Project Description

Previous NSF Support: David Meltzer has received NSF support for four curriculum development projects during the past five years. Three of these supported the development of an elementary physics course targeted at education majors and other nontechnical students.[74] (See Biography section for grant numbers and titles.) Thomas Greenbowe has worked under four contracts with the National Science Foundation during the last five years, including a DUE-ILI grant and a DUE-CCD project under the "Adapt and Adopt" program. In June 2000, Meltzer and Greenbowe were jointly awarded a DUE-CCLI grant to develop active-learning instructional materials for thermodynamics in both physics and chemistry courses. Early in 2001, Greenbowe was awarded another DUE-CCLI grant to develop chemistry curricula.

Project Description: The goal of this project is to investigate the role played by diverse representational modes in the learning of physics and chemistry concepts. We will explore the relationship between the *form of representation* of complex concepts in physics and chemistry, and students' ability to learn these concepts. We will determine the specific learning difficulties that arise as students struggle to master concepts posed in different representational forms, and we will apply our findings immediately to the development of improved curricular materials and instructional methods. We will then assess the effectiveness of these new materials and methods in bringing about greater student mastery of targeted concepts.

We believe that this research may be considered exploratory and relatively "high risk," suitable for support by an SGER grant, because it represents preliminary work with a very novel approach to an important emerging theme: widespread use of multiple representations in college-level physics and chemistry pedagogy.

Much previous research has shown that the use of multiple forms of representation in teaching concepts in physical science both has great potential benefits, and yet poses significant challenges to students and instructors. Facility in the use of more than one representation deepens a student's understanding, but specific learning difficulties arise in the use of diverse representational modes.

By "representational mode" we mean any of the widely diverse forms in which physical concepts may be understood and communicated. For instance, problems or principles may be stated in "verbal" form, using words only, or purely in mathematical form, using equations and special symbols. As an example of the use of diverse representational modes, consider Coulomb's law. In Appendix A, we show four different representations of what is essentially the *identical* problem. These are posed in four distinct representational modes – verbal (V), diagrammatic (D), mathematical/symbolic (M), and graphical (G). An analogous set of questions in a chemistry context is shown in Appendix B. *[Note: These four representational modes will be the focus of this preliminary project.]* Although to the expert these four problems are nearly identical and merely represent four different aspects of the same concept, to an introductory student they may appear very different.

What we are concerned with in this proposal are (1) common, widespread learning difficulties encountered by *many* students, and (2) the *relative degree of difficulty* of different representations in a specific context. It is often assumed by instructors that a representation that they find particularly clear and comprehensible (e.g., a graph) will also be especially clear for the average student. Research and experience shows that this is often *not* the case, but relatively little study has been devoted to this issue.

Theoretical basis for the role of multiple representations in improving student learning: There is no purely abstract understanding of a physical concept – it is always expressed in *some* representational mode. Physical scientists employ a wide variety of representations as means for understanding and working with physical systems and processes[8,35,36,40,60] In some sense, it seems that a range of diverse representations is required to "span" the *conceptual* space associated with a physical idea.[22] There is also much evidence that increasing understanding in one form of representation has significant beneficial effects in improving understanding using other forms of representation.[34,39,56] This then raises two issues: (1) What particular learning difficulties are associated with the various forms of representation? (2) Are any useful generalizations possible regarding the relative degree of difficulty of learning with the various representational modes?

Review of previous research on multiple representations: In many recently developed curricular materials in chemistry and physics there has been a great deal of attention paid to presenting concepts in various representational modes.[3,22,32,33,34,37,49,55,65,69,70] There has yet to be an investigation of the ***comparative*** merits of the various representational modes with regard to their relative effectiveness in aiding student learning in particular contexts. A closely related issue is that of students' relative performance on similar problems making use of different representational forms.[4,10,34,50,51,66] There has also been speculation regarding the role that students' individual learning styles might play, and the possible relevance of gender differences.[10]

Student difficulties in mastering physics concepts using graphical representations have been studied in considerable detail for topics in kinematics.[2,17,46,54] These studies and other, related work in the field of mathematics education (e.g., Hitch et al.[24]) have delineated several broad categories of conceptual difficulties. Conceptual difficulties related to diagrammatic representations of electric circuits and fields have been addressed by several investigators,[14,21,27,47,63,67] as have those in optics.[15,16,58] Difficulties arising from *linguistic* ambiguities ("verbal" representation) have been explored by Jacobs,[25] Kenealy,[30] Touger,[68] and Williams.[75]

Several chemistry educators have investigated learning difficulties in the context of multiple representations.[29,31,61,77,79] Most recently, Kozma[33] and Kozma and Russell[34] have reported on an extensive ongoing project aimed at investigating and developing chemistry students' "representational competence." These authors and their collaborators have reported data on the relative degree of difficulty encountered by novice students presented with a problem posed in one or another form of representation.

