Nuclear Physics Worksheet *Answers*

The nucleus [plural: *nuclei*] is the core of the atom and is composed of particles called "nucleons," of which there are two types:

protons (positively charged); the number of protons in a nucleus is called the ***Atomic Number*** [symbol: Z], and identifies which element corresponds to that particular nucleus.

neutrons (no electric charge); the number of neutrons in a nucleus is called the "neutron number."

The nucleons are held together by the ***"strong"*** force, also called the "nuclear" force.

Different ***isotopes*** of a single element all have the same Z, but different numbers of neutrons.

Radioactive Decay: Certain types of nuclei are said to undergo radioactive decay when they transform into another type of nucleus. (Nuclei that decay are said to be "unstable.") The transformation, or decay, is accompanied by emission of certain types of particles ("alpha" and "beta" particles), or of a gamma ray.

It is not possible to determine when one particular unstable nucleus will decay. However, it ***is*** possible to determine to a very high degree of accuracy what proportion of a ***very large sample*** of unstable nuclei of any given type will decay each second. For instance, if we have 1000 nuclei of type "A," we might find that 4% of them will decay each second. (We will assume here that they each decay into a single nonradioactive nucleus.) In this example, we would find that of the original sample of 1000 nuclei, 40 of them will decay in the first second. For ***any other sample*** consisting of 1000 nuclei of type "A", we can be confident that very close to 40 of them will decay in one second. We would say that the ***"decay constant"*** of this particular nucleus is 4%/s [four percent per second]. Using the symbol "λ" for decay constant, we would say that $\lambda = 0.04/\text{s}$ for this nucleus.

Some symbols we will use:

λ ["decay constant"]: fraction of nuclei present which decay per unit time. ***Note that in this context, λ does <u>not</u> represent a wavelength!*** λ is usually expressed in units of s^{-1}.

N: number of radioactive nuclei of a given type present at a particular moment in time. *N* is a function of time: $N = N(t)$. ***[Note that the number of decays per second is equal to λN.]***

ΔN: change in value of *N*. A ***decrease*** in *N* would correspond to a ***negative*** value of ΔN.

If there are "*n*" decays in time Δt, the value of *N* will ***decrease*** by *n*. Therefore, we can see that:

$$\frac{-\Delta N}{\Delta t} = \text{number of decays per second}$$

1. Consider a sample of N_0 nuclei (where N_0 is the original number, that is, the number at t = 0 s). As time goes on, we continue to make observations on this sample of nuclei.

   a) Will the value of N (i.e., the number of radioactive nuclei present) *increase*, *decrease*, or *remain the same* as time goes on? ***Decrease, due to radioactive decay.***

   b) Will the number of ***decays per second*** *increase, decrease,* or *remain the same* as time goes on? ***Decrease, because the proportion of the total number present that will decay is unchanging, but that total number is itself decreasing.***

2. Consider a radioactive nucleus of a type that has decay constant equal to $\lambda = 0.10$/s [we can also write this as $\lambda = 0.10\ s^{-1}$].

   a) If at a given moment you have a number A of these nuclei, how many of them will decay in one second? ***0.1 A (because this is equal to λA)***

   b) If at another moment you have 0.7 A of these nuclei, how many of them will decay in one second? ***0.07 A (because this is equal to λ times the number present now)***

   c) Is your answer to 1(b) consistent with your answers to (a) and (b) here? ***Yes***

3.

   a) Write an equation relating "decays per second" to λ and N.

   decays per second = λN

   b) Write an equation relating $\frac{\Delta N}{\Delta t}$ to λ and N. *Hint: You need a (–) sign; refer to notes on page one.*

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

4. Suppose a sample of A radioactive nuclei has *"B"* decays per second at a given moment, where "B" represents some specific number such as 10 or 20. When the number of these nuclei has decreased to 0.5 A, how many decays per second will be there be? Express your answer in terms of the number B. *Hint: Review your answer to #2 above.*

   0.5B (Because $\lambda = \frac{B}{A}$, and

   number of decays per second = $\lambda \times$ [number present] = $\frac{B}{A}$(0.5 A) = 0.5 B

5. Given two different samples each containing 1000 nuclei, which one would be the first to have its value of N hit 500: A sample with $\lambda = 0.10$/s, or one with $\lambda = 0.20$/s? Why? ***$\lambda = 0.20$/s, because more will decay each second, thus reaching 500 earlier than the other***

The time required for a sample of N radioactive nuclei of a certain type to decay to ***half*** *of its original value is called the "half-life" [symbol:* $T_{1/2}$*] of that type of nucleus.*

6. Which would have the longer half-life: a nucleus with *large* λ, or one with *small* λ? Does your answer suggest that $T_{1/2} \propto \lambda$, or instead that $T_{1/2} \propto 1/\lambda$? (It's not *exactly* equal to either one.)

