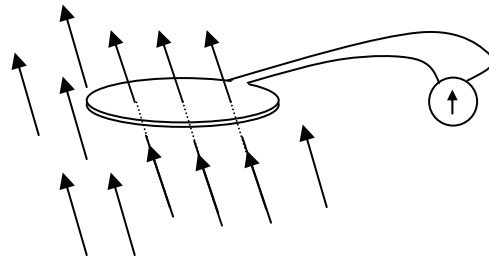


## Chapter 10 Notes: Magnetic Induction

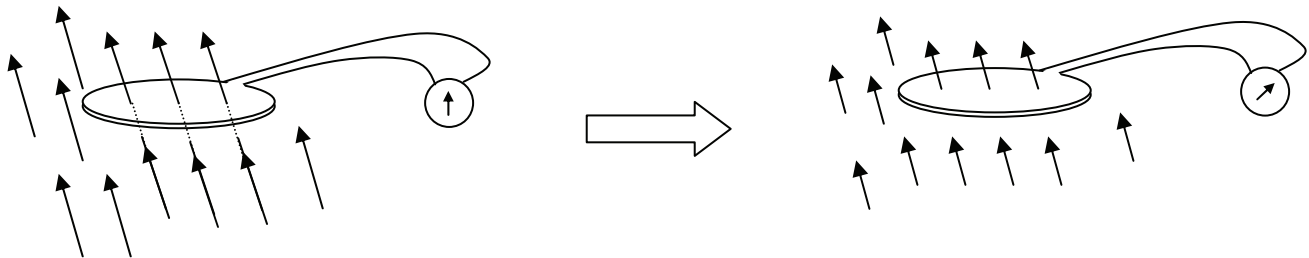
### How can a changing magnetic field cause an electric current to flow?

Eleven years after the connection between magnetism and electricity was first reported by Oersted, the British scientist Michael Faraday made one of the most important discoveries in the history of physics. Oersted had found that an electric current could influence the motion of a compass needle; this showed that an electric current produced a magnetic field. Faraday found that, under certain specific circumstance, a magnet (such as a large compass needle) could itself *produce* an electric current (i.e., it could cause charges to begin to move). Although an electric current *always* produces a magnetic field, Faraday found that a magnet could only produce an electric current under one or more of three basic conditions: (1) the magnetic field varied in *magnitude*; (2) the magnetic field varied in *direction*; (3) the conducting path (which would carry the current) *varied in shape*.

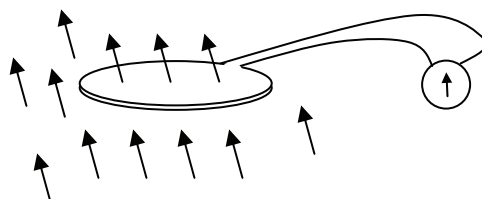
These situations can be illustrated by three different experiments, all involving a magnetic field and a closed loop of conducting material. We could connect a galvanometer (a current-detecting device) to the loop to determine whether or not a current is flowing. We could use a permanent magnet to produce the magnetic field, or instead use the uniform field inside a solenoid. In the diagram below, we have placed a conducting loop in a uniform magnetic field (indicated by the arrows); the loop is connected to a galvanometer. The needle of the galvanometer will deflect (move away from its initial position) if a current is produced in the loop. If the needle is in its initial position (as shown here), there is *no* current flowing in the loop.