This previous work addresses two general learning tasks related to multiple representations of physical science concepts: (1) translating among a "real-world" physical situation and various representations of it,[42] and (2) transforming from one form of representation to another, related one.[26,39] Both mathematics and science educators, such as those cited here, have demonstrated the very significant positive effects on learning that occur when students develop facility in the use of multiple forms of representation.[39] Moreover, it is often the case that thorough understanding of a particular concept may *require* an ability to recognize and manipulate that concept in a variety of representations (a point also made by McDermott[42] and Hestenes[22]).

Numerous physics educators have stressed the importance of students developing an ability to translate among different forms of representation of concepts.[2,66,71,72] As McDermott emphasizes, "Because different representations emphasize different aspects of a concept, the more ways one can represent a concept, the deeper one's understanding is likely to be."[42] Hestenes says, ". . . complete specification (and understanding) of a model [of a physical system] requires *coordination of multiple representations*"[22] [emphasis in original]. This "translation" issue has also been addressed by Clement[9] and Plötzner.[57]

Beyond the above cited investigations, there seem to be few research results available that focus *specifically* on problems that arise in the learning of physics and chemistry concepts with multiple forms of representation. McDermott, after reviewing some of the research discussed above, stated, "To develop appropriate instructional strategies, we need to identify the specific difficulties students have with various representations. Diagrams, graphs, and equations all involve different ways of thinking. The nature of the problems encountered is different in each case . . . What type of instruction can help students make connections between a concept and various representations of that concept, between one representation and another, and between various representations and the real world?"[42]

There appears to have been a resurgence of activity in multiple representations in recent years among mathematics educators and those who focus on the role of computers in education. Numerous reports emphasize the benefits of employing multiple representations in instruction[83,88,90,92] and detail the pedagogical synergies that may be obtained through use and translation among such representations.[86,87] A great deal of discussion has been devoted to the question of just how much of a learning "dividend" may be realized by employment of more than one form of representation.[81,82,84,91] There seems to be only little effort, though, to *compare* representations in terms of their pedagogical effectiveness.[89] A number of current investigations are examining broader issues involved in student learning with diverse representations.[13,34,80] A current large-scale project centered at the University of California at Berkeley, "Project MaRC," is focused on investigations of students' "meta-representational competence," i.e., matters that lie "beyond the knowledge students need to operate standard, schooled representations."[11,64,85] Our project, by contrast, focuses on the "other end" of the instructional problem. That is, we propose to carry out a systematic and intensive study of learning difficulties associated with standard forms of representation across a variety of subject areas. In this we would of course be informed and guided by the ongoing work of these other more broad-based investigations.

Specific questions to be addressed by this project: We propose to investigate the following issues:

1) What *subject-specific* learning difficulties may be identified with various forms of representation of particular concepts in the introductory physics and chemistry curriculum? We will identify and diagnose a large number of learning difficulties commonly found among students in introductory courses, and make direct use of our findings by designing and testing curricular materials that address these learning difficulties.

2) What generalizations may be possible regarding the *relative* degree of difficulty of various representational modes when compared with each other, in learning of particular concepts? That is, given an average class engaging in a typical sequence of instructional activities (i.e., the problems, exercises, homework assignments, readings, etc. that we employ in our courses), do some forms of commonly used representations engender a disproportionately large number of learning difficulties?

3) Do *individual* students do consistently well or poorly with particular forms of representation, with widely varying types of subject matter? If *"Yes,"* one is led to Question #4:

4) Are there any consistent *correlations* among students' relative performance with different representational modes, and parameters such as major field, gender, age, learning style, etc.?