   small **λ;** $\mathbf{T_{1/2} \propto 1/\lambda}$

7. Let's consider a sample of 1000 nuclei with λ = 0.20/s.

   a) How many nuclei would decay in the first second? ***200***

   b) If this same number of decays per second were to continue, how long would it take for *half* of the original number of nuclei to decay away? **2.5 s**

   c) Is the actual half-life of this material *larger than, smaller than,* or *exactly equal to* the your answer for [b]? *Hint: How many nuclei decay between t = 0s and t = 1s? How does this compare to the number that decay between t = 1 s and t = 2s?* ***larger, because 200 decay in the first second, but only around 160 decay in the second second, etc. so it takes longer than 2.5 seconds to get down to 500.***

   d) Work out a more precise estimate of the actual half-life of this material. ***200 decay in the first second, leaving 800. Then around 160 of those will decay, leaving 640. In the third second, around 128 will decay, which gets us down to 512. So we see that the actual half life is longer than three seconds. (An accurate calculation gives*** $T_{1/2}$ ***= 3.47 s.)***

   e) Which would be a better estimate of the ***half-life*** of this material: $\frac{2}{\lambda}$ or $\frac{1}{2\lambda}$? *Hint: Find the* ***numerical*** *values of these two expressions.*

$\frac{2}{\lambda}$ **= 10 s, and** $\frac{1}{2\lambda}$ **= 2.5 s, so** $\frac{1}{2\lambda}$ ***is the closer estimate. (Actually,*** $T_{1/2} = \frac{0.693}{\lambda} = \frac{1}{1.44\lambda}$***)***

8. An isotope of krypton has a half-life of 3 minutes. A sample of this isotope produces 1280 counts per second in a Geiger counter at 3:00 PM. Each count corresponds to the radioactive decay of one nucleus.

   a) As time goes on, will the number of counts per second *increase, decrease,* or *remain the same*? ***decrease***

   b) After one half-life has elapsed, how many counts per second would you expect to observe? *Hint: Refer to your answer to #4.* ***640***

   c) Determine the number of counts per second produced by this sample at 3:15 PM. ***Five half lives will have elapsed; the number of decays starts out at 1280 per second, then drops to 640, then 320, then 160, then 80, and finally 40. So at 3:15 PM, there will be 40 decays per second.***

Radioactive Dating: Radioactive materials are often used to provide estimates of the ages of substances that contain those radioactive materials. A particularly important example is ***radiocarbon dating*****,** which makes use of the radioactive nucleus ^{14}C that has a half-life of 5730 years. (The number "14" refers to the total number of nucleons – protons [6] plus neutrons [8] – in this nucleus. The nonradioactive form of carbon is ^{12}C, and has 6 protons and 6 neutrons.)

The relative proportions of ^{14}C and ^{12}C in the atmosphere have been fairly stable for the past 50,000 years or so. ***In a sample of 10^{12} nuclei, approximately <u>one</u> will be ^{14}C, and the rest will be ^{12}C.*** Plants absorb carbon (in the form of carbon dioxide) from the air and animals eat plants. As long as the organisms are alive, the ratio of ^{14}C to ^{12}C in their tissues remains at the atmospheric ratio of 1 in 10^{12}. However, when the organism dies, the radioactive carbon is no longer replenished from the atmosphere and so its proportion *decreases*. By measuring the relative proportions of the two types of carbon in a sample of biological material, the age of the sample can be determined with great accuracy (up to approximately 50,000 years).

Let's consider a sample of 120 g of carbon extracted from some material of unknown age. This is equal to ten moles, ***each*** of which contains 6×10^{23} atoms. The decay constant of ^{14}C is 4×10^{-12} s^{-1}. Suppose we determine (using some form of Geiger counter, for instance) that the sample produces 10 radioactive decays every second. We want to find out the ***approximate*** age of the material, using this information. How may we proceed?

9. Suppose we had a sample of 120 g of carbon from a ***living*** organism (this sample contains ***both*** types of nuclei). How many decays per second would we expect to detect? *Hint: Determine the number of* ***radioactive*** *nuclei in this original sample (this is much less than the* ***total*** *number of nuclei present); use the decay constant to find the number of decays per second.*

$N = (6 \times 10^{24}) \times (10^{-12}) = 6 \times 10^{12}$; decays per second = $\lambda N = (4 \times 10^{-12}\ /\text{s}) \times (6 \times 10^{12}) = 24/\text{s}$

10. How many decays per second would we expect to detect from that sample 5730 years after the death of the organism? *Hint: Find the number of radioactive nuclei present at this time, and again use the decay constant.*

