in one or another *specific* context. In addition, **we follow the practice recommended by Bing and Redish**

(2009) of "asking open-ended questions that give students a wide range of possible responses [which] will require them to explain their reasoning to a much greater depth," to maximize the likelihood that we fully understand the students' thinking process.

An example of work that is particularly relevant to our project is that of Torigoe and Gladding (2007a,b; 2011). In a series of investigations, they demonstrated convincingly that "confusions of symbolic meaning," and *not* mere manipulation errors of the symbolic equations, are what in fact underlie many of the difficulties students have in understanding physics equations. In fact, they suggest that "an inability to interpret physics equations may be a major contributor to student failure in introductory physics." They showed that even when faced with physics problems that were *precisely* identical, differing only by using common symbols (such as "M") for numerical quantities (such as "1.5 kg"), students displayed sharply decreased levels of correct responses in the symbolic versions. **Thus it is clear that we must probe the role of symbol-meaning confusion in mathematical difficulties manifested by our students.** In order to bridge the gap between quantitative and conceptual questions, these authors offer a set of "question properties" (such as "use of a compound expression" and "[use of] simultaneous equations") that may be used by instructors "to produce quantitative questions that emphasize meaningful symbolic representation." **We will make use of these question properties as we develop assessment materials to probe students' mathematical thinking**.

Also highly relevant to our project is the work of Galle and Meredith (2014); they identified specific mathematical ideas in the context of harmonic motion that were causing difficulties for their students. They developed an intervention in the form of a worksheet administered during the lab period; the worksheet guided students to a conceptual understanding of the variables employed in harmonic-motion equations; pre- and post-instruction diagnostics were administered. These authors reported that "timely intervention makes a significant difference" by improving both students' confidence in *and* the correctness of their answers on questions related to the physical meaning of the harmonic motion equations. This relatively straightforward investigation/intervention provides a model for the point of departure of our own project, as described in the next section. Specifically, **this research/instruction model incorporates identification and analysis of specific student mathematical difficulties; the analysis is used to develop tightly focused instructional materials; use of the materials in targeted instruction is followed by assessment of student learning.**

The specific instructional model followed by the various authors cited here, and that we plan to follow as well, is that identified with "research-based active-learning instruction" as discussed in detail by Meltzer and Thornton (2012). **This model emphasizes active student problem-solving during class time in small groups, using research-based curricular materials, in which students express and reflect on their thinking verbally and in writing, with guidance and rapid feedback provided both by instructors and fellow group members.** A key source and example for this model is *Tutorials in Introductory Physics* developed by the University of Washington group (McDermott, Shaffer, and the Physics Education Group, 2002-2003).

V. PROJECT PLAN

A. Year 1

1. *[Summer 2015]* Initially, experienced course instructors in our algebra-based and calculus-based physics courses will assemble draft lists of mathematics concepts and skills that appear to be particularly difficult or troublesome for students in those courses. **Our initial focus will be trigonometry, graphing, symbolic representation, and vector manipulation.** These drafts will be reviewed and supplemented in the context of the relevant research literature; they will serve as an initial basis for development of diagnostic test items and instruments, to test students' knowledge of the concepts and skills so identified. The diagnostics will incorporate questions on specific concepts posed both in a "physics context" and in a non-physics "mathematics context" to probe whether students' performance differs in the two contexts. Diagnostic questions available in the literature will be examined for appropriateness (e.g., *Calculus Concept Inventory*: Epstein, 2007; 2013; *Test of Understanding of Vectors*: Barniol and Zavala, 2014), and will be considered for use or adaptation. In addition, diagnostic items already developed by project personnel in previous projects (see Section X) will be employed or adapted as well.
2. *[Summer 2015]* Individual one-on-one interviews will be conducted with student volunteers from the physics courses given during the summer session in order to explore their reaction to the draft diagnostics. They will be asked to solve the diagnostic problems while (or just before) explaining their thinking to the interviewer. The interviewer will ask probing follow-up questions to ensure that the students interpret the diagnostic item in the manner intended, and answer the diagnostic in a way that accurately reflects the students' own thinking. Revisions to the draft diagnostics will be made based on these interviews, as well as on interviews carried out in a "focus group" setting (2-5 students working together). The initial number of interviews (including focus group interviews) is anticipated to be on the order of 10, at which point additional interviews may be carried out if necessary.
3. *[Fall 2015]* The diagnostics will be administered to the physics classes near the beginning of the Fall 2015 semester, primarily during lab and/or recitation periods. (The first week's lab and recitation periods normally meet briefly, if at all, due to the absence of material to cover during the first week.) If necessary, revised versions of some or all of the diagnostics may be administered later in the course if initial results are indeterminate or new issues arise during instruction.
4. *[Fall 2015]*The data generated through administration of the diagnostics will undergo an intensive review by project personnel, in order to identify the nature of students' ideas; both potentially-productive aspects of students' ideas as well as potentially troublesome difficulties will be identified. (See following section for examples of data analysis methods.)
5. *[Fall 2015-Spring 2016]* A first, draft set of instructional materials will be developed to address the mathematical learning issues identified through administration of the diagnostics. The nature of the materials will be primarily "guided-inquiry worksheets" in the style of the University of Washington *Tutorials in Introductory Physics*. Previously developed materials by project personnel will be revised as appropriate; other materials in the literature will be examined and considered for use. The materials will include adaptations from past and ongoing research projects by project personnel (e.g., Nguyen and Meltzer, 2003), other CLS math/physics personnel (e.g., Larson and Zandieh, 2013) and from the published literature (e.g., Galle and Meredith, 2014; Barniol and Zavala, 2014). *(See Thermo PER, 2014 for analogous research-based instructional materials developed by the PI and his collaborators.)*

6. *[Spring 2016]*Additional and revised diagnostics will be administered to students enrolled in all introductory physics courses at the beginning of the Spring 2016 semester; some of these may be identical to, or slightly revised versions of, those given during the previous semester. Some, however, will be newly developed or freshly adapted from materials in the published literature. The purpose will be first, to confirm and expand the findings from the first administration of the diagnostics during the previous Fall semester, and second, to target additional topical areas that were not addressed in the first round of diagnostics.
7. *[Spring 2016]* Initial drafts of curricular materials will undergo preliminary assessment through one-on-one and focus-group interviews with student volunteers from targeted courses; appropriate revisions to materials will be made based on these interviews. This process will also be applied to materials drawn from the research literature, in order to ensure that they are well-tuned to the student population in our courses and to make any necessary adjustments.
8. *[Spring 2016]* Instruction based on the new materials will take place in each course, beginning in the Spring semester. Instruction would take place primarily during recitation periods, although first-week lab periods could be used as well. When practical, parts of regular class meeting times will be used for this focused instruction with the new materials.
9. *[Spring 2016]* The same or slightly modified diagnostics used pre-instruction will be administered post-instruction to assess changes in student thinking and ability (if any) on the targeted concepts. Follow-up one-on-one interviews will be carried out.
10. *[Spring 2016]* Overall learning gains on regularly given, standardized concept-based course diagnostics (such as the Force Concept Inventory and the Conceptual Survey in Electricity and Magnetism) will be monitored for possible changes due to the instructional reforms. Possible influence by other instructional or student-population changes will have to be considered as well, and we anticipate only a loose correlation between our project reforms and overall learning gains as measured by these types of conceptual diagnostics.
11. *[Spring 2016]* D/F/W rates for the courses will be monitored and compared to analogous rates for previous course administrations to determine whether any changes are apparent. These changes would need to be considered in the light of any other revisions that are made to course materials and instructional methods, in addition to those targeted by this project. (It may be difficult to trace any changes in D/F/W rates to any particular cause, but it is nonetheless important to monitor this statistic.)

