The Physics Education Research Section PERS publishes articles describing important results from the field of physics education research. Manuscripts should be submitted using the web-based system that can be accessed via the American Journal of Physics home page, http://ajp.dickinson.edu, and will be forwarded to the PERS editor for consideration.

Student understanding of calorimetry in introductory calculus-based physics

Warren M. Christensen[a)]
Department of Physics, North Dakota State University, Fargo, North Dakota 58108

David E. Meltzer[b)]
Mary Lou Fulton Teachers College, Arizona State University, 7271 E. Sonoran Arroyo Mall, Mesa, Arizona 85212

Ngoc-Loan Nguyen[c)]
Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011

(Received 27 November 2010; accepted 2 August 2011)

We report on students' thinking regarding calorimetry concepts in an introductory calculus-based physics course. We found that despite overall good performance, only about half of the students were able to provide correct answers with satisfactory explanations. A number of persistent student difficulties were found to affect approximately 40% of the students even after instruction, including apparent confusion about the meaning of specific heat and misunderstanding of the nature of thermal energy exchange. Student response patterns varied significantly depending on the context of the question and often reasoning did not appear to be consistent among contexts, instead favoring "rule-based" reasoning. Interviews with students suggest that difficulty with algebraic manipulations is a significant contributor to incorrect responses on calorimetry questions.

[DOI: 10.1119/1.3630936]

I. INTRODUCTION

Calorimetry is often one of the first topics discussed in the introductory physics course after basic concepts of heating and temperature are introduced. It is considered to be relatively easy because calorimetry problems require only the straightforward application of a few fundamental concepts along with basic algebraic skills. Students are expected to apply relations involving specific heat (such as $Q = mc\Delta T$) to various problems involving temperature changes of materials in thermal contact. The definition of specific heat does not vary significantly from textbook to textbook, and is most often discussed in a fashion similar to "You can think of specific heat as the heat transfer to one kilogram of the material needed to raise its temperature by one Kelvin."[1] Nonetheless, the topic of calorimetry depends on concepts such as thermal equilibrium and heat transfer, concepts that have been found challenging in previous studies of students' thinking. In view of those conceptual challenges and the central role calorimetry often plays in introductory discussions of thermal physics, investigation of students' thinking in this area merits a focused investigation.

II. PREVIOUS RESEARCH ON STUDENT LEARNING OF CALORIMETRY CONCEPTS

There are numerous studies reporting on the difficulties that students encounter with the concepts of heat and temperature.[2–6] Some of these studies have been in the context of thermochemistry.[7–12] For example, Greenbowe and Meltzer[13] investigated student thinking regarding calorimetry concepts in the context of solution chemistry as studied in introductory chemistry courses. In addition to previously reported student learning difficulties regarding the relation between heat and temperature (such as those cited in Refs. 2–6), they found significant misunderstandings on the role of chemical reactions in the heating process. Jasien and Oberem[14] investigated calorimetry-related ideas among various groups of college students who had diverse backgrounds in the physical sciences. They reported student difficulties with concepts involving thermal equilibrium, heat capacity, and specific heat, with no significant correlation between the observed difficulties and the number of physical science courses that the students had taken.

The most wide-ranging investigation of student thinking about calorimetry in a physics context is that of Cochran at the University of Washington.[15] Consistent with the findings of other investigators,[9,13,14,16,17] it was found that many students have much difficulty distinguishing between heat and temperature. This difficulty may impair their ability to understand other thermal concepts, such as recognizing that objects in thermal equilibrium with each other are at the same temperature. It was also found that students occasionally focused on rates of heat transfer or of temperature change when it was not appropriate to do so,[18] and that students often incorrectly treated the amount of heat transfer as being dependent on only a single property in the interaction. For example, the change in temperature of a hot copper block in water was thought to be due solely to the specific heat or initial temperature of the block, ignoring the role of the block's mass.

III. OBJECTIVES OF THE INVESTIGATION

In this section we discuss the specific concepts that were the targets of our investigation. We also discuss the questions we used to assess students' understanding of these concepts.