In the *initial* situation shown above, where neither the loop nor the magnetic field is changing in any way, *no current* is observed to flow in the loop. However, if we *change the magnitude* of the magnetic field – either an increase or a decrease – then a current *does* flow in the loop, as shown here:

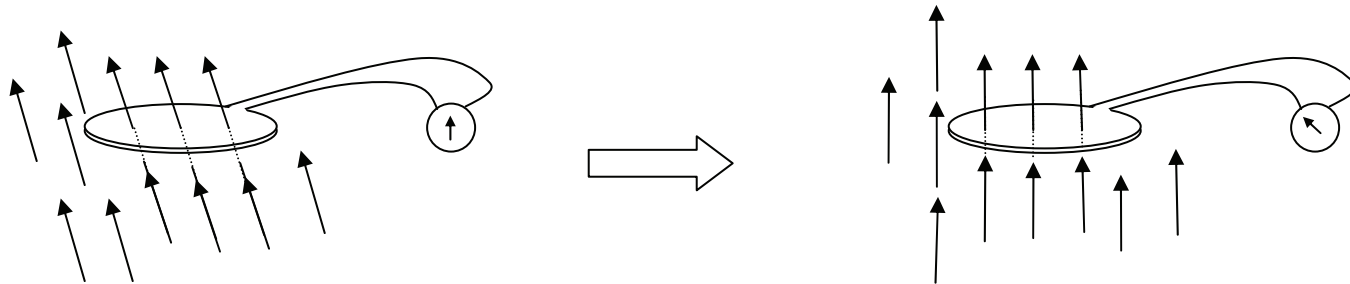


However, if the magnetic field magnitude *stops changing*, the current will abruptly cease flowing and the galvanometer needle will go back to its initial position (again indicating “zero current”):



When the same experiment is done with an **increasing** magnetic field magnitude, the galvanometer needle deflects in the **other** direction (in this example, to the left). This indicates that the direction of the current flow for an **increasing** magnetic field magnitude is opposite to that for a **decreasing** magnetic field magnitude.

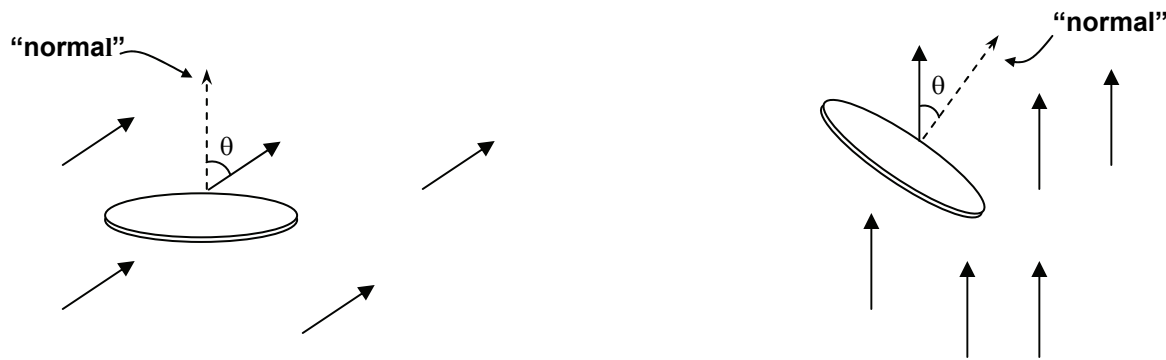
Another experiment keeps the magnitude of the magnetic field constant, but **varies the direction** of the field. In this case also, a current is observed to flow in the loop **as long as the field direction is changing**. (Once again, it is observed that when the field direction remains unchanging, the current ceases to flow.)



(Note that the galvanometer needle has deflected to the left in this situation, indicating that the direction of current flow is **opposite** to that in the situation shown on page one. The reason for this is discussed below.)

One more experiment – which we will not illustrate here – alters the **shape** of the loop, and in particular changes the **area** of the conducting loop. While the area is changing, a current will flow in the loop.

It thus turns out that an alteration in any of three separate quantities can result in current flow: (1) the magnetic field magnitude  $B$ , (2) the area of the current loop  $A$ , and (3) the direction of the magnetic field **with respect to** the loop. Experiments show that it doesn't matter whether it is the field that is redirected, or the orientation of the loop itself that is changed. If the **relative orientation** between the field and the loop is changing, a current will flow in the loop. If one describes the orientation of the loop with a so-called "normal" vector – a vector that points perpendicular to the **plane** of the loop – then the key quantity is the angle  $\theta$  between the direction of the magnetic field, and the direction of the normal. If this angle is changing, a current will flow.