B. Year 2

12. *[Summer 2016-Spring 2017]* The process described in #1-11 above will be iterated during Year 2 of the project. Findings from Year 1 will inform work in Year 2. (Note, however, the distinction that the initial work in Year 1 will be based both on instructors' previous experiences and on the research literature, while Year 2 will start from a basis of project work during the previous year.)
13. *[Summer 2016-Spring 2017]* Research findings, diagnostics, and curriculum materials from the project will be widely disseminated in both mathematics and physics education communities, although the initial focus will be on physics.

We believe that the two-year duration for this project is appropriate in that it allows us to carry out an initial period of drafting and testing, followed by a full year of follow-up work which builds on the findings of the previous period.

VI. FURTHER DETAILS OF THE DATA ANALYSIS

There are three primary categories of raw data to be collected:

1. **Field Notes:** observation notes recorded during and soon after one-on-one and focus-group interviews, by the Graduate Research Assistant (GRA) and PIs.
2. **High-Definition video and audio recordings with partial transcripts:** audio-video recordings of one-on-one interviews and focus groups (2-5 students working together). Partial transcripts of particularly significant sections of the audios and videos will be prepared (not full transcripts). Our target for Year 1 will be to record 20-40 interviews/groups, approximately one hour each. Targets for Years 2 will be determined near the conclusion of Year 1.
3. **Written work by students:** This includes all written work done by student volunteers in one-on-one and focus-group interviews, all pre- and post-instruction diagnostics administered to all students in courses, along with samples of written work on project-related curricular materials by students in courses. It includes responses to multiple-choice questions (usually accompanied by explanations of students' reasoning), free-response questions requiring verbal and/or graphical responses (see samples on pp. 11-15), or combinations of the various types of items.

Data Analysis: The raw data will undergo extensive analysis. Since it is not possible to describe detailed analysis of data from diagnostics that have not yet been created, we instead describe here some examples from the PI's previous projects to illustrate precisely how the analysis is carried out and utilized in the development of curricular materials.

Example of Analysis of Students' Written Work: Students' written responses to each diagnostic item will be categorized and tabulated. For example, in reviewing students' explanations of their responses to Question #7 on the Vector Diagnostic shown on pp. 12-14 below, we first check whether students have given the correct "*smaller than*" answer. Next, we see whether they have given an adequate explanation of this correct response. If, however, they instead give the "*larger than*" or "*equal to*" responses, we attempt to categorize their explanations to better understand their reasoning.

In this case, we found common justifications for the "*larger than*" response were either "*the arrows are further apart*" or "*the angle is greater between the vectors*." The primary justification offered for the "*equal to*" response was that vectors **A** and **B** had equal magnitude. As a specific example of a quantitative finding, we determined that in the first-semester, algebra-based course, 32% of students gave the correct "*smaller than*" response, while approximately 33% gave the incorrect response "*larger than*" and the other 33% gave "*equal to*." Together with the justifications categorized and described above, this allowed us to determine that for the first-semester, algebra-based course, instructional materials would have to address the student idea that equal-magnitude vectors added together—regardless of *how* they are added together— must always yield resultants that have the same magnitude as each other. The instructional materials would *also* have to address—to roughly the same degree—the idea that a larger angle between two vectors implied that their resultant would have a larger magnitude than would be the case with a smaller angle between them. By contrast, our investigation showed that in the *second*-semester algebra-based course, although correct responses increased to around 50% of the total, the incorrect "equal to" response was still a very popular response while the "larger than" response had declined sharply in popularity compared to the first-semester course. This allows us to target the students in the second-semester course with a different set of instructional materials that are more closely tuned to *their* most common student ideas, rather than those found in the first-semester course.

This type of detailed response analysis allows us to design well-focused instructional materials that help students address common reasoning errors. When comparing post-instruction assessments with pre-instruction assessments, we evaluate both the changes in the proportion of "correct" answers *and* changes in the proportion and quality of explanations of those answers, in order to assess the impact of an intervention. That is, we have found that in some cases, student learning is reflected not necessarily by (or solely by) increased proportions of correct responses, but instead *by the decreased proportion of* ***consistently*** *incorrect responses* (For a specific example of this type of analysis in an electricity and magnetism context, see, e.g., Meltzer, 2007.)

Analysis of Video and Audio Recording data: In our practice, although we listen to or watch the entire recording, we prepare partial transcripts only of carefully selected portions of the recording. This allows us to focus our attention on key moments when students are expressing their ideas on issues revealed by the written data to be potentially troublesome for large numbers of students. This allows us to confirm the finding and to extend it by asking follow-up questions of the students in order to reveal their reasoning in greater detail. An example of this type of analysis can be found in Meltzer (2004).

Comparison with Baseline Data and Relation to Ongoing WIDER Project
PI Meltzer and Co-PI Kang are also investigators on the current NSF-funded project “WIDER: EAGER: Recognizing, assessing, and enhancing evidence-based instructional practices in STEM at Arizona State University, Polytechnic.” As a central component of this project, we have been systematically administering pre- and post-instruction diagnostics in introductory physics and calculus courses. The body of data we are assembling will be used directly in analysis of assessment data to be acquired in the present project.

VII. WORK PLAN
Project work will be carried out by the Senior Personnel and a graduate research assistant (GRA) who is a Masters or Ph.D. student in STEM or STEM education. The GRA will handle the logistics of test administration and data collection using various diagnostic instruments, and will assist in carrying out one-on-one interviews to validate and refine diagnostic and curricular materials. The PI and Co-PIs will have primary responsibility for coordinating the drafting, editing, and analysis of diagnostic instruments, and for drafting initial plans for corresponding curricular materials, as well as supervising and carrying out interviews and focus-group interviews. The senior personnel will also participate by helping to draft diagnostic items and analyze diagnostic data from their own and other classes, by providing detailed reviews and comments on initial drafts of project materials including diagnostics and curricular materials, and by reviewing, adapting, and employing evidence-based instructional materials for those classes. Co-PI Kang will also review instructional materials developed in the project for possible incorporation in CLS math courses. Project staff will jointly analyze the data and assemble reports for review and discussion with other CLS faculty (including non-physics/math faculty, as well as adjunct and temporary physics/math faculty), instructional staff, and administrative personnel.