5) [A follow-up to Question #4:] Does the overuse or under-use of certain types of representational forms in standard instruction have any potential impact on members of traditionally underrepresented groups?

Methods of Investigation: Our first step will be to design, test, and validate a set of assessment instruments relevant to each topical area under investigation. Each instrument will consist of a chemistry or physics problem posed in at least two different representational forms. We will acquire three types of data:

1) The instruments will be used as starting points during our clinical interviews. We will ask students to "think out loud" as they work through the problems, and we will be able to probe their thinking more deeply with follow-up questions. This is a powerful tool but it can only be used with relatively small numbers of students. (Both PI's have many years of experience in carrying out such qualitative research, and our research groups have conducted and analyzed hundreds of hours of clinical interviews during recent years.)

In addition to detailed probing of conceptual difficulties related to specific representational forms, the clinical interviews allow us to investigate directly how students attempt to "translate" from one representation to another. (Research in other fields such as systems analysis[28] suggests that, early in the problem-solving process, students often translate a problem posed in one representational form into another that they find personally more appealing.) We also anticipate that specific learning difficulties uncovered during the interviews will form the basis for the design of additional assessment instruments. In addition, we will often have the opportunity to acquire additional data related to individual students' learning styles. This might include evaluations of field dependence/independence using, e.g., the Group Embedded Figures Test,[76] and visual processing ability using, e.g., the Guilford-Zimmerman Aptitude Survey.[19]

2) Some problem sets used in our classroom assessments will be posed in free-response form in which students have to calculate their own numbers or draw their own diagrams or graphs to get credit. On selected problems, students will be required to write explanations of their reasoning. These will be graded to ensure that serious efforts are made and thus will constitute an effective probe of student thinking (albeit one that is cumbersome to evaluate). In conjunction with the clinical interviews, these will provide significant insight into *why* the learning difficulties occur, and help guide us in creating materials that will help students overcome these difficulties.

3) A few of the instruments will be designed in multiple-choice form to permit rapid analysis with large samples. An extra-credit option allows students to signify their confidence in the answer they select, which provides an additional dimension of data for further analysis. Some "two-tiered" multiple-choice problems will be used, which allow a separate choice for explanations.

As noted above, in this preliminary investigation we will focus primarily on four major representational forms: verbal (V), diagrammatic (D), mathematical (M), and graphical (G). We do this for both practical and logistical reasons. Historically, V, D, M, and G have been ubiquitous in scientific work. There are certainly other pedagogically significant representations, e.g. computer animations, haptic (sense of touch) and kinesthetic interfaces and representations, videotapes of actual physical processes, and actual physical objects and systems using laboratory equipment. However, these lack the relative standardization (due to long-term use in instruction) and/or ease and flexibility of implementation that characterize V, D, M, and G. For example, preparation of an assessment instrument incorporating items in V, D, M, and G formats may take a few hours, while addition of a computer animation could easily require several additional *weeks* of work. Nonetheless, our group does have particular expertise in the area of computer animations and whenever possible, we will incorporate these in our clinical interviews to add an extra dimension to the investigation. In addition, when feasible, we will certainly make use of actual laboratory equipment during clinical interviews to help gauge students' interpretation of problems posed in other forms of representation.

One might also consider the possibility that there are "couplings" between and among various modes with respect to effectiveness in promoting student learning. For instance, a student may grasp some concept via a graphical representation that then permits better understanding of (and performance on) a test item using a mathematical representation. Indeed, a primary motivation for use of multiple representations is that precisely such effects are assumed to occur. However, to first order, student learning that occurs with one particular form of representation should be reflected by performance *on items employing that form of representation.* Of course it is possible that higher-order (i.e., more indirect) effects may occur in which student learning with e.g., graphical representations is reflected in performance on test items using mathematical representations but *not* proportionally so on items employing graphs. Such nonlinear effects are subtle and extremely difficult to tease out in any scientific study; we propose to postpone exploration of such effects to a later, more detailed investigation.

Most of the students in our study will be drawn from the large introductory lecture classes for General Physics and General Chemistry; the number of students in these classes is in the 80-300 range. The algebra-based courses enroll primarily life sciences majors, pre-meds, pre-vets, physical therapy majors, pre-pharmacy students, etc.; the calculus-based courses enroll primarily engineering and physical science majors. Test instruments will be administered simultaneously to all students in the class.

