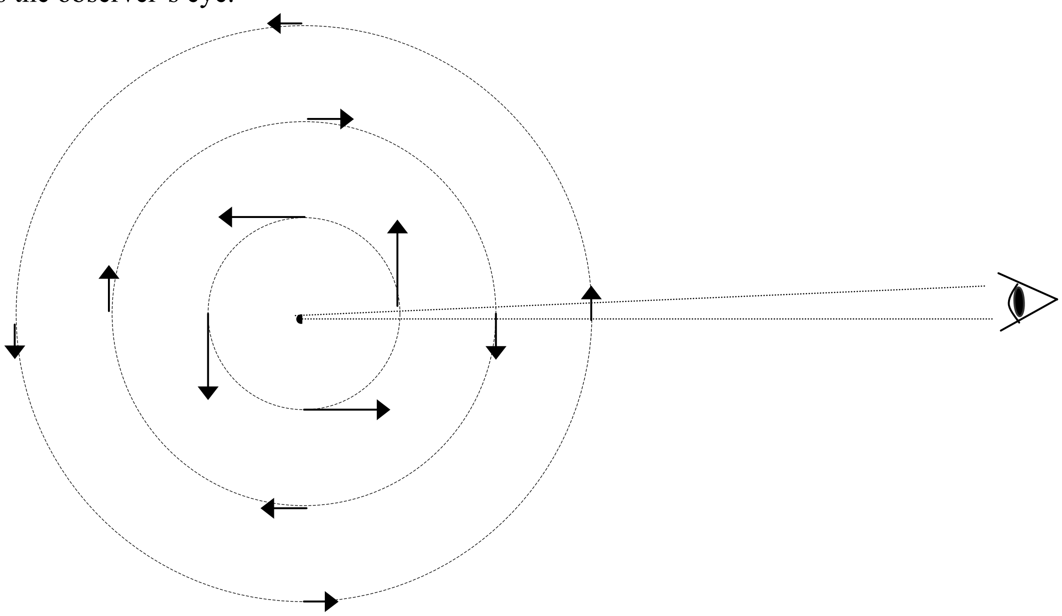
Chapter 12 Notes: Optics

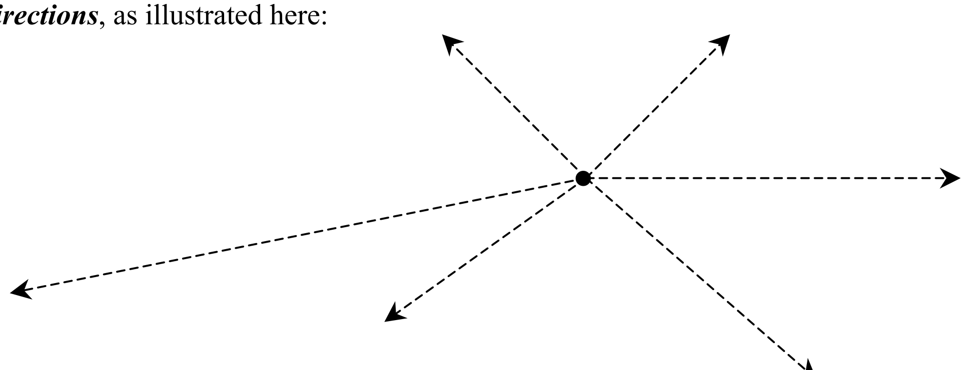
How can the paths traveled by light rays be rearranged in order to form images?

In this chapter we will consider just one form of electromagnetic wave: visible light. We will be examining the path traveled by light as it reflects off of surfaces, and as it propagates through certain types of (transparent) materials. Although everything we discuss here is valid in some form for ***other*** e-m waves, the particular ***materials*** which are relevant in this chapter (such as glass and water) would not be as appropriate for a similar discussion of, for instance, the paths traveled by radio waves. The propagation properties of e-m waves in various materials are strongly dependent on the frequency of the waves. In this chapter we focus on the particular behavior of light waves and their interaction with materials since light is of such great practical importance.