VIII. DISSEMINATION

We expect this project to yield results that will be of direct and immediate benefit both to physics instructors and to physics education researchers, as well as to the mathematics education community. We will prepare reports of our findings both on students’ mathematics ideas and on the nature and assessment of the curricular materials and instructional methods we develop. Presentations will be made to professional organizations such as the American Association of Physics Teachers and American Physical Society, and to the Conference on Research on Undergraduate Mathematics Education (RUME). Research papers will be prepared and submitted to journals such as American Journal of Physics, The Physics Teacher, and RUME Proceedings. We expect our work to provide a valuable addition to the literature cited above.

IX. BROADER IMPACTS

For over 100 years, the general physics course has served as a key component of the professional preparation of future engineers and other technical personnel. The physics courses themselves serve, in effect, as both gateways and potential obstacles to those who aspire to technical careers. It is widely reported that inadequate mathematics preparation has a potentially severe negative impact on students starting these career tracks. For the most part, attempts to address these issues have centered on reforms in college mathematics courses. However, as we have pointed out, such a focus may completely miss the mark since those courses don't necessarily emphasize the concepts and techniques found most troublesome by the average physics student. The model we propose of identifying and addressing mathematics difficulties *in the context of the physics courses themselves* has the potential to alter the common practice of addressing these difficulties, and to provide a model and basis for other institutions that have recognized analogous problems among their own courses and student populations.

X. EXAMPLES OF DIAGNOSTICS, AND PREVIOUS RELATED WORK [See pp. 12-15]

PI Meltzer and his former graduate students carried out extensive studies of student understanding of vectors in introductory algebra- and calculus-based physics courses. Among the materials they developed and administered were diagnostics on addition and subtraction of vectors (Nguyen and Meltzer, 2003), and on dot and cross products (Christensen, Nguyen, and Meltzer, 2004); see pp. 12-15, below. Some of the cross-product questions were administered in both a physics context (involving a magnetic field) and a non-physics context (purely abstract "mathematical"). Differences in responses in the two contexts were found, but significance could not be verified. Since physics course instructors here at ASU (including Meltzer) have noted that student difficulties with vectors appear to be significant issues, these vector diagnostics will be among the first to be deployed in our initial testing during Year 1. As shown in, e.g., Nguyen and Meltzer (2003), clear patterns of specific student ideas may be identified. In that previous work, only preliminary steps were taken to develop instructional materials to address the student ideas that were identified. In the present project, development and testing of these instructional materials will be one of the initial tasks addressed during Year 2.

algebraic and graphical aspects of vector concepts among students in introductory physics courses at several institutions similar to our own. Their results and ours consistently support a conclusion that significant additional instruction on vectors may be needed if introductory physics students are to master those concepts. We suspect that most instructors would be unsatisfied with a situation in which more than half of the students are still unable, after a full semester of study, to carry out two-dimensional vector addition (as we found to be the case in the algebra-based course).

It is clear from our findings that many students have substantial intuitive knowledge of vectors and vector superposition, obtained to some extent by study of mechanics, and yet are unable to apply their knowledge in a precise and therefore fruitful manner. They seem to lack a clear understanding of what is meant by vector direction, of how a vector may be "moved" so long as its magnitude and direction are strictly preserved, and of exactly how to carry out such moves by parallel transport. Many students are confused about the tip-to-tail and parallelogram addition rules.

One way in which vector addition may be introduced is through the use of displacement vectors, because students all have experiences that could allow understanding of how a 50-m walk to the east and subsequent 50-m walk to the north is equivalent to a 71-m walk to the northeast. Students could be guided to determine similar equivalent displacements—perhaps initially by using a grid—when the component displacements are at arbitrary angles. In order to solidify the notion of vector addition, it also would be important for students to practice applying these methods when no grid or other means for quantitative measurement is available. Many of the responses by students in our study (in particular, to problem #7) suggest that an ability to solve vector problems when a grid is available do not always translate to a similar ability in the absence of a grid. Recent interviews carried out by our group lend support to this observation.[12] We believe that curricular materials that guide students through a series of exercises in which they perform vector additions and subtractions (both with and without use of a grid) may be useful in improving their understanding of these ideas.

Further research will be needed to determine whether curricular materials based on such a strategy are effective in improving both students' performance on assessments such as the quiz used in our study, and students' ability to provide explanations of their work with precision (describing a clearly delineated calculational procedure) and accuracy (describing a *correct* calculational procedure). Additional research (such as that initiated by Oritz *et al.*[5]) is necessary to probe students' understanding of more advanced vector concepts such as scalar and vector products.

As a consequence of our findings, we have increased the amount of instructional time we devote specifically to vector concepts. We have developed some instructional materials[13] in a format similar to the problems on our diagnostic quiz, and continue development and assessment of additional materials. Our group has carried out a preliminary series of student interviews to shed additional light on student understanding of vector concepts. We are also extending our research to assess students' understanding of more advanced concepts, such as scalar and vector products, coordinate systems and rotations, etc. In addition, we are examining student understanding of vector ideas, specifically in the context of physics concepts such as superposition of forces and fields.

ACKNOWLEDGMENTS

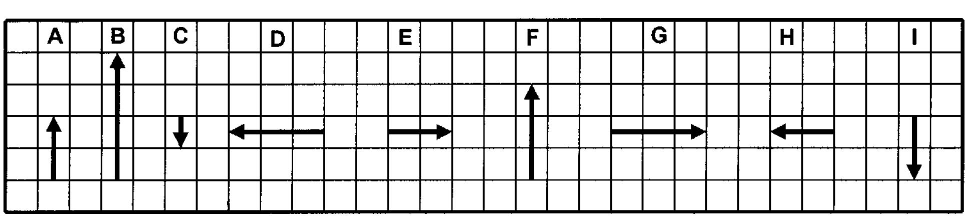
We are grateful for the assistance of Larry Engelhardt, both for his collaboration in the data analysis and for the insight he provided based on the student interviews he has recently carried out. This material is based in part upon work supported by the National Science Foundation under Grant No. REC-0206683.

APPENDIX: VECTOR CONCEPT QUIZ

Name: ______________________

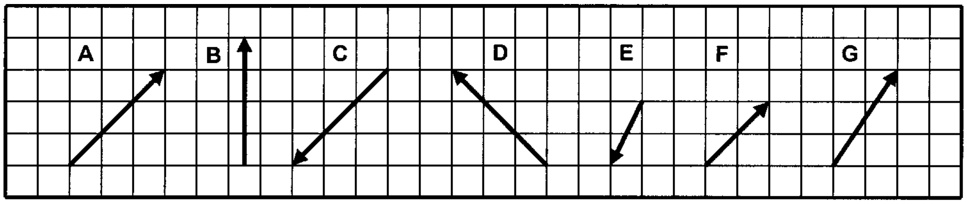
Class: ______________________ Section: ______________________

1. Consider the list below and write down **all** vectors that have the same magnitudes as each other. For instance if vectors $\vec{\mathbf{W}}$ and $\vec{\mathbf{X}}$ had the same magnitude, and the vectors $\vec{\mathbf{Y}}$, $\vec{\mathbf{Z}}$, and $\vec{\mathbf{A}}$ had the same magnitudes as each other (but different from $\vec{\mathbf{W}}$ and $\vec{\mathbf{X}}$) then you should write the following: $|W|=|X|$, $|Y|=|Z|=|A|$.