A. Concepts targeted for assessment

(1) When two objects in thermal contact are in an insulated container, the magnitude of the heat transfer from one object is equal to the magnitude of the heat transfer to the other object.

(2) The specific heat is the amount of energy per unit mass required to change the temperature of an object.

The concept of specific heat as well as recognition of the consequences of energy conservation are fundamental to an understanding of calorimetry. In the context of introductory physics the complication of heating due to chemical reactions is not present; if no phase transitions occur, the solution of calorimetry problems can be accomplished using straightforward algebra.

B. Assessment questions

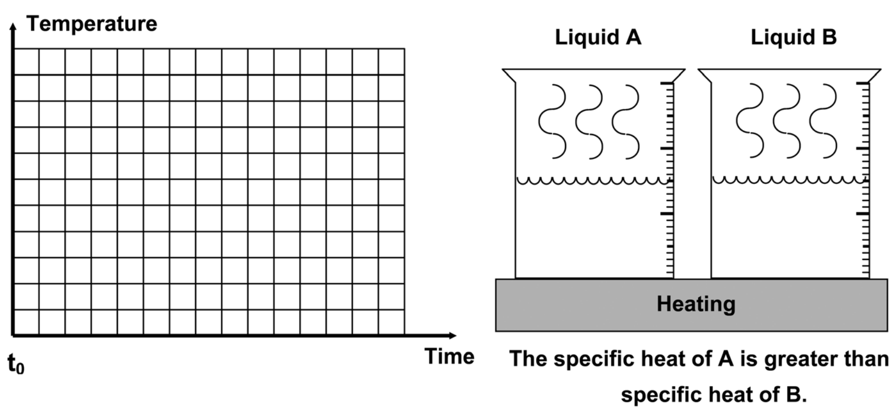
Our investigation focused on student responses to two related questions that were each administered in multiple formats. The "object in liquid" question (see Fig. 1) and the "two-liquid" question (see Fig. 2) were used to probe student thinking regarding substances of equal masses but different specific heats. Both questions require students to apply the idea that energy is conserved during the process, and that energy transfer to a substance changes its temperature by an amount that depends on its specific heat. In the object-in-liquid question, an object of specified material is immersed in a specified liquid with a mass equal to that of the object but with differing specific heat and initial temperature. Students are asked to determine whether the object or liquid will undergo the largest change in temperature. The answer is that whichever has the lower specific heat will undergo the largest temperature change. In the two-liquid question, two liquids in separate containers have identical initial temperatures but different specific heats; they receive the same rate of heating from a hot plate. Students are asked to graph the temperature versus time of each liquid on a common set of axes. A correct answer associates a line of larger slope with the liquid having the lower specific heat.

The two-liquid question involves common initial temperatures and equal rates of heating, and the object-in-liquid

> The specific heat of water is greater than that of copper.
>
> A piece of copper metal is put into an insulated calorimeter which is nearly filled with water. The mass of the copper is the same as the mass of the water, but the initial temperature of the copper is higher than the initial temperature of the water. The calorimeter is left alone for several hours.
>
> During the time it takes for the system to reach equilibrium, will the temperature change (number of degrees Celsius) of the copper be *more than*, *less than*, or *equal to* the temperature change of the water? Please explain your answer.

Fig. 1. Object in liquid, free-response question. Four versions of this question were used, varying both the pair of substances (water and copper or bromoform and aluminum) and the identity of the substance that was specified to have the higher initial temperature.

> Suppose we have two ***separate*** containers: One container holds Liquid A, and another contains Liquid B. The mass and initial temperature of the two liquids are the same, but the ***specific heat*** of Liquid A is ***two times*** that of Liquid B. Each container is placed on a heating plate that delivers the ***same rate of heating*** in joules per second to each liquid beginning at initial time $\mathbf{t_0}$.
>
> a) On the grid below, graph the temperature as a function of time for *each* liquid, A and B. Use a separate line for each liquid, even if they overlap. Make sure to clearly label your lines, and use proper graphing techniques.



> b) Please **explain** the reasoning that you used in drawing your graph.