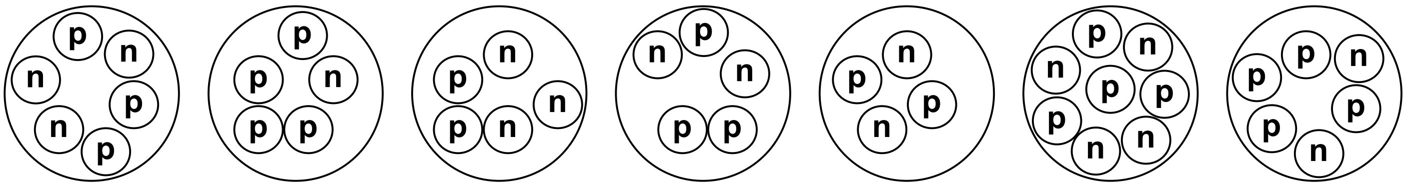
$N = 3 \times 10^{12}$; decays per second = $\lambda N = (4 \times 10^{-12}\ /\text{s}) \times (3 \times 10^{12}) = 12/\text{s}$

11. Find the approximate age of the material. Determine two different ages which "bracket" the actual age:

    material definitely older than: ***5730*** yrs

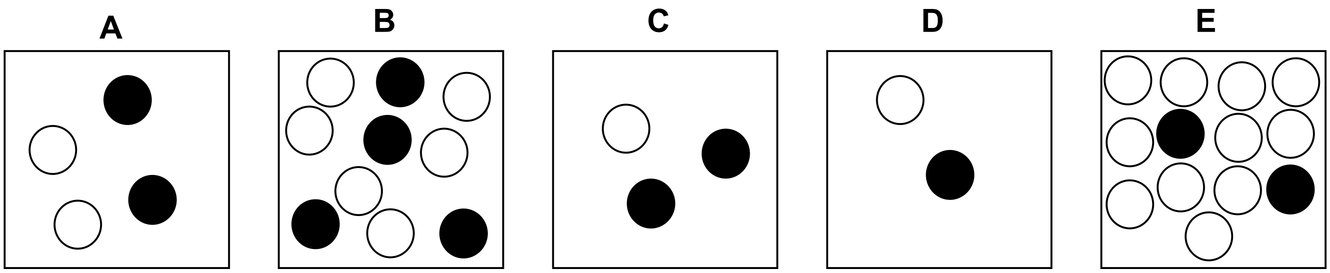
    material definitely younger than: **11,460 yrs (because more than 6 decays per second observed)**

12. Diagrams of several different nuclei are shown. How many different elements are represented? For each element, state how many isotopes of that element are represented.



Z = 2, 2 isotopes; Z = 3, 2 isotopes; Z = 4, 3 isotopes

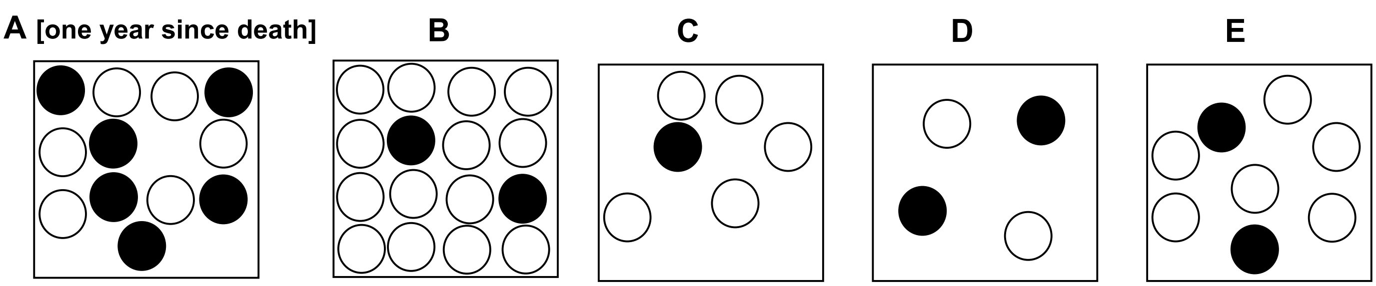
13. In the following diagrams, the white circles represent nonradioactive nuclei. The black circles represent ^{14}C, which is radioactive and has a half-life of 5730 years. Five different samples are shown. Rank order them according to how many nuclei will decay each second, on the average (most to fewest):



Ranking for number of decays per second: B > A = C = E > D

Explain your answer. ***Number of decays per second is proportional to the number of radioactive nuclei present.***

14. These five samples were taken from trees that have been dead for different amounts of time. Again, the black circles represent ^{14}C; when they decay they become white circles. (The proportions of radioactive nuclei are *not* realistic.) Find the approximate ages of samples B, C, D and E.



Hint: Approximately how many decays are likely to have occurred in sample A, given its age?

B = 11,400 yrs; C = approximately 9,000 yrs; D: about one year; E: 5,700 yrs.

Explain your answers. ***A: 6/12 = ½ N are radioactive, with zero half-lives elapsed (so no decays yet); B: 2/16 = 1/8 N are radioactive (so two half-lives elapsed); C: 1/6 N are radioactive (so more than one half-life elapsed); D: 2/4 = ½ N are radioactive (so zero half-lives elapsed); E: 2/8 =1/4 N are radioactive, so one half-life elapsed.***