In addition to these large student samples, a critical part of our study will depend on recruiting ***student volunteers to serve as subjects for extended clinical interviews.*** Our normal procedure has been to solicit volunteers through announcements made by recitation and lab instructors. Although there is necessarily some self-selection in this process, we will attempt to interview students representing a wide range of abilities and demographic parameters, and we will also attempt to recruit individuals who, through previous testing, seem to demonstrate preferences for one or another representational form.

There are three important variables that must be carefully controlled in this study:

1) When problems are presented using different forms of representation, they must be as nearly identical to each other as possible in terms of content and difficulty. This we will control during the design of the assessment instruments; one person will draft the problem set and others will review it.

2) The different problems in a particular set should be posed, to the extent possible, in a "pure" form of each representation. That is, the "verbal" form should use no symbols or equations, etc.

3) The amount of practice that students have in the different forms of representation (on quizzes, homework assignments, exams, in-class exercises, etc.) must be as nearly identical as possible. There can be little doubt that the forms of representation utilized by the instructor and employed in curricular materials may have a significant impact on student learning. We will attempt to control this variable by making extensive use of *all* major forms of representation during instruction. In our position as the course instructors, we will be able to ensure that students have extended and repeated practice with verbal, diagrammatic, mathematical, and graphical forms of representation in all subject areas targeted in our instruction.

For instance, in our second-semester physics class, all curricular materials have been produced by our group. (No standard text is used.) These materials – "lecture notes" [i.e., text], homework, quiz and exam questions, practice questions for in-class use, etc. – all make very extensive use of graphical, diagrammatic, and pictorial representations, as well as more standard verbal and mathematical forms. All materials used during discussion/recitation sessions use these same materials, and teaching assistants get special training in inquiry-based pedagogy that employs these materials. It is made clear to students that they are expected to learn and demonstrate proficiency with all forms of representation utilized during instruction. This thoroughgoing control over the instructional process allows us to establish a nearly "ideal case" scenario in which to compare student learning and performance with different forms of representation. Although one could not generalize the specific levels of performance achievement in our courses to more traditional ones, this instructional format allows us to most effectively probe the *specific representation-related learning difficulties* encountered by a large group of typical students. We contend that the insights gained through this investigation will be tremendously valuable to many other instructors who might make more limited use of multiple representations in their own teaching. Learning difficulties identified in our students, who have *extensive* practice with multiple representations, will almost certainly reflect similar difficulties encountered by students in more traditional classrooms.

The results of this project are expected to yield insights regarding representation-related learning difficulties for a broad range of topics. These will provide the basis for development of improved curriculum and instructional methods. In the present project, our initial target areas are electromagnetism, electrochemistry, and thermodynamics. We point out here that both PI's have a proven track record of over ten years of helping students overcome learning difficulties and correct misconceptions in learning physics and chemistry. Our investigations of student learning, curricular materials based on our research, and assessment data reflecting successful interventions have been reported in over one hundred articles, conference talks, workshops, and invited lectures over the years. We have been awarded six separate NSF grants to carry out our curriculum development work. In particular, our work employing multiple representations has been widely disseminated. All of our instructional activities take place in ***active-learning*** classrooms, with students frequently working in small collaborative groups, guided by Socratic questioning from instructors and teaching assistants. Even in large "lecture" classes – with over 200 students present – we use classroom response techniques to obtain frequent, instantaneous feedback from all students simultaneously, *and* students spend significant fractions of class time in small-group activities.[52,53]

Plan of Work: We will administer our test instruments to many hundreds of students in our courses as part of day-to-day instructional activities. Because these activities themselves constitute ordinary instructional activity, consent from individual students is not required so long as student anonymity is not compromised in the reporting of research results. (In this preliminary study, we will ensure that all enrolled students receive *identical* assessment instruments and curricular materials. Informed consent will of course be required for all students involved in clinical interviews that take place outside of ordinary class activities.)