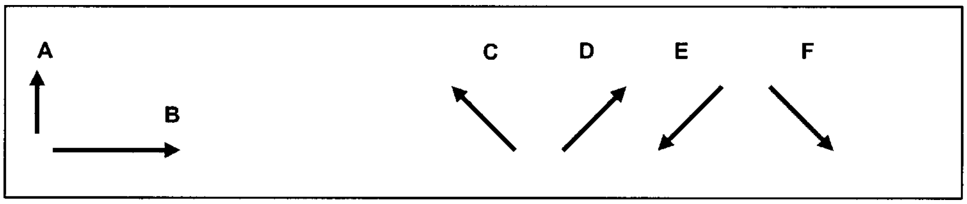
Answer ______________________________

2. List all the vectors that have the same **direction** as the first vector listed, $\vec{\mathbf{A}}$. If there are none, please explain why.



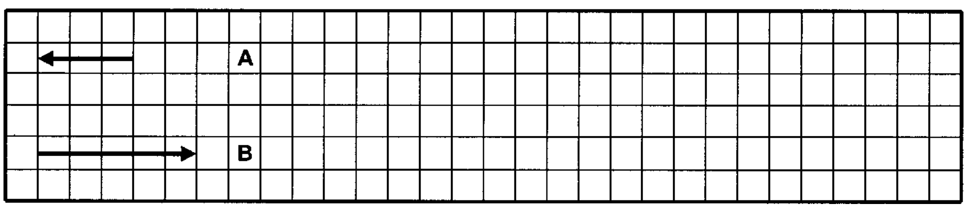
Explain ________________________________

3. Below are shown vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Consider $\vec{\mathbf{R}}$, the vector sum (the "resultant") of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, where $\vec{\mathbf{R}}=\vec{\mathbf{A}}+\vec{\mathbf{B}}$. Which of the four other vectors shown ($\vec{\mathbf{C}}$,$\vec{\mathbf{D}}$,$\vec{\mathbf{E}}$,$\vec{\mathbf{F}}$) has most nearly the ***same direction*** as $\vec{\mathbf{R}}$?



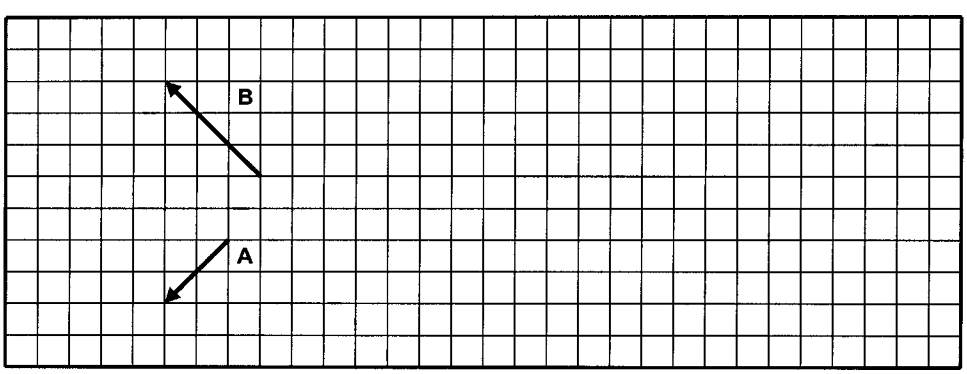
Answer ________________________________

4. In the space to the right, draw $\vec{\mathbf{R}}$ where $\vec{\mathbf{R}}=\vec{\mathbf{A}}+\vec{\mathbf{B}}$. Clearly label it as the vector $\vec{\mathbf{R}}$. Explain your work.

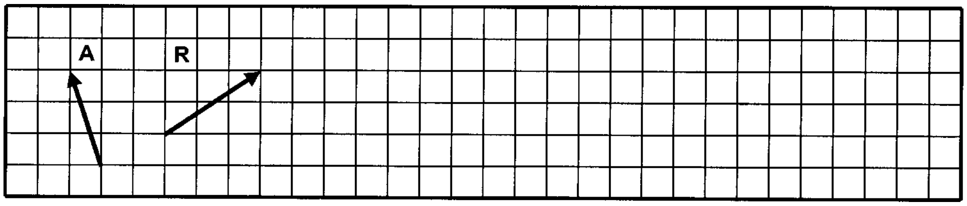


Explain ________________________________

5. In the figure below there are two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Draw a vector $\vec{\mathbf{R}}$ that is the sum of the two, (i.e., $\vec{\mathbf{R}}=\vec{\mathbf{A}}+\vec{\mathbf{B}}$). Clearly label the resultant vector as $\vec{\mathbf{R}}$.



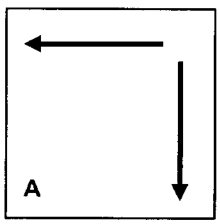
6. In the figure below, a vector $\vec{\mathbf{R}}$ is shown that is the ***net resultant*** of two other vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ (i.e., $\vec{\mathbf{R}}=\vec{\mathbf{A}}+\vec{\mathbf{B}}$). Vector $\vec{\mathbf{A}}$ is given. Find the vector $\vec{\mathbf{B}}$ that when added to $\vec{\mathbf{A}}$ produces $\vec{\mathbf{R}}$; clearly label it $\vec{\mathbf{B}}$. **DO NOT** try to combine or add $\vec{\mathbf{A}}$ and $\vec{\mathbf{R}}$ directly together! Briefly explain your answer.

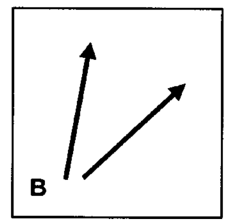


Explain ________________________________

____

7. In the boxes below are two pairs of vectors, pair **A** and pair **B**. (All arrows have the same length.) Consider the magnitude of the ***resultant*** (the vector sum) of each pair of vectors. Is the magnitude of the resultant of pair **A** *larger than, smaller than*, or *equal to* the ***magnitude*** of the resultant of pair **B**? Write an explanation justifying this conclusion.

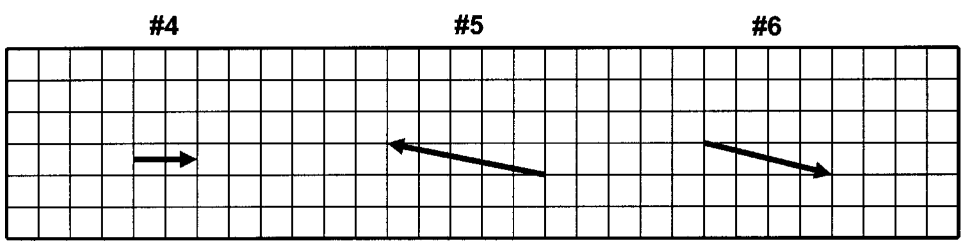




Explain ________________________________

____

Problem solutions:
1. $|A|=|E|=|H|=|I|$, $|D|=|F|=|G|$
2. **F**
3. **D**



7. smaller than.

