## Chapter 1 Notes: Electric Charges and Forces

## What leads us to think that there are such things as "charges," anyway?

Think about what you have learned about "forces." A force can be thought of as a push or a pull that may act on an object. Usually, the force that acts on some object " $A$ " is caused by some other object which is in direct contact with object "A." For instance, a hammer might strike a nail; then the hammer is exerting a force on the nail. According to Newton's third law, the nail also exerts a force on the hammer (equal in magnitude, opposite in direction). This type of force is called a "contact" force.

There is another type of force, in which direct contact between objects is not required. Suppose you hold a book up in the air, and then let it fall. What made it fall? You know that a gravitational force exerted by the earth on the book was responsible for the book's fall. (There is also a gravitational force exerted by the book on the earth.) You have also observed magnets appearing to exert forces on each other, even though they are not in direct contact. These forces are sometimes called "action-at-a-distance" forces.

Now consider some other phenomena you have seen. When you rub a comb through your hair, you may have noticed the comb pick up small bits of paper. You may have seen balloons appearing to "stick" to walls, without any glue. You have probably seen various types of plastic materials (such as thin plastic wrap) adhere to other objects, as if they were "attracted" to each other across space. Magnets don't seem to be involved in this type of phenomena. Could it be due to gravitational forces?

Explain why it is unlikely that these phenomena are caused by gravitational forces. Consider the magnitude of the gravitational force, and how this is related to the masses of the objects involved.

In experiments using certain simple materials such as rubber and glass rods, when they are rubbed with fur or silk, certain phenomena consistently recur: (1) when two of these objects made from identical materials are prepared the same way (e.g., rubber rubbed with fur), the objects appear to exert small repulsive forces on each other. If the objects are suspended by a string, or mounted on a sensitive pivot, this force may result in a visible motion; (2) when two of these objects made from different materials are held near each other (e.g., a rubber rod held near a glass rod), the objects appear to exert attractive forces on each other. These forces are far too strong to be related to the gravitational force - and in any case, the gravitational force is never repulsive, only attractive! So we must conclude that there is another "action-at-a-distance" force at work, which is known as the "electrical" force. It may be both attractive, and repulsive.

The experiments described above, and many other similar ones, lead us to conclude that there are two different properties of matter that, in some sense, "cause" this force. When two objects with property "A" are near each other, they push each other apart (i.e., exert repulsive forces on each other). The same thing happens when two objects with property " B " are near each other. But when an object with property " A " is near an object with property "B," they exert attractive forces on each other. These properties of matter have been called "charge," and instead of "A" charge and "B" charge, the terms "positive" [symbol: +] and "negative" [symbol: -] charge are used. The symbols usually used for charge are $q$ or $Q$, and it is measured in units called "coulombs" [symbol: C].

## Nature of the Electric Force

From the experiments with "charged" objects (i.e., objects with the charge property), we are led to conclude that the magnitude of the electrical force depends strongly on the distance between the objects. The effects of the repulsive and attractive forces are much more noticeable when the charged objects are close together, than when they are far apart. Many careful experiments have led to the following relationship for the magnitude of the electrical force between two objects separated by a distance $r$, when one object has charge $q_{1}$ and the other object has charge $q_{2}$ :

$$
F_{\text {electrical }}=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{r^{2}}
$$

This relationship is called "Coulomb's law." Here, the letter $k$ represents a proportionality constant, which has a specific numerical value that depends on the system of units being used. The absolute value signs on the charge symbols are there because the magnitude of the force does not depend on whether the charges are positive or negative - it only depends on the amount of charge present on the objects. Note that the electrical force does not
depend on the mass of the objects, or any other property - only on the charge. The direction of the force depends on the relative types of the charges. If $q_{1}$ and $q_{2}$ both have the same type of charge - positive or negative - then the force between them is repulsive, and is directed along a straight line connecting the two charges (see figures below, left and center). If one charge is positive is the other is negative, then the force is attractive, but still directed along the line connecting the charges (see right figure below).