A concept that will be very important to us here is that of a "point source" of e-m waves. This is simply an emitter that is so small we can always ignore the detailed pattern of its emitted e-m radiation. We assume that all point sources emit e-m waves in ***all directions.*** The waves spread outward in a three-dimensional pattern, moving away from the source as do water waves from a rock dropped in a lake (or, more precisely, as do the waves from a firecracker exploding underneath the water's surface!). In the diagram below, we show a few of the electric field vectors contained in the e-m waves as they spread out from the point source at the center. We also show an observer (indicated by the "eye" at the right); the dashed cone indicates the path traveled by the small portion of the emitted light that strikes the observer's eye.

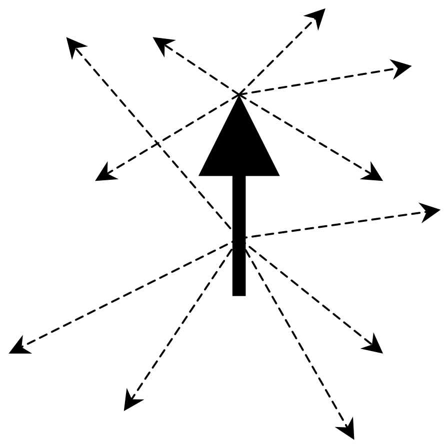


From the observer's point of view, all of the light that is seen has traveled along virtually the ***same*** straight-line path out from the source. An observer at any other point would see the same thing. The path traveled by the light from a point source to an observer is a straight line from the source, perpendicular to the electric and magnetic field vectors that compose the e-m wave. This path is called a "ray." In effect, the point source can be thought of as an emitter of light along an infinite number of rays. We can think of a "light ray" as a narrow beam of light traveling in a straight line. And so, from now on, we will imagine that ***the point source emits "rays" of light outward in all directions***, as illustrated here:



Now, any actual object that is emitting light (or reflecting light) is not actually a point source, but it does behave as if it were entirely composed of ***many*** point sources. That is, ***each point*** on an object emits an infinite number of light rays in ***all directions***. ***<u>It is essential to keep this idea in mind when analyzing optical phenomena!</u>***

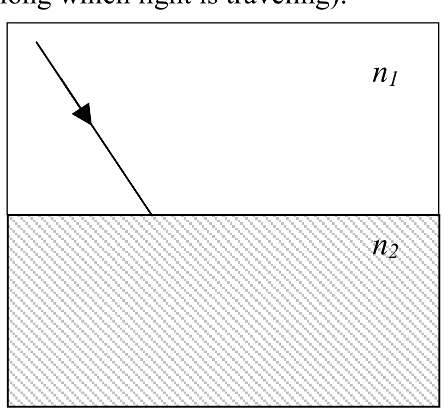
Here we show a typical "object" that is often used in optical experiments. It is simply an arrow, which may be drawn on a piece of paper or might actually be an "arrow-shaped" light source. We have picked out just two points on this arrow: one at the tip, and one in the middle of the tail. We have drawn just ***a few*** of the light rays that are emitted from these two points on the object:



Very often in discussions of optics, the paths of a few light rays from selected points on an object will be shown. However, it is very important to remember that any object is really a collection of an infinite number of points, and each of those points emits an infinite number of light rays in every possible direction. We will see later on how this idea will help us understand the way in which images are formed by lenses.

Part I: Path traveled by a <u>single</u> light ray. Before we can discuss images formed from the many light rays emitted by a luminous object, we need to analyze the path followed by a ***single*** ray of light as it travels from one material to another. That leads to our first question:

What happens when light traveling in one material encounters <u>another</u> material? As was discussed in Chapter 11, the speed of all types of e-m waves in a vacuum is equal to *c*. However when traveling through material substances, the speed of e-m waves is ***less*** than *c*, and the precise speed will depend on the frequency of the particular e-m wave. In the context of light waves, the ratio between *c* and the speed in the material is called *"n,"* or the "index of refraction" of the material: $n_A = \dfrac{c}{v_A}$, where v_A is the speed in material *A*, and n_A is the index of refraction of material *A*. What happens, then, when light traveling in material #1 with index of refraction n_1 encounters a different material #2 that has index of refraction n_2? This situation is represented in the diagram below (where the arrow indicates the path along which light is traveling):