The results of all of these experiments can be summed up by considering one single quantity, which is called the **magnetic flux** [symbol:  $\Phi$ ]. The magnetic flux is simply the product of  $B$ ,  $A$ , and the cosine of the angle  $\theta$ :  $\Phi = BA \cos \theta$ . (We need only consider angles from  $0^\circ$  to  $180^\circ$ .) If the magnetic flux is **changing**, a current will flow; if it stops changing, the current ceases to flow. The current that is produced by the changing magnetic flux is called an **"induced"** current.

**Question:** Consider the diagrams at the top of this page, and assume that the normal to the loop points up. Is the flux in the loop on the right **greater than, less than, or equal to** the flux in the loop on the left? **Answer:** The flux in the loop on the right is greater, because in that case  $\theta = 0^\circ$  and so  $\cos \theta$  has its maximum value of 1.

The precise relationship between the amount of current flow  $I$  and the rate of change of flux can be determined by other experiments. The result, for a loop with resistance  $R$  that obeys Ohm's law, is as follows:

$$I = \frac{1}{R} \frac{\Delta \Phi}{\Delta t} \quad [ \text{"Faraday's law"} ]$$

Here,  $\Delta t$  represents the period of time over which the flux is changing ( $\Delta t = t_{final} - t_{initial}$ ) and  $\Delta\Phi$  represents the net change in the flux during this time period ( $\Delta\Phi = \Phi_{final} - \Phi_{initial}$ ). The experiments show that if the flux is **decreasing** (i.e.,  $\Delta\Phi$  is **negative**), the direction of current flow is **opposite** to that which occurs when the flux is **increasing** (i.e., when  $\Delta\Phi$  is **positive**). For this reason, it is important to keep track of the sign of  $\Delta\Phi$  in each situation, and to interpret positive or negative values of  $I$  as representing opposite directions of current flow.

**Question:** In the diagrams on page one, the galvanometer needle deflected to the right. Why is the galvanometer needle shown in the diagram at the top of page two with a deflection to the left? **Answer:** In the page one diagram, the magnetic flux was **decreasing**; in the page two diagram, the flux was **increasing** and so the direction of current flow was opposite to that in the page one example. Therefore, the galvanometer deflects in the opposite direction from that shown on page one.

If we write out Faraday's law in more detail, we have:

$$I = \frac{1}{R} \frac{\Delta\Phi}{\Delta t} = \frac{1}{R} \frac{(\Phi_{final} - \Phi_{initial})}{\Delta t} = \frac{1}{R} \frac{([B_{final} A_{final} \cos \theta_{final}] - [B_{initial} A_{initial} \cos \theta_{initial}])}{\Delta t}$$

This is a somewhat complicated equation, and in fact if all three quantities – magnetic field magnitude, direction, and loop area – are varying at the same time, it can be quite difficult to analyze the situation. However, we will normally consider systems in which only **one** of the variables is changing at any one time, while the other two are constant. For instance, let us consider a case where the shape of the loop is unchanging (and so its area is constant), and the orientation of the loop with respect to the magnetic field also does not change. Then  $A$  and  $\theta$  (and so also  $\cos \theta$ ) will both have an unchanging value. We will assume that only magnetic field magnitude  $B$  is changing with time. Then our expression for the current undergoes a considerable simplification.

