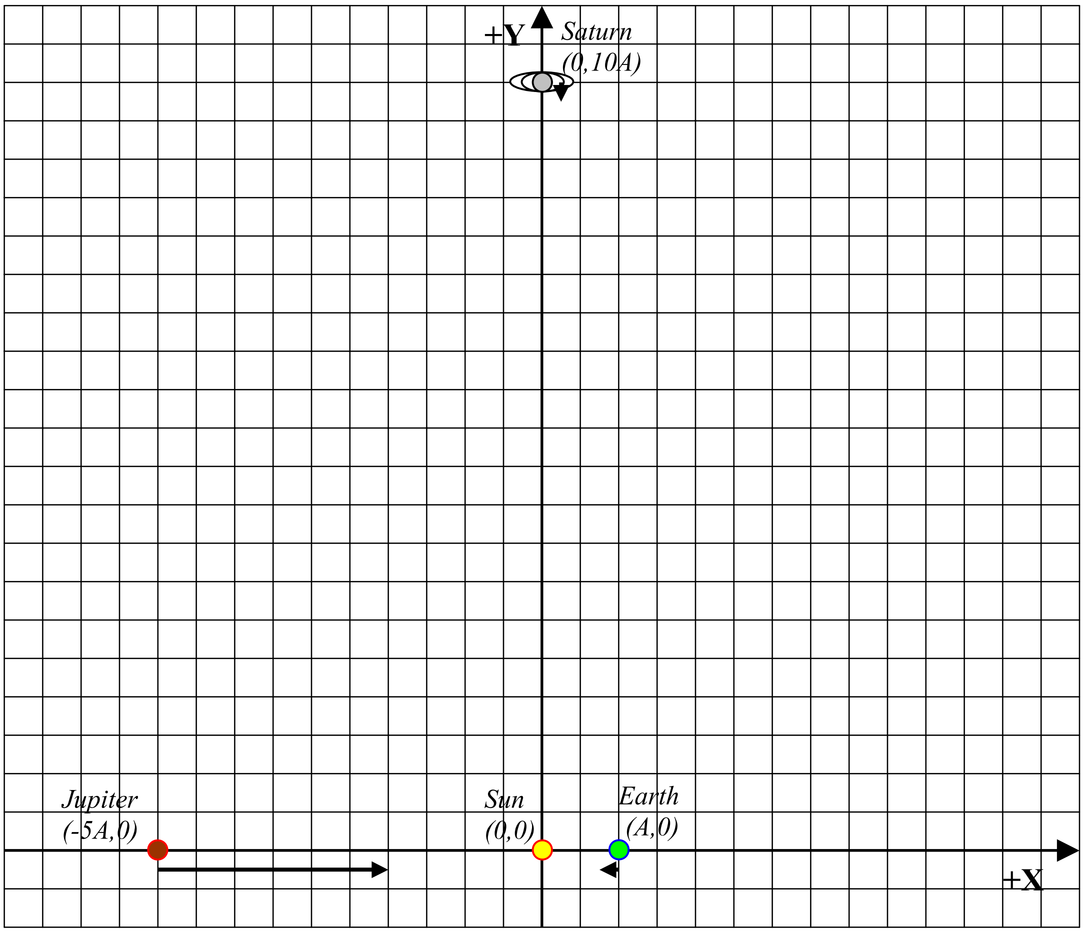
# Physics 112
# Homework #1: Gravitation
# SOLUTION by N. L. Nguyen

## Total value: 5 points

Draw an x-y coordinate system with the sun at the origin: ***Use graph paper!***




First to solve the problem, one has to realize that for Questions 1-3 ***only the sun-planet forces are significant***. The rest are too small in comparison to matter.