[a]Electronic mail: nguyenn@iastate.edu
[b]Electronic mail: dem@iastate.edu

[1]Randall D. Knight, "Vector knowledge of beginning physics students," Phys. Teach. **33**, 74–78 (1995).
[2]Stephen Emile Kanim, "An investigation of student difficulties in qualitative and quantitative problem solving: Examples from electric circuits and electrostatics," Ph.D. dissertation, University of Washington (UMI, Ann Arbor, MI, 1999), UMI #9936436, Chaps. 6 and 7.
[3]Jose M. Aguirre, "Student preconceptions about vector kinematics," Phys. Teach. **26**, 212–216 (1988).
[4]Jose M. Aguirre and Graham Rankin, "College students' conceptions about vector kinematics," Phys. Educ. **24**, 290–294 (1989).
[5]Luanna G. Ortiz, Paula R. L. Heron, Peter S. Shaffer, and Lillian C. McDermott, "Identifying student reasoning difficulties with the mathematical formalism of rotational mechanics," AAPT Announcer **31**(4), 103 (2001).
[6]ACT, Inc., ACT research data at ⟨www.act.org⟩.
[7]John Roche, "Introducing vectors," Phys. Educ. **32**, 339–345 (1997).
[8]J. P. Guilford, *Fundamental Statistics in Psychology and Education*, 4th ed. (McGraw-Hill, New York, 1965), pp. 188–189.
[9]T. Entwistle, D. Gentile, E. Graf, S. Hulme, J. Schoch, A. Strassenburg, C. Swartz, C. Chiaverina, R. B. Clark, T. Durkin, D. Gavenda, F. Peterson, C. Robertson, and R. Sears, "Survey of high-school physics texts," Phys. Teach. **37**, 284–296 (1999).
[10]Arthur Eisenkraft, *Active Physics* (It's About Time, Armonk, NY, 1998).
[11]See Ref. 2, p. 156, Fig. 6-8.
[12]Larry Engelhardt (private communication).
[13]David E. Meltzer and Kandiah Manivannan, "Workbook for introductory physics, additional materials: Vector exercise," CD-ROM available from DEM.

Student difficulties with graphical representation of vector products: crossing and dotting beyond *t*'s and *i*'s*

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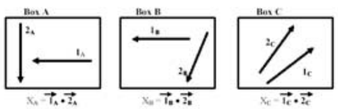
*Supported in part by NSF REC #0206683

In an effort to test students' understanding of the graphical representation of scalar and vector products, a four-question quiz was administered to students in a first-semester calculus-based physics course [221] during the spring and summer of 2004, as well to as students in a second semester calculus-based physics course [222] during the summer of 2004. The questions and results are below. (Questions were administered during the final week of the spring course, and near the mid-point of the summer courses.)

1. In each of the three boxes below (Box A, Box B, Box C) there is a pair of vectors, $\vec{1}$ and $\vec{2}$. All arrows have the same length. Consider the dot product ("scalar product") of each pair of vectors.

X_A is the dot product of the vectors in Box A.
X_B is the dot product of the vectors in Box B.
X_C is the dot product of the vectors in Box C.



Choose the answer that best describes the dot products: X_A, X_B, X_C.

A. $X_A = X_B = X_C$
B. $X_A > X_C > X_B$
C. $X_A > X_B > X_C$
D. $X_B > X_C > X_A$
E. $X_B > X_A > X_C$
F. $X_B = X_A > X_C$
G. $X_C > X_A > X_B$
H. $X_C > X_B > X_A$ (correct)
I. $X_C > X_B = X_A$
J. Cannot be determined from the given information.

$\vec{1}_i \bullet \vec{2}_i = |1_i||2_i|\cos(\theta_i)$
Since all vectors are of equal length:
$X_i \propto \cos(\theta_i)$;
$\cos(\theta_C) > \cos(\theta_B) > \cos(\theta_A)$
$\therefore\ X_C > X_B > X_A$

Correct Responses

	N	% of *N*
221 Spring	168	68%
221 Summer	36	64%
222 Summer	41	76%

Students failing to recognize X_A is smallest (i.e., responding with answers A, B, C, E , F, or G):

	N	% of *N*
221 Spring	168	28%
221 Summer	36	22%
222 Summer	41	20%

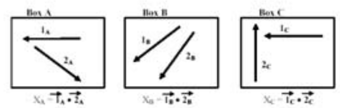
Students failing to recognize X_C is the greatest (i.e., responding with answers A, B, C, D, E, or F):

	N	% of *N*
221 Spring	168	27%
221 Summer	36	22%
222 Summer	41	17%

Those students who appeared to utilize a component method for calculating the scalar products were successful in obtaining a correct answer. Students often abandoned a component method in favor of some equation representation [i.e., $|1_A||2_A|\cos(\theta)$], with varying degrees of success.

2. In each of the three boxes below (Box A, Box B, Box C) there is a pair of vectors, $\vec{1}$ and $\vec{2}$. All arrows have the same length. Consider the dot product ("scalar product") of each pair of vectors.

X_A is the dot product of the vectors in Box A.
X_B is the dot product of the vectors in Box B.
X_C is the dot product of the vectors in Box C.



Choose the answer that best describes the dot products: X_A, X_B, X_C.

A. $X_A > 0, X_B > 0, X_C > 0$
B. $X_A > 0, X_B > 0, X_C = 0$
C. $X_A > 0, X_B > 0, X_C < 0$
D. $X_A > 0, X_B = 0, X_C > 0$
E. $X_A > 0, X_B = 0, X_C < 0$
F. $X_A < 0, X_B > 0, X_C > 0$
G. $X_A < 0, X_B > 0, X_C = 0$ (correct)
H. $X_A < 0, X_B > 0, X_C < 0$
I. $X_A < 0, X_B = 0, X_C > 0$
J. Cannot be determined from the given information.

$\vec{1}_i \bullet \vec{2}_i = |1_i||2_i|\cos(\theta_i)$
Since all vectors are of equal length:
$X_i \propto \cos(\theta_i)$
$\cos(\theta_A) < 0,\ \cos(\theta_B) > 0,\ \cos(\theta_C) = 0$
$\therefore\ X_A < 0,\ X_B > 0,\ X_C = 0$

Correct Responses

	N	% of *N*
221 Spring	168	52%
221 Summer	36	58%
222 Summer	41	61%

Students failing to recognize X_C is zero (i.e., responding with answers A, C, D, E, F, H, or I):

	N	% of *N*
221 Spring	168	28%
221 Summer	36	17%
222 Summer	41	20%

Students failing to recognize X_A is negative (i.e., responding with answers A, B, C, D, or E):

	N	% of *N*
221 Spring	168	32%
221 Summer	36	33%
222 Summer	41	27%

Typical student response when failing to recognize X_A is negative (seen in 221 and 222 students):

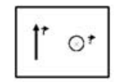
"I know C has to be 0, because cos(90) = 0, and you use the absolute values so [the magnitudes] must be >0. The angle isn't negative because it's the angle between the two vectors."