1) A large number of "multi-representation-mode" problem sets will be designed and administered. The relative success of the students in solving these apparently different problems provides evidence of the relative difficulty of different representational modes in learning specific topics. ***See Appendices A and B.***
2) Numerous problems will be designed and administered, and the results analyzed, in which students are required to provide written explanations of their reasoning with a specific focus on issues related to the form of representation. Whenever possible, very similar problems will be presented simultaneously in two or more representations. ***A sample assessment instrument of this type is in Appendix C.***
3) By carrying out these investigations among numerous student population samples (from different courses at ISU, as well as at other institutions), we will determine the *consistency and reproducibility* of the observations. ***A letter from Prof. K. Manivannan (SMSU) confirming his collaboration is in Section I .***
4) To obtain in-depth understanding of students' cognitive difficulties, numerous clinical interviews will be conducted with individual students. Students will be selected to represent the range of variability in the class. During these interviews, a carefully structured series of questions is posed in which one focus is on representation-related issues. ***See Appendix D.***
5) Results provided by phases #1-4 will be used to generate curricular materials, including exercises and sequences of conceptual questions that specifically target learning difficulties by promoting a process of conceptual change. ***Letters from Prof. M. Sanger (University of Northern Iowa) and Prof. D. McCarthy (SLU) confirming their commitment to class-test materials developed by this project are included in Section I. Additional faculty at ISU have also agreed to class-test materials developed in this project.***

Appendix A: Sample of Instrument to Compare Learning Difficulties with Different Forms of Representation (Physics)

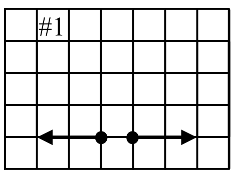
IF YOU WANT A QUESTION GRADED OUT OF THREE POINTS (-1 [MINUS ONE] FOR WRONG ANSWER!!) INSTEAD OF 2.5 POINTS (ZERO FOR WRONG ANSWER) WRITE "3" IN SPACE PROVIDED ON EACH QUESTION.

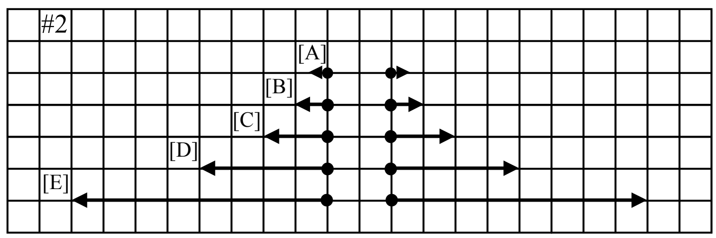
1. When two identical, isolated charges are separated by two centimeters, the magnitude of the force exerted by each charge on the other is eight newtons. If the charges are moved to a separation of eight centimeters, what will be the magnitude of that force now?

A. one-half of a newton
B. two newtons
C. eight newtons
D. thirty-two newtons
E. one hundred twenty-eight newtons

Grade out of three? Write "3" here: ☐

2. Figure #1 shows two identical, isolated charges separated by a certain distance. The arrows indicate the forces exerted by each charge on the other. The same charges are shown in Figure #2. Which diagram in Figure #2 would be correct?





A.
B.
C.
D.
E.

Grade out of three? Write "3" here: ☐

3. Isolated charges q_1 and q_2 are separated by distance r, and each exerts force F on the other. $q_1^{initial} = q_1^{final}$ and $q_2^{initial} = q_2^{final}$; $r^{initial} = 10m$; $r^{final} = 2m$. $F^{initial} = 25N$; $F^{final} = ?$

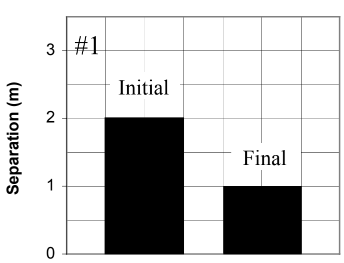
A. 1 N
B. 5 N
C. 25 N
D. 125 N
E. 625 N

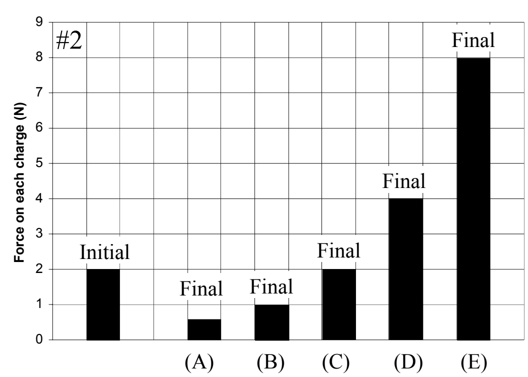
Grade out of three? Write "3" here: ☐

4. Graph #1 refers to the initial and final separation between two identical, isolated charges. Graph #2 refers to the initial and final forces exerted by each charge on the other. Which bar is correct?