$$F_{sun,earth} = F_{\Theta,E} = \frac{GM_{\Theta}M_E}{r_{\Theta,E}^2} = \frac{G(3\times10^5 M_E)M_E}{r_{\Theta,E}^2} = (3\times10^5)\frac{GM_E^2}{r_{\Theta,E}^2}$$

$$F_{sun,jupiter} = F_{\Theta,J} = \frac{GM_{\Theta}M_J}{r_{\Theta,J}^2} = \frac{G(3\times10^5 M_E)(300M_E)}{(5r_{\Theta,E})^2} = \frac{300}{25}(3\times10^5)\frac{GM_E^2}{r_{\Theta,E}^2} = 12\cdot(3\times10^5)\frac{GM_E^2}{r_{\Theta,E}^2}$$

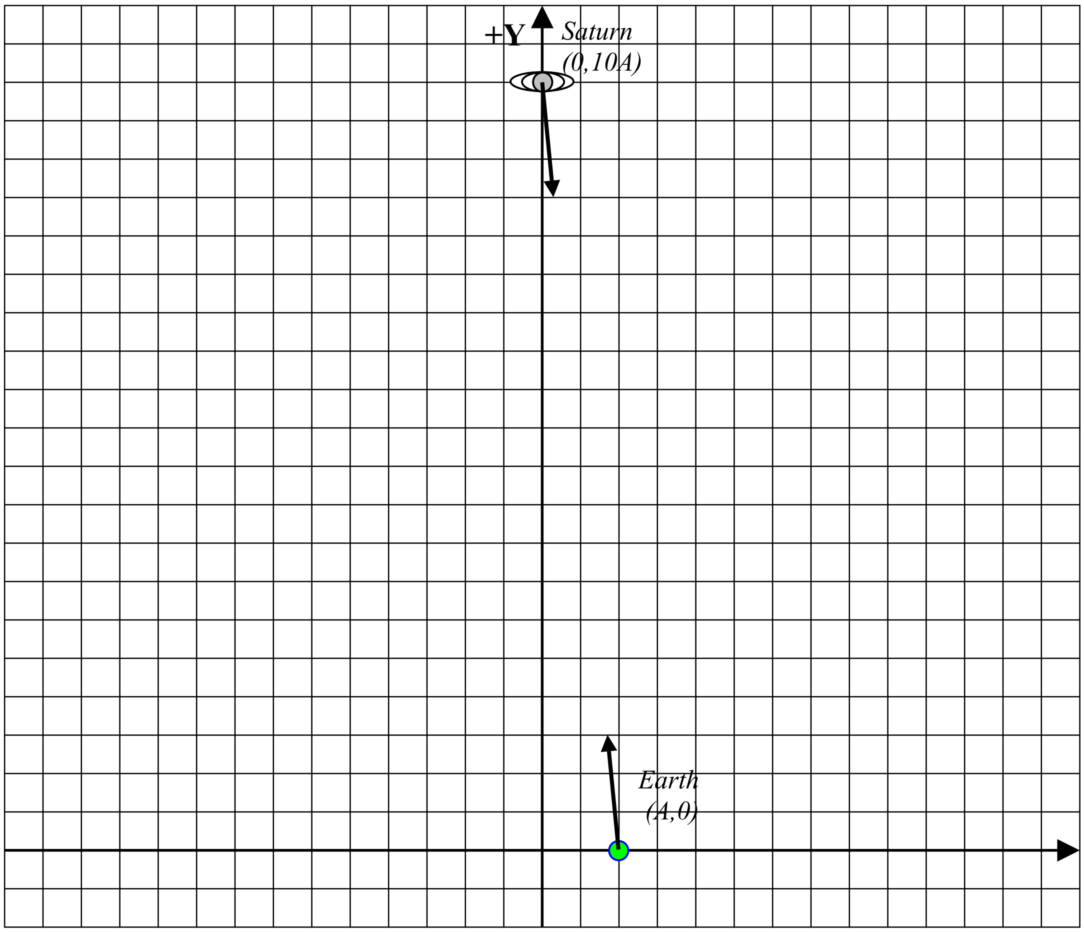
$$F_{sun,saturn} = F_{\Theta,S} = \frac{GM_{\Theta}M_S}{r_{\Theta,S}^2} = \frac{G(3\times10^5 M_E)(100M_E)}{(10r_{\Theta,E})^2} = \frac{100}{100}(3\times10^5)\frac{GM_E^2}{r_{\Theta,E}^2} = (3\times10^5)\frac{GM_E^2}{r_{\Theta,E}^2}$$