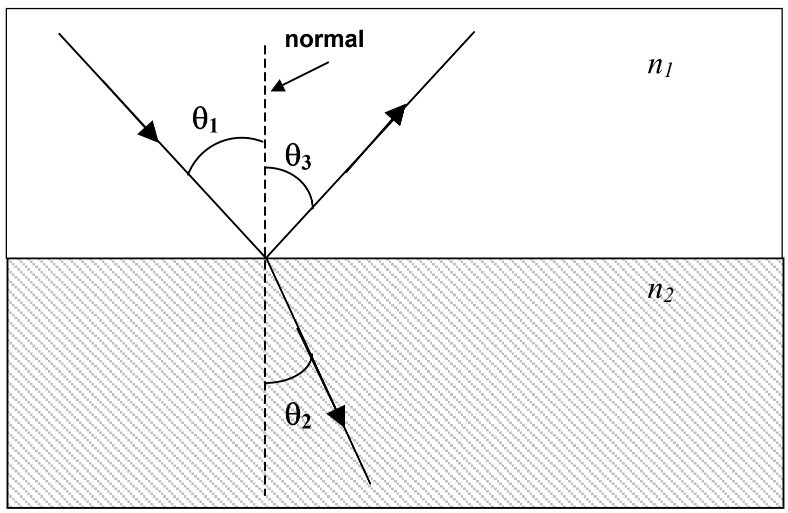


Examples of *n*:

$n_{air} = 1.0$
$n_{water} \approx 1.3$
$n_{glass} \approx 1.5$
$n_{diamond} = 2.4$

The answer to this question, which is of great practical importance, can be provided by Maxwell's theory of electromagnetism. (The mathematical formulations of the basic principles of Maxwell's theory are called "Maxwell's equations.") Here we will just state the results without going into any of the mathematical derivations.

When light strikes an interface, some light is reflected and some is transmitted. In general, what happens when light encounters an interface between two different materials is this: some of the light goes on through into the new material, and some of it is "reflected" off of the interface and continues traveling in the first material along an altered path. The path of the reflected light, as well as that of the transmitted light, will depend on the precise path of the incoming, or "incident," ray. In order to specify all of the different directions involved, it is again convenient to draw a "normal" line perpendicular to the surface. We'll indicate that "normal" with a dashed line. (Note that all three rays are represented as being in the same plane [here, the plane of the page]. Maxwell's equations prove that this will always be true.)



Some terminology:

(1) The incoming ray is called the "incident" ray. The transmitted ray (traveling in material #2) is called the "refracted" [bent] ray because, in general, it does not continue along the same path as the incident ray.

(2) Angle θ_1 is called the "angle of incidence." It is the angle between the incident ray and the normal to the surface.

(3) Angle θ_3 is called the "angle of reflection." It is the angle between the reflected ray and the normal to the surface.

(4) Angle θ_2 is called the "angle of refraction." It is the angle between the refracted ray and the normal to the surface.

The relationship between the angles on this diagram was found by experiment long before Maxwell's equations provided a theoretical explanation for them. These are the results:

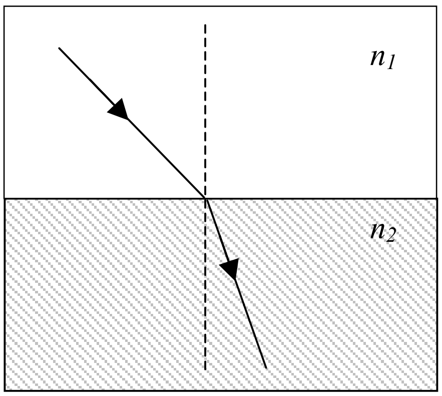
The Law of Reflection: The angle of incidence is equal to the angle of reflection: $\theta_1 = \theta_3$

Snell's law: The relationship between the path of the incident ray and that of the transmitted ray is determined by the ratio of the indices of refraction of the two materials, according to this equation:

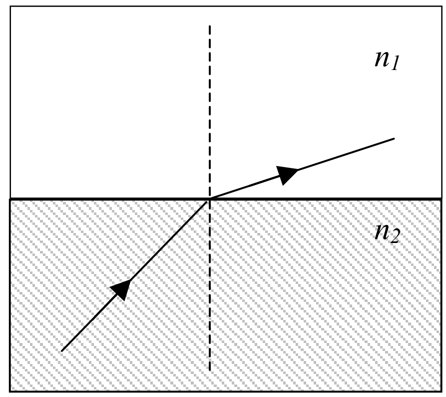
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Question: *In the diagram at the top of page 3, is* n_2 ***greater than, less than,*** *or* ***equal to*** n_1? ***Answer:*** n_2 *is* ***greater*** *than* n_1. *We can see that angle* θ_2 *is smaller than angle* θ_1, *and both angles are less than* $90°$*. Since the value of* $\sin\theta$ ***increases*** *as* θ *increases from* $0°$ *(*$\sin 0° = 0$*) to* $90°$ *(*$\sin 90° = 1$*), we get in this case that* $\sin\theta_2 < \sin\theta_1$*. Snell's law can be written as* $\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1}$*, and so we see that* $n_1 < n_2$.

A convenient way to remember how this works is this: when light travels into a medium with ***higher*** index of refraction, the light ray bends ***toward*** the normal; when it travels into a medium with ***lower*** index of refraction, it bends ***away*** from the normal. The phrases "toward" and "away from" the normal are just ways to describe the shape of the path taken by the light in the two cases.



$n_2 > n_1$

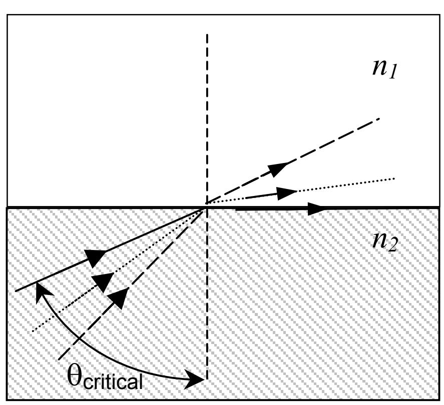


Total Internal Reflection: In the case where the light is traveling ***into*** a material with ***lower*** index of refraction (for instance, from glass into air), it is possible to find an angle of incidence large enough that the angle of refraction is exactly 90°. For this angle of incidence, and any angle that is ***larger*** than it, the transmitted wave ***completely disappears.*** All of the incoming light will be reflected, and so this situation is called ***"total internal reflection."*** It is as if the light were "trapped" inside the material with the higher refractive index. The angle of incidence which results in a 90° angle of refraction is called the ***"critical angle."*** *Note that the critical angle depends on the refractive indices of* ***both*** *materials!* If we assume again that $n_2 > n_1$, and that the light is traveling ***from*** n_2 ***into*** n_1, we can find the critical angle this way:

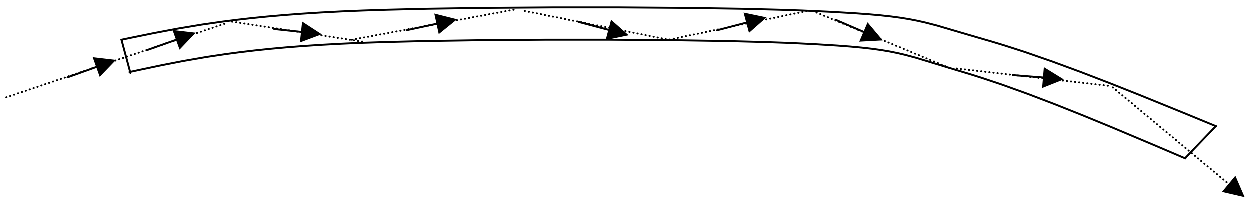
$$n_1 \sin 90^\circ = n_2 \sin\theta_{critical} \quad \Rightarrow \quad n_1 = n_2 \sin\theta_{critical} \quad \Rightarrow \quad \frac{n_1}{n_2} = \sin\theta_{critical}$$

$$\Rightarrow \qquad \theta_{critical} = \arcsin\left(\frac{n_1}{n_2}\right)$$

$n_2 > n_1$



Total internal reflection is the basis of ***fiber optics***. Optical fibers are long, thin glass tubes. Light beams are directed into one end and continue on through the entire fiber while losing essentially no energy, finally exiting the other end. Each time the light beam strikes an internal surface of the fiber, it is totally internally reflected and so no light leaves the tube until the very end. These light beams (usually produced by lasers) are used to carry voice and data signals with very high efficiency.