Fig. 2. Two-liquid, free-response question. Three versions of this question were used in which the ratio of specific heats, c_A/c_B, was specified as being 2, 3, or 4.

question involves two substances at different initial temperatures in an insulated container;[19] the two-liquid question includes a graphical component which is absent from the other question.[20] Both questions require application of simple algebra or proportional reasoning. The questions were administered before all instruction, after lecture instruction but before recitation instruction, and after all instruction was completed. Both questions were administered simultaneously on a single sheet of paper unless otherwise noted.

Most of this study was conducted with students in the second semester of a year-long calculus-based introductory physics course at Iowa State University. This sequence usually enrolls 700–800 students per calendar year. Most of the students are engineering majors, but a few physics majors and computer science majors are included. The semester-to-semester variations in course content are usually minor; some instructors cover certain topics in greater depth than do other instructors, but the overall list of topics is stable. During this study, the first semester usually covered mechanics and electric fields, and the second semester covered magnetism, ac circuits, waves, fluids, and thermal physics.

IV. RESULTS OF ASSESSMENTS

A. Before all instruction

We administered the object-in-liquid question prior to all instruction on thermodynamics to all students attending the first week of recitation in Fall 2005. (In other semesters this question was almost always used after lecture instruction but before recitation instruction, and frequently was coupled with a separate calorimetry question.) We found that before any instruction, students' previous exposure to this material was evident; 50% of the 479 students answered correctly that the substance with the smaller specific heat would have a greater temperature change than the substance with the larger specific heat. Moreover, 80% of those who gave a correct answer provided acceptable explanations (see the following section for sample explanations).[21]

B. After lecture instruction

After lecture instruction, 63% of students correctly answered the object-in-liquid question by writing that the substance with the smaller specific heat would have a greater temperature change than the substance with the larger specific heat ($N = 1036$ over three semesters; see Tables I and II). [In Fall 2005 this question was administered both before all instruction and after lecture instruction.] Students giving a correct answer with a correct explanation (53% of all students) relied on the equation $Q = mc\Delta T$ or the definition of specific heat to explain their answer. A representative explanation is that "Object A will change less than liquid B because the specific heat of object A is greater so it takes more heat to change its temperature by one degree."

Nearly one quarter of all students (22%) stated that the temperature change of the object and the liquid would be the same. Explanations for this response include the idea that equal energy transfer is assumed to imply equal temperature change. For example, "The temperature change of the copper and the water will be the same. Any heat lost by the copper will be gained by the water, or any heat gained by the copper will be lost from the water. So ΔT of both is the same."

The remaining 18% answered that the substance with the smaller specific heat would have a smaller temperature change than the substance with the greater specific heat. Most students offering this response stated that the temperature change was proportional to the specific heat. For instance, "The temperature change of copper will be less than that of the ΔT of the water [*sic*], because the specific heat of water is greater, and the masses are the same."

Approximately one third of those students who said that the temperature changes for the object and the liquid would be equal justified their answer by stating that the object and the liquid go to "equilibrium." Although the definition of this term was explicitly discussed in the course texts and, presumably, in the lectures, we were uncertain of the meaning attributed to it by the students who used it in their explanations. Reports in the literature have suggested significant difficulties with this concept (see, for example, Ref. 14). To address this issue and to minimize potential confusion, we changed the wording of the object-in-liquid question for the Fall 2005 and Spring 2006 courses. "During the time it takes for the system to reach equilibrium…" was changed to "During the time it takes for the object and the liquid to reach a common final temperature…" This re-wording affected only the manner in which students expressed their explanations, and did not lead to any significant changes in responses for the different answer options (see Table I). Students didn't use the term equilibrium in their explanations as frequently as they had done with the original form of the question. Instead, some used the term "common final temperature" to justify their responses that the temperature changes of the object and liquid would be equal.

Because obtaining a correct answer to the two-liquid question depends, to some extent, on students' ability to properly graph two lines, we decided to accept as correct any response that showed the slope of line B (representing the substance with smaller specific heat) as being greater than that of line A. Interview data supported this criterion because, although many students initially failed to draw graphs that reflected a

Table I. Object in liquid, free-response question; after lecture instruction. "Greater c, smaller ΔT" corresponds to the correct response that the substance with the greater specific heat would have a smaller change in temperature, and "greater c, greater ΔT" associates that same substance with the greater change in temperature. The statistics in the All Semesters column represent the 95% confidence interval of student performance for each answer category, based on score variances among the three semesters.