To draw net gravitational forces, then compare ratios by dividing by the *smallest* force.

$$\frac{F_{\Theta,J}}{F_{\Theta,E}} = 12; \frac{F_{\Theta,S}}{F_{\Theta,E}} = 1$$

Choose the Sun-Saturn and Sun-Earth gravitational vectors to be **1/2** a grid square long. Then Sun-Jupiter vector is **6** grid squares long.

Question 4:
Now assume that the Sun and Jupiter disappear. On a second diagram, draw a new set of force vectors representing the gravitational forces now acting on the Earth and Saturn.




We could recalculate the forces using the equation provided by *Newton's Universal Law of Gravitation* but this is unnecessary. To solve this problem we only need to remember two things.

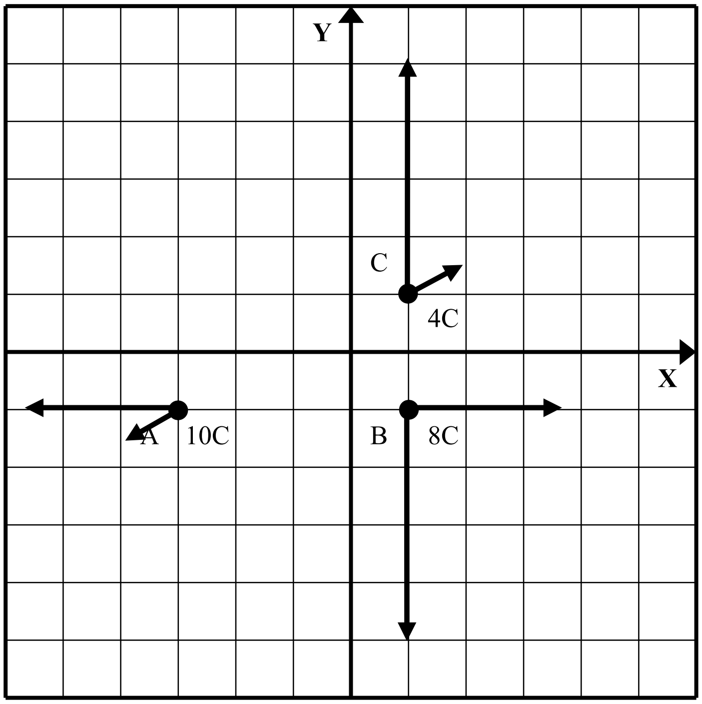
1. Gravitation always acts to attract different masses toward each other in a straight line from their center of mass.

2. *Newton's Third Law* states that two interacting objects will exert **equal** (in magnitude) and **opposite** (in direction) forces on each other.

Since we don't have to compare the force arrows to any other arrow, we are free to choose them to be any length as long as they are of the **same length** and are **pointing toward** and not away from each other.

## Homework Exercises

1. In this figure, charged particles A, B, and C are labeled with their respective charges. Draw and label arrows representing the electrical forces on each charge, due to the other two charges. Label the force on charge "a" due to charge "b" as "$F_{AB}$", etc. Make sure the relative lengths of the arrows correspond to the relative magnitude of the respective forces.