Total internal reflection also explains the color and sparkle of diamonds. The index of refraction of diamond is 2.4 – very nearly the highest of any substance. Because of the high index of refraction, the critical angle for diamond is very low: only around 24°. When light enters one face of the diamond and strikes an internal surface, it is very likely to have an incidence angle larger than that, in which case it will be totally internally reflected. Then, it strikes *another* internal face – and so on. It is almost as if a collection of mirrors were trapped inside the diamond. Whatever the angle from which the diamond is viewed, an observer is likely to see one of the light beams that has been bouncing around inside. In addition, as the light beams travel inside the diamond, light of different colors follow slightly different paths. (This is due to the fact that the indices of refraction for different frequencies of light have slightly different values.) By the time the light exits the diamond, it has "spread out" into a small rainbow of different colors. Because of its great hardness, diamond can be cut into intricate shapes that enhance these effects.

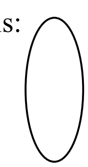
At this point we can go on to discuss the formation of images.

Part II: Formation of an "image" by <u>many</u> light rays. The most important practical applications of optical principles are associated with the formation of ***images.*** Images of objects can be created by carefully designing systems to redirect the paths of light rays, making use of reflection (with mirrors) and refraction (with lenses).

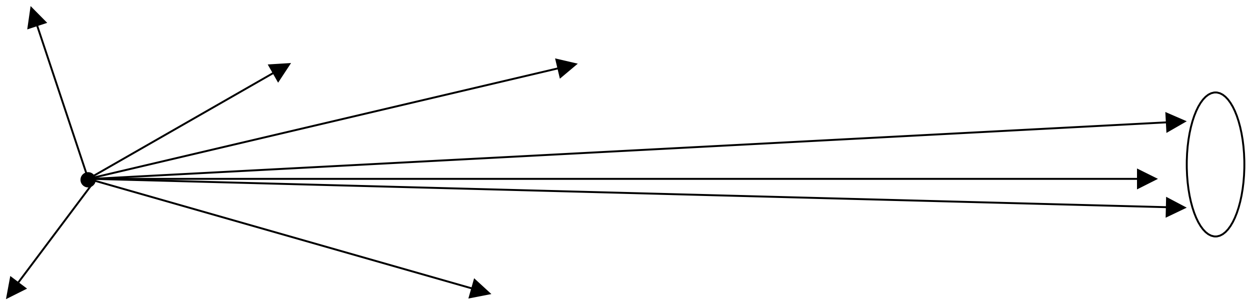
We may describe an "image" of an object as a sort of "map" – a map composed of light rays, formed from an object that is emitting or reflecting light. We call it a "map" because each point on the object corresponds to a point on the image, and some of the geometrical "shape" relationships of the object are retained in the map. But, like any map, there may also be some distortion of the original object. It may be magnified or reduced, turned upside down, inverted "left-to-right," or have its shape changed in other ways. Images are formed by microscopes, telescopes, cameras, projectors, and many other devices. And, of course, there is another extremely important use of images: the images formed on our retinas by the lenses in our eyes!

The principles behind all of these methods of forming images are very similar. The image is formed at a specific location, e.g. on the retina, on a screen, on a sheet of film, etc. Light rays from the original object are redirected to the image location, either by reflection from mirror-like surfaces or refraction through transparent materials. A ***lens*** is a piece of transparent material shaped so as to form images through refraction. Here we will discuss some of the basic principles behind the operation of a lens.

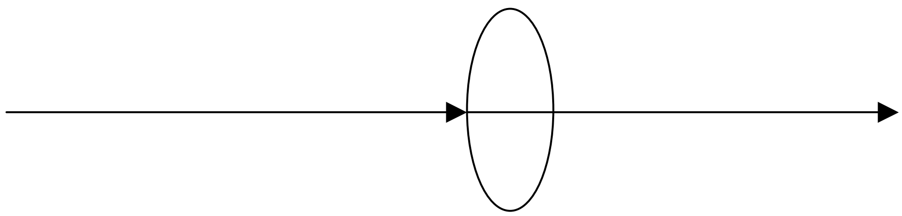
A ***converging lens*** is shaped to be thicker at the middle than at its ends; it might be composed of two pieces of a spherical surface "stuck" together. We will consider a so-called "thin" lens, which has a diameter that is much smaller than the radius of curvature of the spheres from which it was formed. It looks something like this:



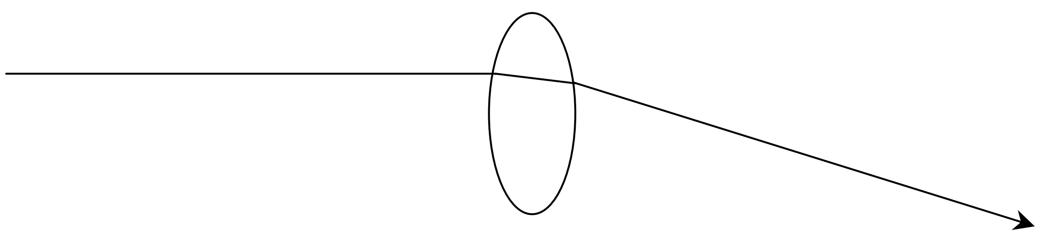
Let's first consider the image that would be formed of a point source located far from the lens.



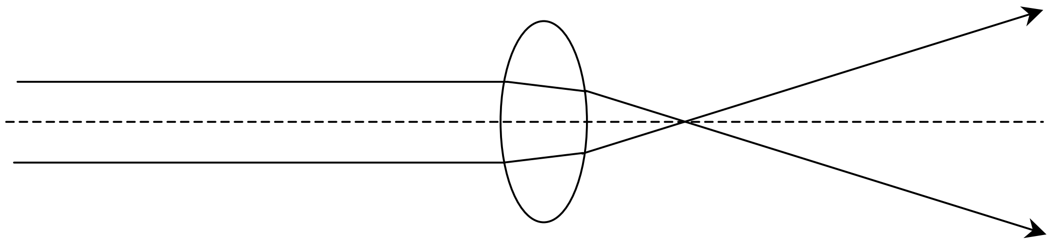
The first thing we can notice is that all of the light rays that enter the lens from the point source are *nearly parallel* to each other; only a small "bundle" of rays make it to the lens, and they all followed pretty nearly the same path to get there. So what will happen to the parallel rays as they go through the lens? Well, a ray that strikes the lens right at the center has an angle of incidence of 0°; that means its angle of refraction will ***also*** be 0°and so it will pass through unchanged, moving along its original path:



A ray that comes in parallel to that central axis, but not right on it, will be refracted twice – once at each of the surfaces.

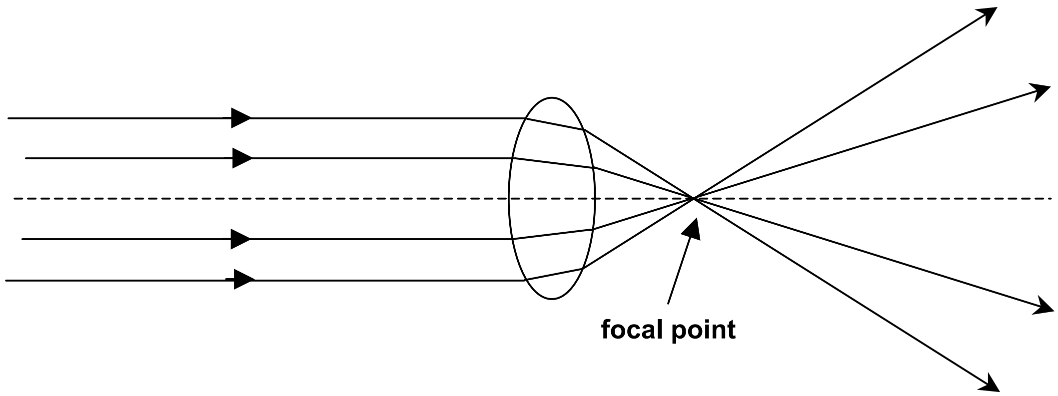


Now, if another parallel ray enters from the same distance ***below*** the axis, it is easy to see that it will bend ***upward***, instead of downward. So it will meet the first ray at some point along the central axis:



Well, that tells us about just *two* of the rays coming from the distance source. But there are many more – an infinite number! – and they would strike the lens at all possible points along its left surface. When they pass through the lens, what path will they follow? It is possible that they will meet up at the same point as these first two rays that we've drawn?