	Spring 2003 $N = 359$	Fall 2005 $N = 427$	Spring 2006[a] $N = 250$	All Semesters $N = 1036$
Greater c, smaller ΔT	64%	61%	64%	63 ± 4%
Correct with correct explanation	*55%*	*51%*	*53%*	*53 ± 5%*
Equal ΔT	21%	25%	20%	22 ± 7%
Greater c, greater ΔT	15%	14%	16%	15 ± 2%

[a]In Spring 2006, the object-in-liquid question was administered without the two-liquid question.

quantitatively accurate ratio of the slopes, they were able to recognize this defect and almost always were able to correct it when pressed to do so.

After lecture instruction, using the "B-slope > A-slope" criterion for correctness, we found that 72% of the students gave correct responses ($N = 788$ over two semesters; see Table III); 58% of the students gave a correct explanation along with the correct response; 27% stated that the slope of B would be less than the slope of A, and there were almost no students who answered that the slope of the two liquids would be the same, despite the fact that 22% had given an analogous answer on the object-in-liquid question (see Table I).

We have no convincing explanation for the sharp disparity in proportions of students who chose the "temperature changes are equal" response on the two questions.

We tracked correlations of student responses between the two questions to determine the consistency of student thinking[22] and found that 82% of those students who answered the object-in-liquid question correctly also answered the two-liquid question correctly, and only 15% selected an incorrect answer of "greater c, greater ΔT" on the two-liquid question.

Students who stated that the temperature changes were equal for the object-in-liquid question split their answers on the two-liquid question between the correct answer "greater c, smaller ΔT" (48%) and the incorrect answer "greater c, greater ΔT" (45%). None of these students offered an answer on the second question that was consistent with their incorrect answer on the first question.

Similarly, students who gave an incorrect answer consistent with "greater c, greater ΔT" on the object-in-liquid question split their answers almost evenly between the correct answer "greater c, smaller ΔT" (51%) and the incorrect answer "greater c, greater ΔT" (47%) on the two-liquid question; none of them gave an "equal ΔT" response.

These findings suggest that students employ reasoning that is strongly context-dependent. We call this reasoning "rule-based" because students typically justify their answers by citing one or more "rules" which they tend to employ instead of trying to arrive at an answer by reasoning from basic principles (see Sec. V).

C. Post-instruction results

We probed student thinking after all instruction was finished during the summer offering of the same course during 2002. We administered the two questions to all 32 students during one of the recitation sessions; the responses[23] were consistent with those obtained after lecture but before recitation instruction in subsequent offerings of the same course (as reflected in Tables II and III).

We created a multiple-choice equivalent of the two-liquid question (see Fig. 3), and administered it on a midterm exam after all instruction was complete in Spring 2004. Due to the subtle difference between choices A and B on this question, we group the responses by combining all those students who gave an answer that was consistent with greater specific heat corresponding to a smaller change in temperature; this combination is the sum of those who answered either A or B. Similarly, we categorized both C and D responses as being incorrect under the common heading of "greater specific heat implies greater temperature change." Response E corresponds to equal temperature change.[24]

Table II. Object in liquid, free-response question; after lecture instruction. This is a breakdown of the responses given in Table I. Some of the students justified an "equal ΔT" response by claiming that the system goes either to "equilibrium" or to a "common final temperature."

		All Semesters (3 samples) $N = 1036$	
Correct (greater c, smaller ΔT)	**63%**		
correct explanation		53%	
incorrect explanation		10%	
initial temperature higher (or lower)			*2%*
other explanations			*7%*
Incorrect (equal ΔT)	**22%**		
energy transfers are equal			*5%*
equilibrium/common final temperature			*7%*
masses are equal			*4%*
other explanations			*6%*
Incorrect (greater c, greater ΔT)	**15%**		
specific heat proportional to ΔT			*5%*
"correct" explanation, incorrect answer			*1%*
other explanations			*8%*

This grouping of answers on the multiple-choice version of the two-liquid question yields a response pattern that is almost identical to that obtained on the free-response version of the same question given after lecture instruction, across all three answer categories, as well as being similar to the response pattern obtained on the free-response version after all instruction during summer 2002.[25] As before, responses that are consistent with the liquids having equal changes in temperature were non-existent, although such answers were often given in the context of the object-in-liquid question.