Grade out of three? Write "3" here: ☐

A.
B.
C.
D.
E

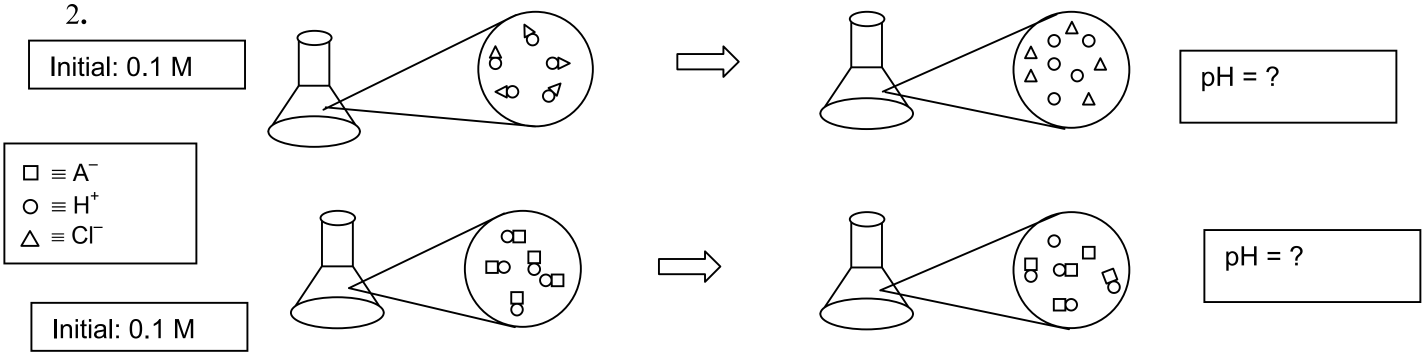




Appendix B: Sample of Multi-Representation Instrument (Chemistry)

1. Hydrogen chloride gas is bubbled into water, resulting in a one-tenth molar hydrochloric acid solution. In that solution, after dissociation, all of the chlorine atoms become chloride ions, and all of the hydrogen atoms become hydronium ions. In a separate container, HA acid is added to water creating an initial concentration of one-tenth molar HA-acid solution. In that solution (at equilibrium), twenty percent of the H atoms becomes hydronium ions, and twenty percent of the A atoms become A^- ions. Find the pH of (a) the hydrochloric acid solution, and (b) the HA-acid solution.

2.



3.