Many students chose θ to be the tip-to-tail angle, without recognizing the need to use parallel vector transport.

One of the questions administered to the students in the Spring 221 class was given to the Summer 221 and 222 students as a question on an exam. Due to the constraints of the exam we were forced to condense the responses from 10 down to 5. The question for the 222 class was put into the context of a charged particle in a magnetic field.

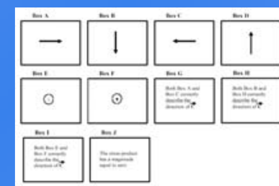
In the figure below there are two vectors, $\vec{1}$ and $\vec{2}$. There exists a cross-product ("vector product"), $\vec{C}$, of the vectors (i.e., $\vec{C} = \vec{1} \times \vec{2}$). Calculate the direction of $\vec{C}$.

Note: ⊙ represents a vector pointing **out** of the page.
⊗ represents a vector pointing **into** the page.



Which of the following boxes best describes the direction of $\vec{C}$?

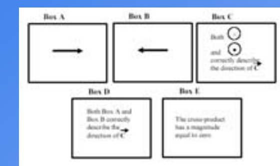
Multiple choice options for Spring 221



Spring 221 (*N* = 168)									
A	B	C	D	E	F	G	H	I	J
18%	0%	40%	1%	6%	4%	17%	1%	3%	5%

In order to get down to five choices, we removed B, D, E, F, and H. Even though choices E and F had more responses than choice I, studies have shown that some students have difficulty distinguishing the direction of a vector from that of a vector in the opposite direction (Nguyen and Meltzer, 2003). The substantial number of students selecting response G seems to support that notion. Therefore, we retained response I as a choice for the summer exam question, renaming it response C.

Multiple choice options for Summer 221/222



Summer 221 (*N* = 48)				
A	B	C	D	E
23%	50%	4%	6%	17%

Summer 222 (*N* = 56)				
A	B	C	D	E
25%	68%	4%	4%	0%

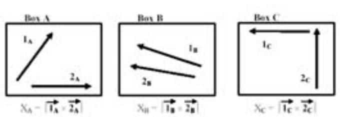
One sixth (17%) of 221 students responded that the vector product has a magnitude of zero.

On Question 3, 15% of 222 students had explicitly given "zero" for the magnitude of the vector product of two perpendicular vectors (i.e., stated that $X_C = 0$ on that question). On this exam question, by contrast, none gave that response. It is possible that the magnetic-field context of the 222 exam question was responsible for this difference.

Both 221 and 222 students seem to have significant difficulty in applying the right-hand rule, as ~25% of both classes chose the direction *opposite* to the correct response on the exam question. This is consistent with the responses to Question 4.

3. In each of the three boxes below (Box A, Box B, Box C) there is a pair of vectors, $\vec{1}$ and $\vec{2}$. All arrows have the same length. Consider the cross product ("vector product") of each pair of vectors.

X_A is the magnitude of the cross product of the vectors in Box A.
X_B is the magnitude of the cross product of the vectors in Box B.
X_C is the magnitude of the cross product of the vectors in Box C.



Choose the answer that best describes the **magnitudes** of the cross products: X_A, X_B, X_C.

A. $X_A = X_B = X_C$
B. $X_A > X_B > X_C$
C. $X_A > X_C > X_B$
D. $X_A > X_C = X_B$
E. $X_B > X_C > X_A$
F. $X_B > X_A > X_C$
G. $X_C > X_A > X_B$ (correct)
H. $X_C > X_B > X_A$
I. $X_C = X_B > X_A$
J. Cannot be determined from the given information.

$|\vec{1}_i \times \vec{2}_i| = |1_i||2_i|\sin(\theta_i)$
Since all vectors are of equal length:
$X_i \propto \sin(\theta_i)$
$\sin(\theta_C) > \sin(\theta_A) > \sin(\theta_B)$
$\therefore\ X_C > X_A > X_B$

Correct Responses

	N	% of *N*
221 Spring	206	58%
221 Summer	36	50%
222 Summer	41	56%

Students failing to recognize X_C is the greatest (i.e., responding with answers A, B, C, D, E, or F):

	N	% of *N*
221 Spring	206	36%
221 Summer	36	42%
222 Summer	41	37%

Students failing to recognize X_B is smallest (i.e., responding with answers A, B, E, F, H, or I):

	N	% of *N*
221 Spring	206	35%
221 Summer	36	42%
222 Summer	41	39%

Typical student response for an incorrect calculation of the magnitude of the vector product:

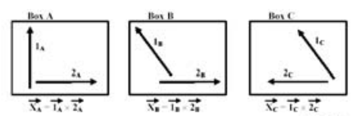
"Because for cross prod[uct] it is (1)(2)cos θ and you can factor out the (1)(2)"

Many students used a similar "cos θ" reasoning; they not only failed to recognize X_C as being the greatest quantity, but most often determined that it was zero.

Several students attempted to use a matrix method to calculate the cross product but there were no apparent successes.

4. In each of the three boxes below (Box A, Box B, Box C) there is a pair of vectors, $\vec{1}$ and $\vec{2}$. All arrows have the same length. Consider the cross product of each pair of vectors.

$\vec{X}_A$ is the cross product of the vectors in Box A.
$\vec{X}_B$ is the cross product of the vectors in Box B.
$\vec{X}_C$ is the cross product of the vectors in Box C.



Choose the answer that best describes the **direction** of the cross products: $\vec{X}_A$, $\vec{X}_B$, $\vec{X}_C$

Note: ⊙ represents a vector pointing **out** of the page.
⊗ represents a vector pointing **into** the page.

A. $\vec{X}_A$: ⊗, $\vec{X}_B$: ⊗, $\vec{X}_C$: ⊗
B. $\vec{X}_A$: ⊗, $\vec{X}_B$: ⊗, $\vec{X}_C$: ⊙ (correct)
C. $\vec{X}_A$: ⊗, $\vec{X}_B$: ⊙, $\vec{X}_C$: ⊙
D. $\vec{X}_A$: ⊗, $\vec{X}_B$: ⊙, $\vec{X}_C$: ⊗
E. $\vec{X}_A$: ⊙, $\vec{X}_B$: ⊙, $\vec{X}_C$: ⊙
F. $\vec{X}_A$: ⊙, $\vec{X}_B$: ⊙, $\vec{X}_C$: ⊗
G. $\vec{X}_A$: ⊙, $\vec{X}_B$: ⊗, $\vec{X}_C$: ⊙
H. $\vec{X}_A$: ⊙, $\vec{X}_B$: ⊗, $\vec{X}_C$: ⊗
I. Cannot be determined from the given information.