A further follow-up using the object-in-liquid question in a multiple-choice format ["text multiple choice," Fig. 4(a)] was done in spring of 2003 and 2004; the wording of this version was similar to that of the free-response version of this same question. Another version of this multiple-choice question, which uses common mathematical symbols instead of text ["symbol multiple choice," Fig. 4(b)], was administered in spring of 2004 and 2006.[26,27] The text version was administered on the midterm exam during 2004, and the symbol version on the final exam during the same semester; only minor differences in responses were found.[28] Despite all instruction having been completed (including lecture, recitation, homework, and exam preparation), student performance on both multiple-choice versions of this question is

Table III. Two-liquid question, free response; after lecture instruction. "Greater c, smaller ΔT" is the correct response that the liquid with the greater specific heat corresponds to a smaller slope on the temperature-time graph (corresponding to a smaller rate of temperature change). An "equal ΔT" response refers to graphs with equal slopes for both liquids; "greater c, greater ΔT" associates the liquid having the greater specific heat with the larger slope on the graph.

	Spring 2003 $N = 361$	Fall 2005 $N = 427$
Greater c, smaller ΔT	70%	73%
Correct with correct explanation	*50%*	*65%*
Equal ΔT	0%	0%
Greater c, greater ΔT	28%	26%

Suppose we have two *separate* containers: One container holds Liquid A, and another contains Liquid B. The mass and initial temperature of the two liquids are the same, but the specific heat of Liquid A is *four times* that of Liquid B. Each container is placed on a heating plate that delivers the same rate of heating in joules per second to each liquid beginning at initial time t_0.

On the grids below are four graphs that represent the temperature-versus-time plots for liquid A and liquid B, with liquid A represented by a solid line and liquid B by a dashed line. Indicate the graph whose temperatures are plotted most accurately for liquid A versus liquid B.

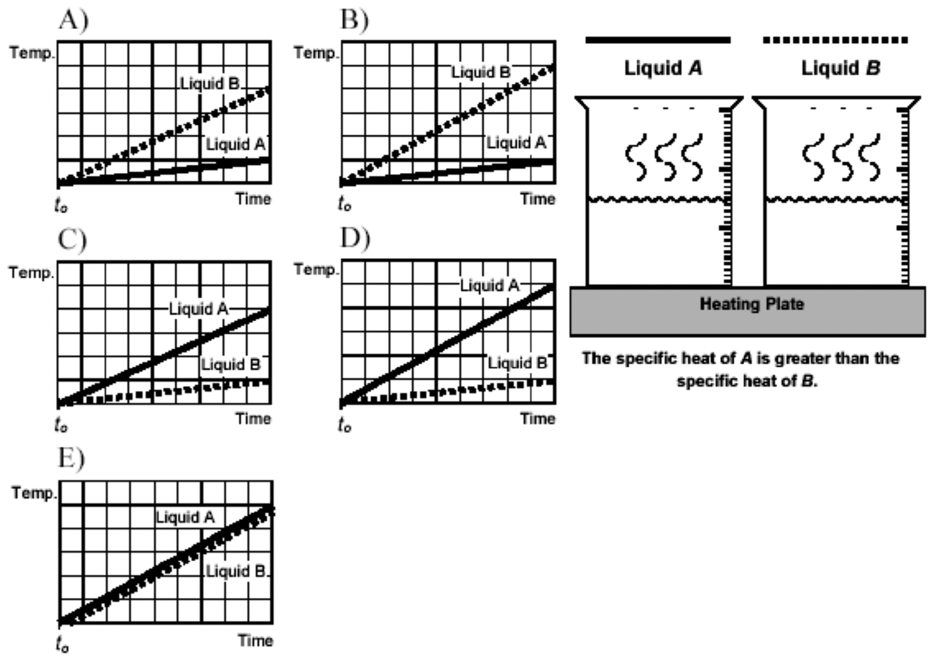


Fig. 3. Two-liquid question, multiple-choice version.