If  $A$  and  $\cos \theta$  are not changing, this means that their **final** values are identical to their **initial** values. Instead of writing, for instance, " $A_{final}$ " we can just write " $A$ ," and we can also use " $A$ " to represent the **initial** value of the area. In the same way, we need only write " $\cos \theta$ " and not worry about final or initial values of the angle. Then we get the following simplified expression:

$$\begin{aligned} I &= \frac{1}{R} \frac{\Delta\Phi}{\Delta t} = \frac{1}{R} \frac{([B_{final} A_{final} \cos \theta_{final}] - [B_{initial} A_{initial} \cos \theta_{initial}])}{\Delta t} = \frac{1}{R} \frac{([B_{final} A \cos \theta] - [B_{initial} A \cos \theta])}{\Delta t} \\ &= \frac{1}{R} A \cos \theta \frac{(B_{final} - B_{initial})}{\Delta t} \\ &= \frac{1}{R} A \cos \theta \frac{\Delta B}{\Delta t} \end{aligned}$$

In this way, we can see that when the loop area and the relative orientation of field and loop are not changing, the magnitude of the induced current is directly proportional to the rate of change of the magnetic field magnitude. If a different pair of quantities is constant – for instance,  $B$  and  $A$  constant, but  $\cos \theta$  varying with time – then  $I$  will be proportional to the rate of change of the third quantity, e.g.:  $I = (1/R) (BA) [\Delta(\cos \theta) / \Delta t]$ .

Up to this point, we have been assuming that the magnetic field, although it may be changing with time, is **uniform** at any given time. Of course we know that the conducting loop may be in the presence of a nonuniform magnetic field; how does one calculate magnetic flux in such a case?

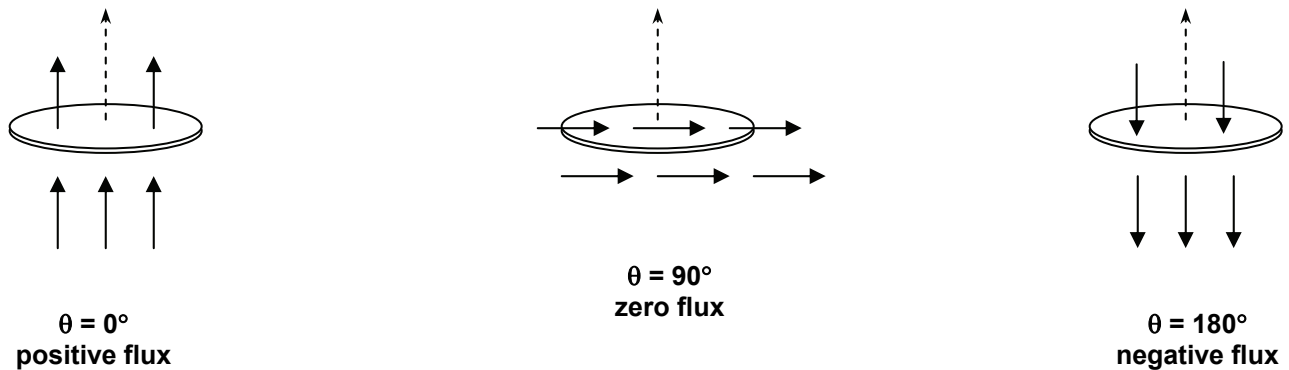
This is easy to do as long as we can assume that the magnetic field has the same magnitude and direction at all points **within** the loop itself. We can imagine a very thin sheet stretched within the loop, where the loop itself is the boundary for the sheet. Examples similar to this would be a soap film stretched within a wire loop, or a drum head stretched on a metal frame. The magnetic flux refers to the magnetic field at all points located **on** that imaginary surface (analogous to the soap film, or drum head). If the field has approximately the same magnitude and direction at all of those points, we can easily calculate the flux: we simply need to know the magnitude  $B$  and

the angle  $\theta$  between the direction of the field and the normal to the plane of the loop. (If the field can not be assumed uniform over that surface, we would need to use more advanced mathematical techniques to find the flux.)

Note carefully that as long as the magnetic field magnitude is not zero, there **will** be some amount of flux within the loop **as long as the angle  $\theta$  is not equal to  $90^\circ$** . (The value of the cosine is nonzero for any other angle between  $0^\circ$  and  $180^\circ$ .) This means that as long as the direction of the field is not **parallel** to the plane of the loop, there will be flux within the loop for a nonzero field. This does not mean, however, that a **current** will be induced! A current is induced only when the flux within the loop is **changing**. There may be a very large magnetic flux in the loop, and yet no current flowing within the loop if that flux has a constant value.