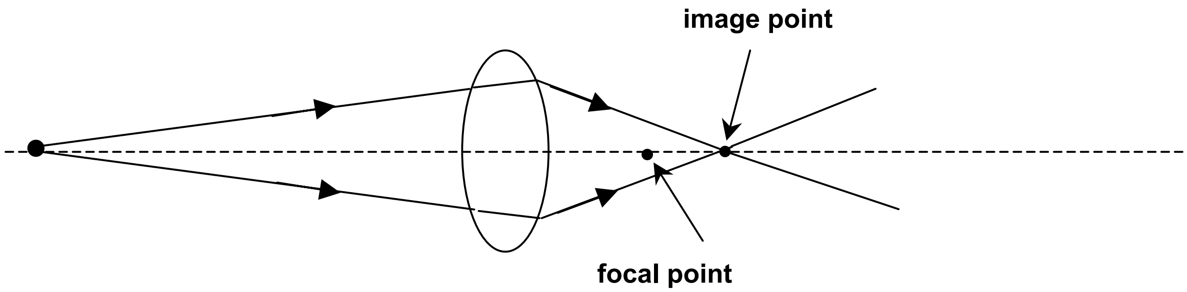
In fact, it can be shown that in the case of a thin lens such as this one, all of the incoming parallel rays ***do*** in fact converge at that one special point, as we'll see on the next page.



The point at which all of these parallel rays converge is called the "focal point" of the lens. The distance from the focal point to the lens is called the ***focal length*** [symbol: *f*], and it will be determined by the precise shape of the lens, and by its index of refraction.

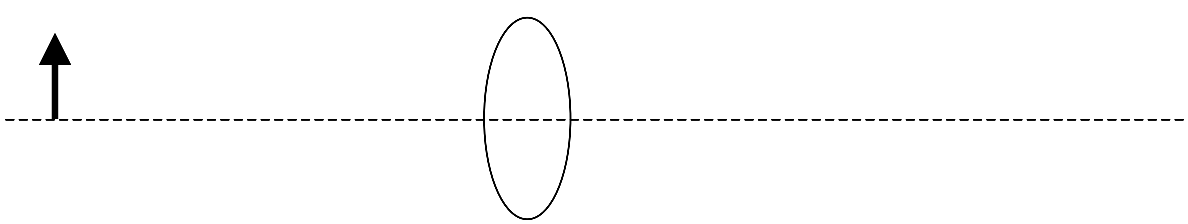
Every ray originating from that point source that strikes the lens will meet at the focal point. There will be a bright spot at that point due to the convergence of all the light rays. We have formed a "map" or *image* of the point source at the focal point. If you have ever focused the sun's rays on a leaf or piece of paper with a magnifying lens, you have formed an image of a distant point source at the focal point of your lens. (The sun is so far away it *acts* as if it were a point source in this case.) If you've done this, you know that you can create heat so intense that the paper or leaf will burn.

Now, in itself this is not all that useful. We are mainly interested in forming images of objects, not of distant point sources! Let us first see what happens if we bring the point source closer in to the lens, but keep it on the axis:

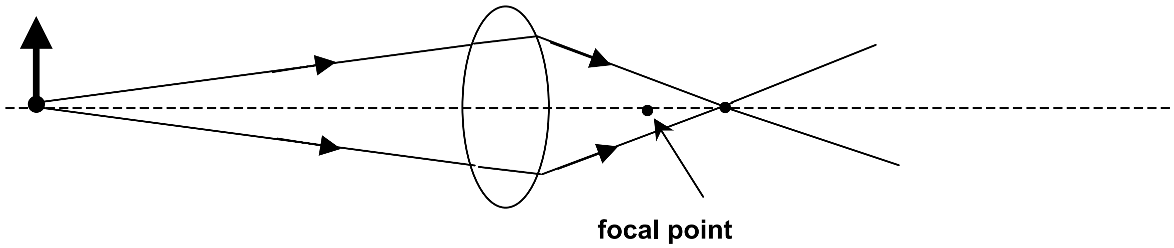


What happens is that an image of the point source is still formed along the axis, but ***not*** at the focal point. Instead, the "image point" is somewhat farther away from the lens than is the focal point.