Let *r* be the distance of one grid square along the x or y axis. Then:

$$F_{AB} = k\frac{(10C)(8C)}{(4r)^2} = \frac{80}{16}\frac{k}{r^2}\cdot C^2 = \frac{20}{4}\frac{k}{r^2}\cdot C^2 = (5)\frac{k}{r^2}\cdot C^2$$

$$F_{BC} = k\frac{(8C)(4C)}{(2r)^2} = \frac{32}{4}\frac{k}{r^2}\cdot C^2 = 8\frac{k}{r^2}\cdot C^2$$

$$F_{AC} = k\frac{(10C)(4C)}{(\sqrt{20}\cdot r)^2} = \frac{40}{20}\frac{k}{r^2}\cdot C^2 = 2\frac{k}{r^2}\cdot C^2$$

To get appropriate vectors divide all the forces by the smallest one. Since all charges are positive, and *like charges repel* then the force arrows will point away from each other.

2. Two identical particles are separated by 1 cm; they repel each other with a force of $10^{10}$ newtons. What is the magnitude of the charge on *each* particle?

$$F = 10^{10}\,N = k\frac{q_1 q_2}{r^2} = k\frac{q^2}{r^2}$$

$$q^2 = (10^{10}\,N)\,r^2/k$$

$$q = \sqrt{\frac{r^2\cdot 10^{10}\,N}{k}} = \sqrt{\frac{(0.01m)^2(10^{10}\,N)}{9\times 10^9\,Nm^2/C^2}} = 0.0105C$$

3. Two protons are separated by two meters. Determine the magnitude of the electrical force that they exert on each other, as well as the magnitude of their mutual gravitational force.

$$F_{elec} = k\frac{q_1 q_2}{r^2} = (9\times10^9\ Nm^2/C^2)\cdot\frac{(1.6\times10^{-19}\ C)^2}{(2m)^2} = 5.76\times10^{-29}\ N$$

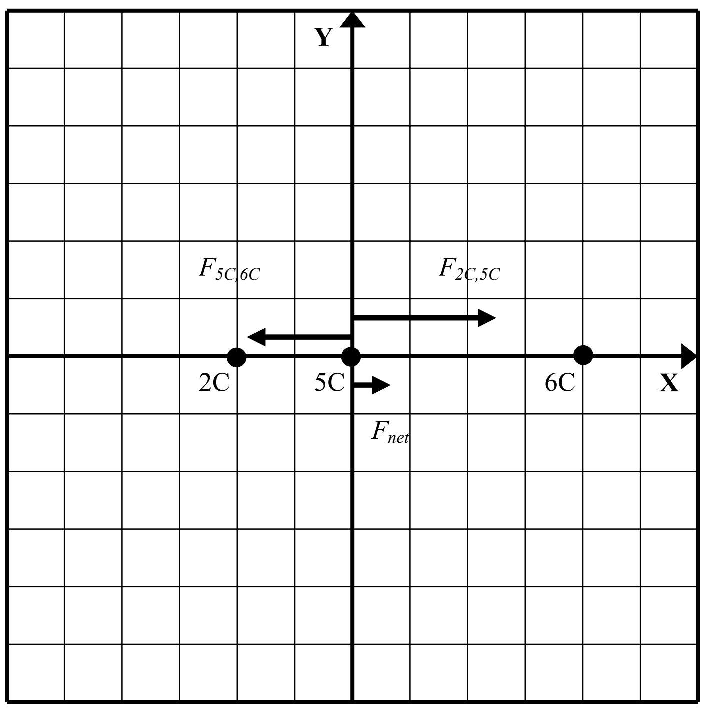
$$F_{grav} = G\frac{m_1 m_2}{r^2} = (6.67\times10^{-11}\ Nm^2/kg^2)\cdot\frac{(1.67\times10^{-27}\ kg)^2}{(2m)^2} = 4.67\times10^{-65}\ N$$

What is the ratio of the magnitude of the electrical force acting between them compared to the magnitude of the gravitational force?

$$\frac{F_{elec}}{F_{grav}} = \frac{5.76\times10^{-29}\ N}{4.67\times10^{-65}\ N} = 1.23\times10^{36}$$

4. Three negative charges are sitting on the x axis, as shown in the figure below. A 2-C charge is at x = -2 m; a 5-C charge is at the origin, and a 6-C charge is at x = 4m.