$HA(s) + H_2O(l) \Leftrightarrow H_3O^+(aq) + A^-(aq)$

ionization = 20%
initial concentration = 0.1 M

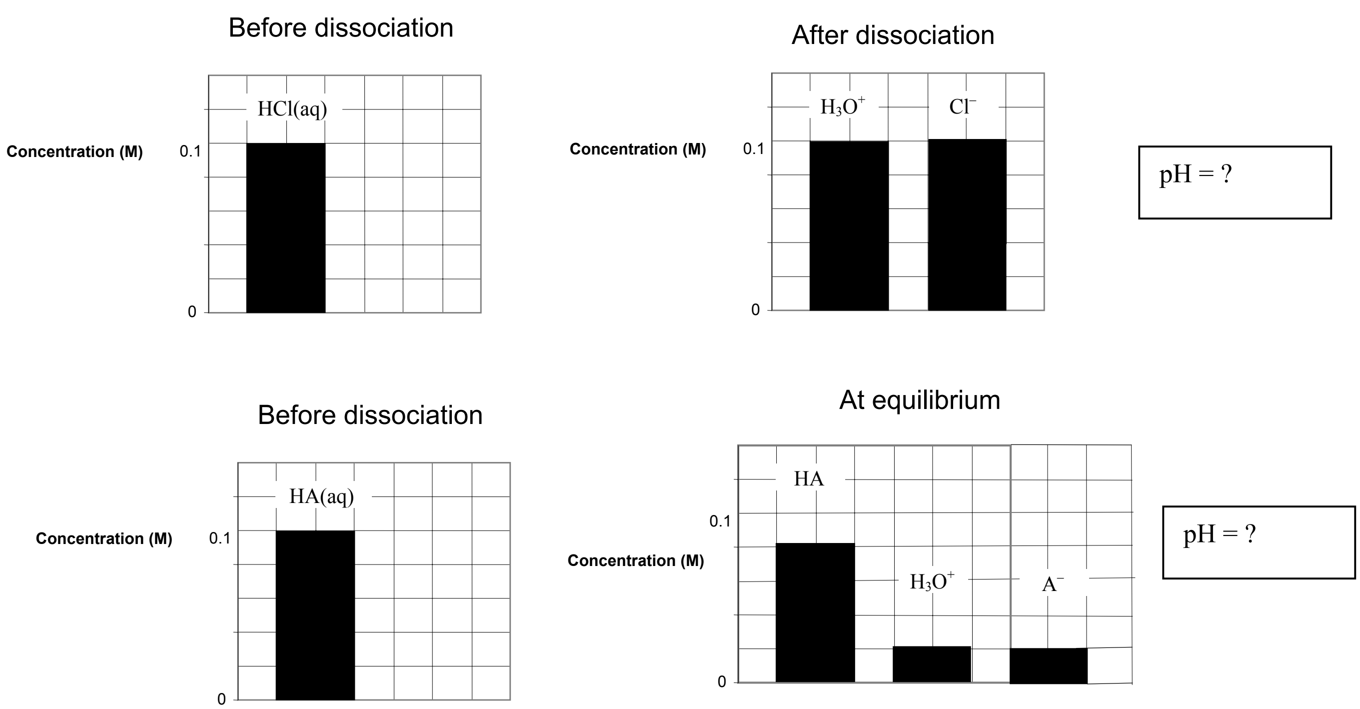
pH = ?

$HCl(aq) + H_2O(l) \rightarrow H_3O^+(aq) + Cl^-(aq)$

ionization = 100%
initial concentration = 0.1 M

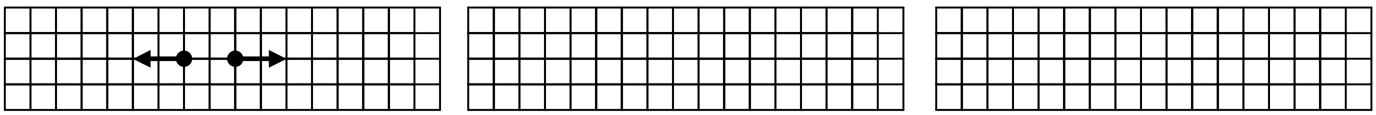
pH = ?

4.



Appendix C: Example of Instrument to Diagnose Learning Difficulties with Diagrams and Mathematical Symbols, in the Context of Coulomb's law

1. The diagram on the left shows two isolated particles with equal magnitude charges, along with the electrical forces acting on those particles due to their mutual interactions. The ***same*** charges are to be repositioned in the ***center*** diagram, this time separated from each other by ***half*** the distance that separated them in the diagram on the left. In the diagram on the ***right***, the separation distance is the same as in the center diagram, but the charge on the ***left*** now has ***half*** its original magnitude. Complete the figures in the center and right diagrams to represent the new positions and forces.



Explain for each case:

1) How did you decide where to locate the ***tails*** of the arrows?

2) How did you decide on the ***directions*** of the arrows?

3) How did you decide on the ***lengths*** of the arrows?

2.

a) Isolated particles with charges q_1 and q_2 ($q_1 = q_2$) are separated by distance r, and initially experience mutual interaction forces $\boldsymbol{F_1}^{\boldsymbol{initial}}$ = 25N $\hat{\boldsymbol{i}}$ and $\boldsymbol{F_2}^{\boldsymbol{initial}}$ = –25N $\hat{\boldsymbol{i}}$, where $\hat{\boldsymbol{i}}$ represents a unit vector; $r^{initial}$ = 10m. The particles are repositioned so $r^{final} = 0.2\, r^{initial}$;

$\boldsymbol{F_1}^{\boldsymbol{final}}$ = ?