The word “flux” is suggestive of something “flowing” through the loop. In the case of magnetic field, nothing is actually flowing through the loop. However, it might be helpful to imagine for a moment that the arrows representing the magnetic field vectors instead represent the velocity of a stream of water that **is** flowing through the loop. The maximum “water flux” – which one might measure in liters per second – would occur when the direction of flow was perpendicular to the plane of the loop; this corresponds to a value of  $0^\circ$  for angle  $\theta$ . If the loop were oriented so that the water flowed **parallel** to the plane of the loop, there would be **no** water flowing through the loop; this corresponds to  $\theta = 90^\circ$ . The results found for water flow in this analogy are consistent with what is found for the case of magnetic flux.

Now, what is the significance of a **negative** flux? If angle  $\theta$  is equal to e.g.  $0^\circ$ , the value of  $\cos \theta$  is  $+1$ . If  $\theta = 180^\circ$ , then  $\cos \theta$  is  $-1$  and we have a negative value for the flux. Going back to the water analogy, we could interpret this as water flowing in the **opposite** direction through the loop. We can illustrate these three different situations for the case of magnetic flux with these diagrams (the normal is pointing “up” in all three):



The direction chosen for the normal is arbitrary; it could point up **or** down. However, we imagine that the normal is “stuck” in the surface of the loop; if the loop turns, the normal turns with it. If the magnetic field points in the same direction as the normal (or **any** direction for which  $\theta < 90^\circ$ ), we say that flux is “positive”; if the magnetic field points in **any** direction for which  $\theta > 90^\circ$  (e.g., directly **opposite** to the normal), we must then interpret that as “negative” flux.

The reason that these distinctions in direction are important goes back to the significance of positive and negative values of  $\Delta\Phi$ ; if the sign of  $\Delta\Phi$  changes (e.g., from positive to negative) then the direction of current flow will be reversed. An **increasing** magnitude of positive flux corresponds to  $\Delta\Phi > 0$ , while a **decreasing** magnitude of positive flux corresponds to  $\Delta\Phi < 0$ . **However**, an increasing magnitude [absolute value] of **negative** flux corresponds to **decreasing** flux, or  $\Delta\Phi < 0$ . (For instance, if the flux changes from  $-5 \text{ Tm}^2$  to  $-10 \text{ Tm}^2$ , the flux is becoming “more negative” and so is **decreasing**.) What this means is that an increasing magnitude of **positive** flux will cause current to flow in the **opposite** direction to that which results from an increasing magnitude of **negative** flux.

**Question:** Note that in the diagrams on page one, the magnitude of flux has **decreased** and the galvanometer needle has deflected to the right. Suppose that the magnetic field was pointing **downward** in these diagrams, instead of upward. Would the galvanometer needle have deflected to the right (as shown), or to the left?  
**Answer:** It would have deflected to the left. If the field were pointing downward, we would have observed a decreasing magnitude of **negative** flux; this corresponds to  $\Delta\Phi > 0$  [i.e., **increasing** flux]. This produces a current flow in the opposite direction from a decreasing magnitude of **positive** flux.

It is important to keep in mind that the ratio  $\Delta\Phi/\Delta t$ , which is the **rate of change** of flux, can itself vary. A larger value of this ratio leads to larger currents, and when this ratio is zero (because  $\Phi$  is constant and so  $\Delta\Phi = 0$ ) there is **no** induced current. To calculate the correct numerical value of the current at a particular moment, we must use the value of  $\Delta\Phi/\Delta t$  that is “in effect” at that moment. **Question:** At  $t = 0$  s,  $\Phi = 0$  Tm<sup>2</sup>; then,  $\Phi$  increases by 3 Tm<sup>2</sup> each second for two seconds. At  $t = 2$  s,  $\Phi$  holds constant at 6 Tm<sup>2</sup> for two more seconds. Assume  $R = 1$  ohm. (i) What is the current at  $t = 1$  s? (ii) What is the current at  $t = 3$  s? **Answer:** (i) At  $t = 1$  s,  $I = (1/1)(3) = 3$  A; (ii) At  $t = 3$  s,  $I = 0$  A. Here we must use  $\Delta\Phi/\Delta t = 0$  Tm<sup>2</sup>/s and **not** the “average” value  $\Delta\Phi/\Delta t = 6$  Tm<sup>2</sup>/3 s = 2 Tm<sup>2</sup>/s.