In the SI system of units, distance is measured in meters (m), charge is measured in coulombs (C), and force is measured in newtons $(\mathrm{N})$. In these units, the constant $k$ has the value $9 \times 10^{9} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}^{2}$. So, for instance, the magnitude of the electrical force between an object with a charge of 3 C and one with a charge of 6 C , separated by a distance of 4 m , is:

$$
F=\frac{\left(9 \times 10^{9} N m^{2} / C^{2}\right)(3 C)(6 C)}{(4 m)^{2}}=\frac{\left(9 \times 10^{9}\right)(18)}{16} N=1.01 \times 10^{10} N
$$

This is a huge force, but a charge of 3 C is far larger than would be found on any ordinary object. Although typical amounts of charge are much smaller than that, it turns out that charge never appears in quantities below a certain minimum value. This value, symbolized by the letter $e$, is called the "elementary charge." It is equal to $1.6 \times$ $10^{-19} \mathrm{C}$. This can be thought of as the "minimum package size" for charge. Charge always appears in quantities that are some integer multiple of $e$, e.g. $35 e, 17984 e, 3 \times 10^{6} e$, etc. So, can you obtain a quantity of charge equal to 3.5 $e$ ? [Answer: No].

Atoms are composed of smaller, "sub-atomic" particles, such as the proton (p), neutron (n), and electron (e). It turns out that while the neutron has no charge (and so does not experience electrical forces), both the proton and the electron have an amount of charge whose magnitude is $e$. However, the proton has $+e$ (a positive charge) and the electron has $-e$ (a negative charge). The fact that the charges on these two particles are the same magnitude is interesting, in view of the very large mass difference between them (the proton has a mass nearly 2000 times larger than that of the electron).

## Properties of Electrical Charges and Forces

An important principle about charge that that has been discovered through many experiments is called "conservation" of charge. This principle states that the total amount of charge in any closed system never changes. A "closed" system is one in which charges can neither enter nor leave. By "total amount of charge," we mean the algebraic sum of all positive and negative charge quantities.

Example: If a sealed box initially contains positive charges equal to $6 e$, and negative charges equal to -11 e, the total amount of charge in that box must always remain at $-5 e$. It is possible that some of the positively charged particles and some of the negatively charged particles may actually disappear such phenomena do occur (and energy is then given off in some form) - but nonetheless, the charge on the particles that do remain must always sum up to -5 e.

An important property of electrical forces is known as the "superposition" principle. This states that the net electrical force acting on any charged object is equal to the vector sum of all the individual forces on that object, where each individual force results from the interaction of the object and one other charged object. Each individual interaction is unaffected by any of the other interactions. Algebraically, this can be expressed as:

$$
\vec{F}_{n e t}=\vec{F}_{1}+\vec{F}_{2}+\ldots+\vec{F}_{n}
$$

Here, the net force on the object is found from the vector sum of the forces from $n$ other charged objects. This equation implies that the net $x$ component of the force equals the sum of the individual $x$ components, and that the net $y$ component equals the sum of the individual $y$ components.

Example: Suppose each grid square below is one meter long, and suppose that all three charges are identical, that is: $Q_{1}=Q_{2}=Q_{3}$. What is the net force acting on charge $Q_{2}$ ?


Answer: Net force equals vector sum of [force from $Q_{3}$ ] and [force from $Q_{I}$ ]. These forces are represented by left and right arrows, respectively, in the diagram below:


Since all three charges are identical, we can use the same symbol $Q$ to represent all of them; that is: $Q_{1}=Q_{2}=Q_{3} \equiv Q$. The arrow pointing to the right is the force due to charge $Q_{1}$; its magnitude is $k Q_{1} Q_{2} /(12)^{2}=k Q^{2} /(144)$; the arrow pointing to the left is the force due to the charge $Q_{3}$; its magnitude is $k Q_{3} Q_{2} /(6)^{2}=k Q^{2} /(36)=4 k Q^{2} /(144)$, so it is four times larger than the magnitude of the rightwardpointing force. These forces are directed along the line connecting the charges, and so we can see that each force in this case only has an $x$ component - the $y$ components are zero. The rightward pointing force has a positive $x$ component, and the leftward pointing force has a negative $x$ component. Then the net $x$ component is equal to $k Q^{2} /(144)+\left[-4 k Q^{2} /(144)\right]=\left[-3 k Q^{2} /(144)\right]$. This would be represented by an arrow pointing to the left (negative $x$ direction) with a length of three grid squares (three times the length of the rightward pointing force). That arrow would then represent the net electrical force acting on $Q_{2}$, the middle charge.

Question: Suppose an object with zero net charge (i.e., exactly equal quantities of positive and negative charges) is located in the neighborhood of another object with zero net charge. What will be the net electrical force experienced by either object?
[Answer: There will be virtually no net electrical force on either object, because all of the repulsive and attractive forces will cancel each other out (i.e., their vector sum will be nearly zero). Show this by considering two objects, each containing two protons and two electrons, and drawing all of the force vectors on all of the charged particles. You should be able to see that the net force on each object is nearly zero, as long as they are not located too close together.]