In black, draw and label two arrows representing the electrical forces on the charge at the origin due to the other two charges. Label the force due to the 2-C charge "$F_2$", and the other one "$F_6$". In red, draw an arrow representing the *net* electrical force on the charge at the origin. Make sure the lengths of the arrows correspond to the relative magnitudes of the forces.




Let '*r*' be the distance of one grid square along the x or y axis.

$$F_{5C,6C} = k\frac{(5C)(6C)}{(4r)^2} = \frac{30}{16}\cdot\frac{k\cdot C^2}{r^2} = \frac{15}{8}\cdot\frac{k\cdot C^2}{r^2} = (1.875)\cdot\frac{k\cdot C^2}{r^2}$$

$$F_{2C,5C} = k\frac{(2C)(5C)}{(2r)^2} = \frac{10}{4}\cdot\frac{k\cdot C^2}{r^2} = \frac{5}{2}\cdot\frac{k\cdot C^2}{r^2} = (2.5)\cdot\frac{k\cdot C^2}{r^2}$$

Since all are like charges, the forces will repel and the net force will be the ***difference*** of the two forces and point in the same direction as the stronger force.

5. A 6-C charge and a particle with a 9-C charge are separated by 3 m. Suppose they are now moved to a separation of 1 m; the repulsive force on the 6-C charge will now be different from the original value. The force on the 6-C charge will go back to the original value if the charge on the other particle is changed to what value?

$$F_{original} = k\frac{(6C)(9C)}{(3m)^2} = 6\frac{k\cdot C^2}{m^2}$$

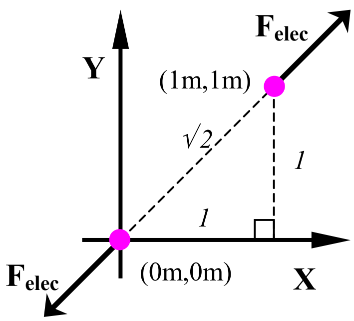
$$F_{new} = k\frac{(6C)q_?}{(1m)^2} = 6\cdot q_?\cdot\frac{k\cdot C}{m^2} \qquad F_{original} = F_{new}$$

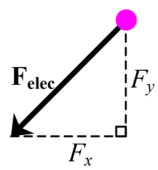
$$\therefore 6\frac{k\cdot C^2}{m^2} = 6q_?\frac{k\cdot C}{m^2}$$

$$\therefore q_? = 1C$$

New value of 9-C charge = ___1 C___

6. A 1-C charge is located at the origin and another 1-C charge is located at the point (1m, 1m). What are the x and y components of the electrical force ($F_x$ and $F_y$) on the charge at the origin?




First it is necessary to solve for **F<sub></sub>**

First it is necessary to solve for $\mathbf{F_{elec}}$ :

$$F_{elec} = k\frac{q_1q_2}{r^2} = k\frac{C^2}{2} = 4.5\times10^9\,N$$

But $F_x$ and $F_y$ are ***not*** merely one-half of the total electrical force. Instead they are related by a *one-one-square root of two* triangle with the total electric force just as the x and y components are related to the hypotenuse of the triangle by *Pythagoras's* theorem.

$$a^2 + b^2 = c^2$$

The angles involved are $45^o$,$45^o$, and $90^o$ in the triangle. Because of this we know that $F_x$ and $F_y$ will be equal in magnitude. To figure out exactly how large they are, we can use the trigonometric relation that the ***sine*** of an angle is equal to the opposite side divided by the hypotenuse which is just $\mathbf{F_{elec}}$ in this case.

$$\sin(45^o) = \frac{1}{\sqrt{2}} = \frac{F_y}{F_{elec}}$$

$$\therefore F_x = F_y = 3.18\times10^9\,N$$