- 1. See sheets.
- 2. See sheets.
- 3. If a positive test charge (for example) is placed at rest near q_1 or q_2 and then allowed to move freely, the test charge will accelerate due to the electrical force. Therefore, its *KE* will increase and its *PE* will decrease ($\Delta TE = 0$). If a positive charge goes toward *lower PE*, then it is also going toward lower *V* (potential), since V = PE/q. In this case, we can see that the test charges will be moving *away* from both q_1 and q_2 (i.e., moving toward lower *V*), so there must be a repulsive force, and therefore q_1 and q_2 are both *positive*.
- 4. For a test charge q_{test} , we know that its potential energy near q_1 is given by $PE = kq_1 q_{test} / r$, so we can see that the *potential* V that it experiences near q_1 is given by

$$V_{q_1} = \frac{PE(q_{test})}{q_{test}} = \frac{kq_1}{r} \quad [potential near q_1]$$

For q_2 we will have

$$V_{q_2} = \frac{kq_2}{r}$$
 [potential near q_2]

If we compare point D (3 meters away from q_1) and point F (6 meters away from q_2), we see that the potential is 1.0 volt at *both* of these points. This means that

$$1.0 \ volt = \frac{kq_2}{6} = \frac{kq_1}{3}$$

And from this we can figure out that $q_2 = 2 q_1$.

- 5. As shown in #4, $q_1 / q_2 = 0.5$
- 6. We can read off from the diagram the following values:

 $V_A > 2.0 \text{ V}$ $V_B = 1.0 \text{ V}$ $V_C = 2.0 \text{ V}$ $V_D = 1.0 \text{ V}$ $V_E \approx 0.6 \text{ V} \text{ [approximately]}$ $V_F = 1.0 \text{ V}$ $V_G \approx 0.75 \text{ V} \text{ [approximately, but certainly larger than } V_E \text{)}$

From this list, we can rank them as follows: A > C > B = D = F > G > E

[A, C, B = D = F, G, E]

7. The magnitude of the electric field in the vicinity of an isolated point charge Q (such as q_1 and q_2) is given by the equation $E = kQ/r^2$. Using this formula, and our result from #5 that $q_2 = 2 q_1$, we can determine the value of E at all of the given points, in terms of k and q_1 , as follows:

$$E_{A} = k q_{2} / (2.5 \text{ m})^{2} = k q_{2} / (6.25 \text{ m}^{2}) = k (2q_{1}) / (6.25 \text{ m}^{2}) = k q_{1} / (3.125 \text{ m}^{2})$$

$$E_{B} = k q_{2} / (6 \text{ m})^{2} = k q_{2} / (36 \text{ m}^{2}) = k (2q_{1}) / (36 \text{ m}^{2}) = k q_{1} / (18 \text{ m}^{2})$$

$$E_{C} = k q_{2} / (3 \text{ m})^{2} = k q_{2} / (9 \text{ m}^{2}) = k (2q_{1}) / (9 \text{ m}^{2}) = k q_{1} / (4.5 \text{ m}^{2})$$

$$E_{D} = k q_{1} / (3 \text{ m})^{2} = k q_{1} / (9 \text{ m}^{2})$$

$$E_{E} \approx k q_{1} / (5.5 \text{m})^{2} = k q_{1} / (30.25 \text{m}^{2}) [\text{approximately}]$$

$$E_{F} = k q_{2} / (6 \text{ m})^{2} = k q_{2} / (36 \text{ m}^{2}) = k (2q_{1}) / (36 \text{ m}^{2}) = k q_{1} / (18 \text{ m}^{2})$$

$$E_{G} \approx k q_{1} / (4.6 \text{m})^{2} = k q_{1} / (21.16 \text{m}^{2}) [\text{approximately}]$$
This now gives the following ranking: $E_{A} > E_{C} > E_{D} > E_{B} = E_{F} > E_{G} > E_{E}$

$$[A, C, D, B = F, G, E]$$

8. From the list in #6, we can determine the following:

A > (2.0 V - 1.0 V) > 1.0 V B = (1.0 V - 1.0 V) = 0 V C = |1.0 V - 2.0 V| = 1.0 V $D = (1.0 V - 0.6 V) \approx 0.4 V$ E = (1.0 V - 1.0 V) = 0 V

This gives the following ranking: A > C > D > B = E[A, C, D, B = E]

9. Since only the electrical force is acting in this case, the total energy *TE* is constant, so $\Delta TE = 0$, which means that $\Delta KE + \Delta PE = 0$. The protons are all the same mass, so whichever acquires the most kinetic energy will be the one with the fastest speed (since $KE = \frac{1}{2} mv^2$). Since $\Delta KE = -\Delta PE$, the proton that loses the largest amount of potential energy will acquire the largest amount of kinetic energy. Now, since $\Delta PE = \frac{\Delta V}{q}$, and these are all positive charges of the same magnitude, the largest loss of potential energy will be associated with the largest loss of *potential*. Then all we have to do is find out which proton experiences the largest decrease in potential.

Proton A goes from point A to point B, and so experiences a change in potential from ≈ 2.5 V to 1.0 V; therefore its decrease in potential is approximately 1.5 V.

Proton B goes from point D to point E; its potential goes from 1.0 V to approximately 0.6 V, so its decrease in potential is approximately 0.4 V.

Proton C goes from point D out to a point 100 km from the origin. That far away from the origin, the potential will be very close to 0 V, so this proton experiences a decrease in potential of nearly 1.0 V.

From this, we can rank the final speeds of the protons as follows: A > C > B.

10. In this situation, the kinetic energy of the protons does not change. However, external work is being done by a nonconservative force (you, pushing). Since $W_{nonconservative} = \Delta TE$, in this case the amount of work you have to do is equal to the *increase* in potential energy, ΔPE (because here, $\Delta KE = 0$). For the same reasons as discussed in #9, the largest increase in potential energy will be associated with the largest increase in *potential*. Therefore, the *most* work will have to be done on the proton that experiences the largest increase in *potential*, which is proton A. Again, the correct ranking will be A > C > B.