Correct Responses

	N	% of *N*
221 Spring	206	58%
221 Summer	34	53%
222 Summer	41	61%

Students responding with answer F (the directions of the vector products are reversed):

	N	% of *N*
221 Spring	206	0%
221 Summer	34	22%
222 Summer	41	20%

Students responding with answer E (all vector products are pointing out of the page):

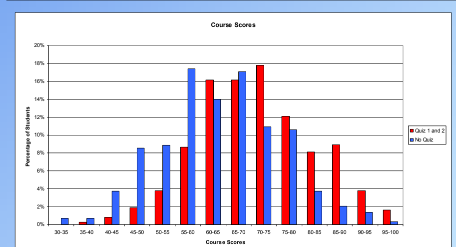
	N	% of *N*
221 Spring	206	16%
221 Summer	34	11%
222 Summer	41	5%

The absence of "F" responses in the spring 221 class is rather troublesome. Before the quiz was administered we speculated that F would be the most common incorrect answer. Our expectations were confirmed during the summer classes for both 221 and 222, but the absence of such responses in the spring 221 class is unexplained.

None of the students who selected response "E" provided an explanation.

The biased nature of a "random" sample when using an online medium

In the process of testing students' understanding of vector and scalar products, we were offered an opportunity to use an online medium, WebCT, to administer a quiz. Complying with the instructor's request, we divided our six question quiz into two 3-question quizzes. At the end of the semester, we analyzed the overall class scores (final numerical grade) of every student in the class. Below is the score distribution for the two groups that took quizzes (combined) and the one that did not.



Statistical analysis shows the following:

Descriptives

SCORE

	N	Mean	Std. Error	95% Confidence Interval for Mean: Lower Bound	95% Confidence Interval for Mean: Upper Bound
No Quiz	293	63.8	.687	62.4	65.1
Quiz 1	167	71.3	.844	69.6	72.9
Quiz 2	204	70.9	.818	69.3	72.6

The mean course score for students who took Quiz 1 (71.3) is statistically identical to the score of those who took Quiz 2 (70.9), but significantly larger ($p < 0.0001$) than that of those who took no quiz (63.8) [a difference equivalent to one full letter grade].

References

1. Ambrose, B. S. (2004). “Investigating student understanding in intermediate mechanics: Identifying the need for a tutorial approach to instruction,” American Journal of Physics **72**, 453-459.

2. Ambrose, B. S., and Michael C. Wittmann. (2014). *Intermediate Mechanics Tutorials* http://faculty.gvsu.edu/ambroseb/research/IMT.html

3. Barniol, Pablo, and Genaro Zavala, “Test of understanding of vectors: A reliable multiple-choice vector concept test,” Phys. Rev. ST Phys. Educ. Res. **10**, 010121-1–010121-14 (2014).

4. Bing, Thomas J., and E. F. Redish. (2009). “Analyzing problem solving using math in physics: Epistemological framing via warrants,” Physical Review Special Topics – Physics Education Research **5**, 020108.

5. Christensen, Warren, David E. Meltzer, and C.A. Ogilvie. (2009). “Student ideas regarding entropy and the second law of thermodynamics in an introductory physics course,” American Journal of Physics **77**, 907-917.

6. Christensen, Warren M., Ngoc-Loan Nguyen, and David E. Meltzer. (2004). “Student difficulties with graphical representation of vector products: crossing and dotting beyond t's and i's,” poster presentation at the 2004 Physics Education Research Conference, Sacramento, California, August 4-5, 2004. Direct download: http://www.physicseducation.net/talks/PERC_Vector_poster.ppt

7. Christensen, Warren M., and John R. Thompson. (2010). “Investigating student understanding of physics concepts and the underlying calculus concepts in thermodynamics,” *Proceedings of the 13th Annual Conference on Research in Undergraduate Mathematics Education* (Mathematical Association of America, Oberlin, OH).

8. Christensen, Warren M., and John R. Thompson. (2012). “Investigating graphical representations of slope and derivative without a physics context,” Physical Review Special Topics - Physics Education Research **8** 023101.

9. Coletta, V. P., & Phillips, J. A. (2005). “Interpreting FCI scores: Normalized gain, preinstruction scores, and scientific reasoning ability,” American Journal of Physics **73**, 1172.

10. Coletta, V. P., J. A. Phillips, and J. J. Steinert. (2007). “Interpreting force concept inventory scores: Normalized gain and SAT scores,” Physical Review Special Topics-Physics Education Research **3**, 010106.

11. Ding, Lin, Ruth Chabay, Bruce Sherwood, and Robert Beichner. (2006). “Evaluating an electricity and magnetism assessment tool: Brief electricity and magnetism assessment,” Physical Review Special Topics - Physics Education Research **2**, 010105.

12. Dray, Tevian, and Corinne A. Manogue. (1999). “The Vector Calculus Gap: Mathematics ≠ Physics,” PRIMUS [Problems, Resources, and Issues in Mathematics Undergraduate Studies] **9**, 21-28.

13. Dray, T., and C. A. Manogue. (2003). "Using differentials to bridge the vector calculus gap," College Mathematics Journal, **34**(4), 283-290.

14. Dray, T., and C. A. Manogue. (2004). "Bridging the gap between mathematics and physics," APS Forum on Education Newsletter Spring, 13-14.

15. Epstein, Jerome, "The Calculus Concept Inventory," in *Proceedings of the National STEM Assessment Conference [Science, Technology, Engineering, and Mathematics], October 19-21, 2006, Washington, D.C.*, edited by Donald Deeds and Bruce Callen (Drury University, Springfield, MO, 2007), pp. 60-67; Epstein, Jerome, "Development and Validation of the Calculus Concept Inventory," in *Proceedings of the Ninth International Conference on Mathematics Education in a Global Community*, 7-12 September 2007, edited by Pugalee, Rogerson, & Schinck; online at http://math.unipa.it/~grim/21_project/21_charlotte_EpsteinPaperEdit.pdf.

16. Epstein, Jerome. (2013). "The Calculus Concept Inventory: Measurement of the effect of teaching methodology in mathematics," *Notices of the AMS* **60**, 1018-1026.

17. Flores, Sergio, Stephen E. Kanim, and Christian H. Kautz, "Student use of vectors in introductory mechanics," Am. J. Phys. **72**, 460-468 (2004).

18. Galle, Gillian, and Dawn Meredith. (2014). "The trouble with trig," The Physics Teacher **52**, 112-114.

19. Gire, Elizabeth, and Edward Price. (2013). "Arrows as anchors: Conceptual blending and student use of electric field vector arrows," *2012 Physics Education Research Conference*, edited by P. V. Engelhardt et al. AIP Conf. Proc. **1513**, pp. 150–153.

20. Gupta, Ayush, and Andrew Elby. (2011). "Beyond epistemological deficits: Dynamic explanations of engineering students' difficulties with mathematical sense-making," International Journal of Science Education **33**, 2463-2488.