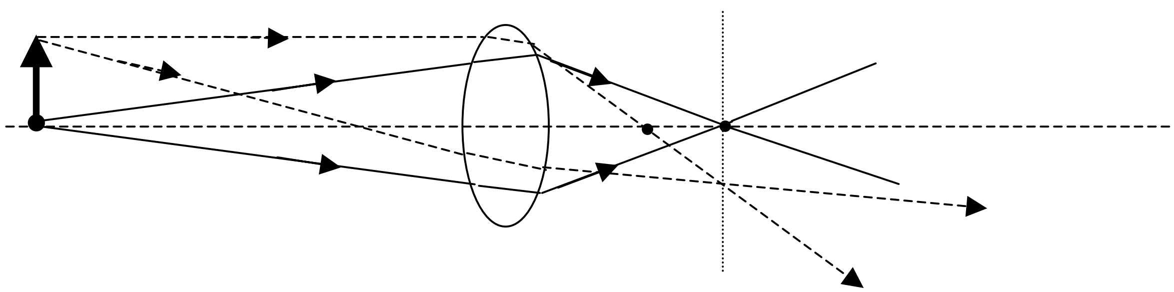
Now, finally, let's consider an actual object that is composed of an infinite number of points. Suppose we put it at the same location as the point source above. Will it form an image, and if so, where?



Well, the point at the very tail of the arrow is just a point source on the axis. We know where the rays from ***that*** point will converge: at the "image point" found for the point source in the example just above.

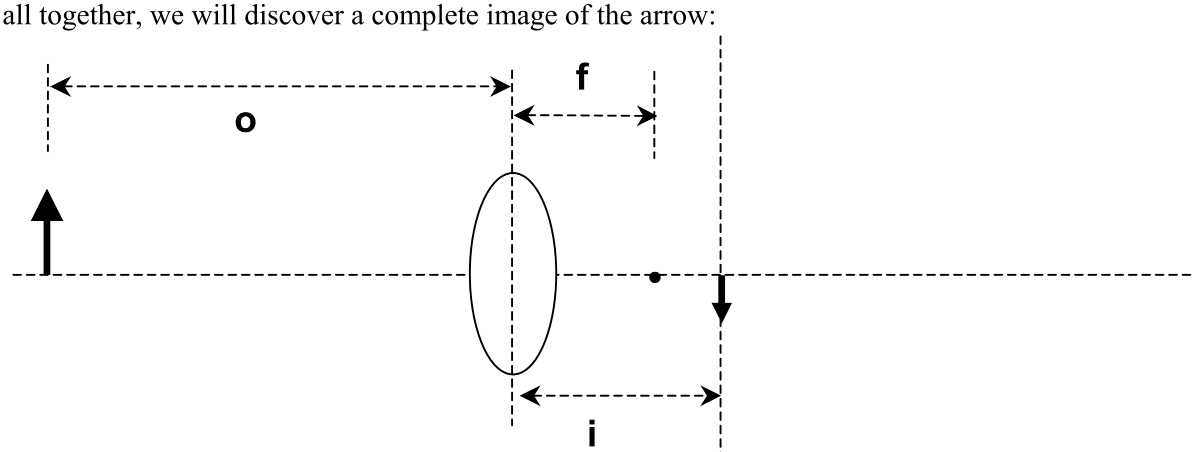


Now what about the rays from, say, the tip of the arrowhead? Once again, it is possible to show that all the rays from the tip of the arrow will ***also*** converge at a single point – ***not*** at the same image point as before, but at another point that is just as far from the lens as is that previous image point. This time, the image point will not be on the axis:



Remember that we have only drawn two out of the infinite number of rays spreading out from the tip of the arrow – and already the diagram is too complicated! But it does turn out that ***all*** of the rays from the tip of the arrow will converge at the same point as these two – that is where the image of the arrowhead will be formed.

Of course we're not going to try to draw rays from all the ***other*** points along the arrow – the diagram would become hopelessly complicated. It should not be surprising to learn, though, that images of all the other points along the arrow will form at the same distance from the lens, at other points along the vertical dotted line. Finally, if we put it all together, we will discover a complete image of the arrow:



The image (which we see is inverted) is formed some distance away from the focal point. The distance of the image to the lens is called the ***image distance*** [symbol: *i*]; the distance of the object to the lens is called the ***object distance*** [symbol: *o*]. There is a very simple relationship between the image distance, the object distance, and the focal length:

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

The ***magnification*** [symbol: *m*] is the ratio of the image height to the object height: $m = (\text{image height}) \div (\text{object height})$. It can be shown that $m = i/o$.