$\boldsymbol{F_2}^{\boldsymbol{final}}$ = ?

b) Isolated particles with charges q_1 and q_2 ($q_1^{initial} = q_2^{initial}$) are separated by distance r, and initially experience mutual interaction forces $\boldsymbol{F_1}^{\boldsymbol{initial}}$ = 25N $\hat{\boldsymbol{i}}$ and $\boldsymbol{F_2}^{\boldsymbol{initial}}$ = –25N $\hat{\boldsymbol{i}}$, where $\hat{\boldsymbol{i}}$ represents a unit vector; $r^{initial}$ = 10m. The particles are repositioned so $r^{final} = 0.2\, r^{initial}$, but the magnitude of ***one*** charge is now cut in half: $q_1^{final} = 0.5 q_1^{initial}$; $q_2^{final} = q_2^{initial}$.

$\boldsymbol{F_1}^{\boldsymbol{final}}$ = ?

$\boldsymbol{F_2}^{\boldsymbol{final}}$ = ?

Explain in detail how you obtained your answers in both cases (a) and (b).

Appendix D: Sample Interview Protocol Outline

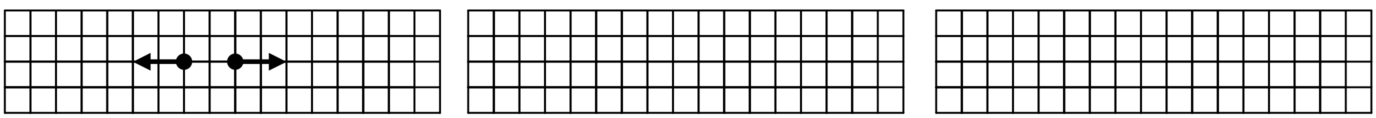
1. Could you please read through this problem and then go ahead and solve it. If you can, explain to me what you're doing while you work out the answer.

When two identical, isolated charges are separated by two centimeters, the magnitude of the force exerted by each charge on the other is eight newtons. If the charges are moved to a separation of eight centimeters, what will be the magnitude of that force now?

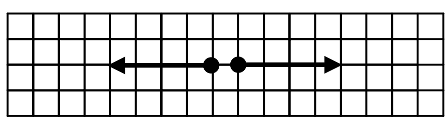
2. Could you please draw me a diagram (or more than one, if you want) to help explain what's happening in question #1.

3. Now I'd like you to go ahead and solve this next problem. Once again, try to explain to me what you're doing as you solve it.

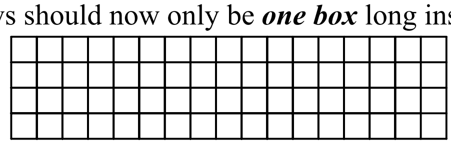
The diagram on the left shows two isolated particles with equal magnitude charges, along with the electrical forces acting on those particles due to their mutual interactions. The ***same*** *charges are to be repositioned in the* ***center*** *diagram, this time separated from each other by* ***half*** *the distance that separated them in the diagram on the left. In the diagram on the* ***right****, the separation distance is the same as in the center diagram, but the charge on the* ***left*** *now has* ***half*** *its original magnitude. Complete the figures in the center and right diagrams to represent the new positions and forces.*



4. Could you please tell me how you decided where to put the dots in the center and right diagrams.
5. What did you do to figure out how long the arrows should be in each case?
6. Why did you draw the arrows pointing in the directions you've drawn?
7. Suppose, in the center diagram, both arrows point in the ***opposite*** direction to what you've drawn. What would that tell you?
8. Suppose that the original diagram (on the left) had looked like this:



In this box, could you please draw a diagram showing the same two charges but now with the charges located such that the arrows should now only be ***one box*** long instead of four as in #8.



9. Suppose I let q_{left} represent the charge on the left, and q_{right} represent the charge on the right in the top diagram in #8. Please write a mathematical equation for the electrical force experienced by q_{right}. If you need to use any other symbols, explain what they are.
10. Now I'd like you to write another mathematical equation to represent the electrical force experienced by q_{left} in the ***bottom*** diagram in #8. Please express your answer in terms of $F_{\#9}$, where $F_{\#9}$ represents the magnitude of your answer to Question #9.