In the discussion up until now we have looked at the current induced in a single loop of conducting material, which we have portrayed as a circular loop. This raises two separate questions: (1) What happens if the loop is not circular? (2) If there is **more** than a single loop, would the current flow be larger or smaller given the **same** value of  $\Delta\Phi/\Delta t$ , and of  $R$ ?

Both of these questions have been answered by experiments: (1) Faraday’s law applies to any conducting loop, of **any** shape. (The particular way in which we have written it does, however, depend on Ohm’s law being obeyed. There is a more general formulation of Faraday’s law that does not have this limitation.) (2) If the path of conducting material is composed of **more** than one loop, the induced current is **larger** than in the single-loop case so long as  $\Delta\Phi/\Delta t$  and  $R$  don’t change. In fact, if there are  $N$  loops in the path, Faraday’s law takes on this form:

$$I = \frac{N \Delta\Phi}{R \Delta t} \quad [\text{Faraday's law for } N \text{ loops}]$$

The fact that the induced current is larger when more loops are in the path is of very great practical importance. It means that one can wind a coil of relatively small area  $A$  with many thousands of “turns” of wire to produce a large induced current. (Although the resistance of the coil increases in proportion to the number of loops, in practical applications only a very small part of the total resistance  $R$  is due to the wire in the loops themselves. Since the total resistance  $R$  is not increased very much by the additional loops, the net current  $I$  becomes much larger.)

**Electrical Generators:** This brings us to one of the most important direct practical applications of Faraday’s law: the generation of electrical power. The basic principle of the electrical generator boils down to this: put a coil of wire in the presence of a strong magnetic field (produced either by a permanent magnet, or an electromagnet); then, rotate the coil by mechanical means (e.g., hand power, steam power or water power). As the loop is rotated, the angle  $\theta$  between the magnetic field direction and the normal to the loop keeps changing. This results in a constantly changing magnetic flux, which will produce an induced current in the loop so long as the loop is connected in a complete circuit. If you connect some useful devices in the circuit with the loop – light bulbs, heaters, etc. – the mechanical energy that powers the generator will be converted into electrical energy, and then into light, heat, or some other useful form of energy. Now, the engineering details that go into designing a practical and efficient generator are **far** more complicated than this very simple description suggests. However, the fundamental principle at work is really nothing more than Faraday’s law as we have described it here.

**Postscript: Lenz’s law.** We have left out one piece of the puzzle in the discussion above: How can we tell in which **direction** the current will flow in the loop? If we have already observed the direction of flow when the flux **increases**, then we can predict that the current will flow in the opposite direction when the flux **decreases**. However, there is a way to predict the direction of current flow without that preliminary observation. The rule that allows this prediction was formulated by Heinrich Lenz, and so it is called “Lenz’s law.”

*This is how it works: The induced current that flows in the loop will produce its own magnetic field, and so there will be an “induced” magnetic flux. This “induced” flux could be either a positive flux, or a negative flux. Lenz’s law states that if the **external** magnetic flux is increasing, the induced flux will be negative; if the external flux is decreasing, the induced flux will be positive. In other words, the induced flux “counters” the **change** in flux produced by the external field. In a particular loop, the direction of the induced flux depends on the direction of the induced current; if you know one, you can figure out the other by making use of one of our right-hand rules. In this way, the direction of induced current flow can be predicted for any given situation, as long as one knows whether the flux due to the external magnetic field will be increasing or decreasing.*