21. Knight, Randall D., "Vector knowledge of beginning physics students," Phys. Teach. **33**, 74-77 (1995).

22. Larson, Christine, and Michelle Zandieh. (2013). "Three interpretations of the matrix equation A**x**=**b**," For the Learning of Mathematics **33**(2), 11-17.

23. Manogue, C. A., K. Browne, T. Dray, and B. Edwards. (2006). "Why is Ampère's law so hard? A look at middle-division physics," American Journal of Physics **74**, 344.

24. McDermott, L. C., P. S. Shaffer, and the Physics Education Group. (2002-2003). *Tutorials in Introductory Physics; Homework for Tutorials in Introductory Physics; Instructor's Guide for Tutorials in Introductory Physics,* (Prentice-Hall, Upper Saddle River, NJ).

25. Meltzer, David E. (2002). "The relationship between mathematics preparation and conceptual learning gains in physics: A possible 'hidden variable' in diagnostic pretest scores," American Journal of Physics **70**, 1259-1268.

26. Meltzer, David E. (2004). "Investigation of students' reasoning regarding heat, work, and the first law of thermodynamics in an introductory calculus-based general physics course," Am. J. Phys. **72**, 1432-1446 (2004).

27. Meltzer, David E. (2007). "Analysis of shifts in students' reasoning regarding electric field and potential concepts," in *2006 Physics Education Research Conference [Syracuse, New York (USA), 26-27 July 2006]*, edited by Laura McCullough, Leon Hsu, and Paula Heron [American Institute of Physics Conference Proceedings **883**, 177-180.

28. Meltzer, David E., and Ronald K. Thornton. (2012). "Resource Letter ALIP-1: Active-learning instruction in physics," American Journal of Physics **80**, 478-496; *direct download*: http://www.physicseducation.net/docs/Meltzer_and_Thornton_2012.pdf.

29. Meltzer, David E., and Ronald K. Thornton. (2013a). *Research-based Active-Learning Instruction in Physics*, at the 2013 Winter Meeting of the American Association of Physics Teachers, New Orleans, Louisiana, January 7, 2013 [Session AD]. Available at: http://www.physicseducation.net/talks/contributed.php.

30. Meltzer, David E., and Ronald K. Thornton. (2013b). *Research-based Active-Learning Instruction in Physics*, at the 2013 American Physical Society April Meeting, Denver, Colorado, April 13, 2013 [Bulletin of the American Physical Society II, **58 (4)**, Session B15 (2013)]. Available at: http://www.physicseducation.net/talks/contributed.php.

31. Meltzer, David E. (2013a). *The "Fully Interactive" Physics Lecture: Active-Learning Instruction in a Large-Enrollment Setting*, invited talk at the Canadian Association of Physicists Congress 2013, Université de Montréal, Montréal, Québec, Canada, May 30, 2013 [Session R-TEACH-1]. Available at: http://www.physicseducation.net/talks/index.php.

32. Meltzer, David E. (2013b). *The Development of Physics Education Research and Research-Based Physics Instruction in the United States*, plenary talk at the 2013 conference on Foundations and Frontiers of Physics Education Research, Bar Harbor, Maine, June 17, 2013. Available at: http://www.physicseducation.net/talks/index.php.

33. Nguyen, Ngoc-Loan P., and David E. Meltzer. (2003). "Initial understanding of vector concepts among students in introductory physics courses," American Journal of Physics **71**, 630-638; *direct download:* http://www.physicseducation.net/docs/AJP-71-630-638.pdf.

34. Pepper, R. E., S. V. Chasteen, S. J. Pollock, and K. K. Perkins. (2012). "Observations on student difficulties with mathematics in upper-division electricity and magnetism," Physical Review Special Topics-Physics Education Research **8**, 010111.

35. Pollock, E. B., J. R. Thompson, and D. B. Mountcastle. (2007). "Student understanding of the physics and mathematics of process variables in P-V diagrams," AIP Conf. Proc. **951**, 168.

36. Pollock, Steven J., and S. V. Chasteen. (2009). "Longer term impacts of transformed courses on student conceptual understanding of E&M," in *2009 Physics Education Research Conference [Ann Arbor, MI, 29-30 July 2009]*, edited by M. Sabella, C. Henderson, and C. Singh, AIP Conference Proceedings Volume **1179** (AIP, Melville, NY, 2009), pp. 237-240.

37. Sherin, B. L. (2001). "How students understand physics equations," Cognition and instruction **19**, 479-541.

38. Steinberg, Richard N., Michael C. Wittmann, and E. F. Redish. (1997). "Mathematical tutorials in introductory physics," AIP Conference Proceedings **399**, 1075; doi: 10.1063/1.53110

39. Thermo PER. (2014); Project materials available at: http://thermoper.wikispaces.com/, and the most recent project description is at http://www.physicseducation.net/current/Description_0817282.pdf.

40. Thompson, John R., Brandon R. Bucy, and Donald B. Mountcastle. (2006). "Assessing student understanding of partial derivatives in thermodynamics," in *2005 Physics Education Research Conference*, AIP Proceedings **818**, 77-80.

41. Thompson, John R., Corinne A. Manogue, David J. Roundy, and Donald B. Mountcastle. (2012). "Representations of partial derivatives in thermodynamics," in *2011 Physics Education Research Conference*, AIP Proceedings **1413**, 85-88.

42. Torigoe, Eugene, and Gary Gladding. (2007a). "Same to us, different to them: Numeric computation versus symbolic representation," in *2006 Physics Education Research Conference*, edited by L. McCullough et al. AIP Conf. Proc. **883**, pp. 153–156.

43. Torigoe, Eugene, and Gary Gladding. (2007b). "Symbols: Weapons of math destruction," in *2007 Physics Education Research Conference*, edited by L. Hsu et al., AIP Conf. Proc. **951**, pp. 200–203.

44. Torigoe, Eugene T., and Gary E. Gladding. (2011). "Connecting symbolic difficulties with failure in physics," American Journal of Physics **79**, 133-140.

45. University of Colorado (CU): E&M. (2014). **Electrostatics:** http://www.colorado.edu/physics/EducationIssues/Electrostatics/materials.html and **Electrodynamics:** http://www.colorado.edu/physics/EducationIssues/Electrodynamics/tutorials.html

46. University of Colorado (CU): Mechanics. (2014). http://www.colorado.edu/physics/EducationIssues/ClassicalMechanics/materials.html

47. Wagner, Joseph F., Corinne A. Manogue, and John R.Thompson. (2012). "Representation issues: Using mathematics in upper-division physics," in *2011 Physics Education Research Conference*, AIP Proceedings **1413**, 89-92.

48. Wawro, Megan, Chris Rasmussen, Michelle Zandieh, George Franklin Sweeney, and Christine Larson. (2012). "An Inquiry-Oriented Approach to Span and Linear Independence: The Case of the Magic Carpet Ride Sequence," PRIMUS **22**, 577-599.

49. Wittmann, M. C., R. N. Steinberg, E. F. Redish, and University of Maryland Physics Education Research Group. (2004). *Activity-Based Tutorials* (Vols. 1 and 2). (Wiley, New York.)