similar to that on the free-response version that had been given as a quiz after lecture instruction only, suggesting that little to no learning occurred on these concepts as a result of the additional instructional activities following the lecture. The proportion of correct responses on all multiple-choice versions of the object-in-liquid question after instruction (68%, $N = 1491$ over four samples)[29] is consistent with that on the corresponding free-response version of that question (63%, $N = 1036$ over three samples) which was given after lecture instruction but before recitation (see Table IV). However, on the multiple-choice questions, the "equal ΔT" response rate of 11% was less than on the free-response question (22%). In contrast, the "greater c, greater ΔT" response was slightly more popular than it was on the free-response question (22% compared to 15%). These differences might be due in part to the order in which the answer options appear in the question (lower in the order might correspond to lower probability of response); however, we have no direct evidence that this effect occurred in our case.

D. Interview data

We conducted 26 one-on-one student interviews during three semesters. We do not have data that would allow us to compare the performance of these students to that of the other students enrolled in the course. In previous projects involving self-selected interview volunteers from the same or similar student populations taking this same course, it was found that interview volunteers performed above the average of all enrolled students.[30]

During the interviews, students were asked the questions described in Sec. III B, and related questions on energy, temperature, and specific heat. The object-in-liquid and two-liquid questions were among the first given during the interview. Students were asked to work through the problems and explain their thinking along the way, and clarifying questions were asked when appropriate. An example of a clarifying question was whether the slopes of the graphs drawn by the student in the two-liquid question were precisely matched to the quantities stated in the problem. This question often led to students admitting that they had just sketched the graph without attempting to achieve quantitative precision with their slopes. Apart from this sort of clarifying question, there were no explicit cues given by the interviewers; for example, at no point were students asked to use mathematics in their solution. The first problem was the object-in-liquid question, which does not require calculations to obtain a correct answer.

The perspective that emerged from the interviews was substantially different and more nuanced than what we

(a)

An object is immersed in a liquid within a sealed and insulated container. The mass of the object is the same as the mass of the liquid. The initial temperature of the object is lower than the initial temperature of the liquid, but the specific heat of the object is ***greater*** than that of the liquid. The calorimeter is left alone for several hours until it reaches equilibrium. Which of the following is true? *Note: Here, "temperature change" means "number of degrees Kelvin increased **or** decreased."*

A. The energy transfer to the object is *not* equal to the energy transfer away from the liquid, and the temperature change of the object is greater than the temperature change of the liquid.

B. The energy transfer to the object is *not* equal to the energy transfer away from the liquid, and the temperature change of the object is less than the temperature change of the liquid.

C. The energy transfer to the object is equal to the energy transfer away from the liquid, and the temperature change of the object is greater than the temperature change of the liquid.

D. The energy transfer to the object is equal to the energy transfer away from the liquid, and the temperature change of the object is equal to the temperature change of the liquid.

E. The energy transfer to the object is equal to the energy transfer away from the liquid, and the temperature change of the object is less than the temperature change of the liquid.

(b)

Object A has mass m_A, specific heat c_A, and the initial temperature $T_{initial\ A}$. Liquid B has mass m_B, specific heat c_B, and initial temperature $T_{initial\ B}$. Object A is immersed in Liquid B within a sealed and insulated container (i.e., a calorimeter). We are given the following information:

$$m_A = m_B$$
$$c_A > c_B$$

$T_{initial\ A} < T_{initial\ B}$ but after a long time, $T_{final\ A} = T_{final\ B}$

Which of the following is true? [Q is heat transfer; $\Delta T \equiv T_{final} - T_{initial}$]

A. $Q_{to\ A} \neq Q_{away\ from\ B}$; $|\Delta T_A| > |\Delta T_B|$

B. $Q_{to\ A} \neq Q_{away\ from\ B}$; $|\Delta T_A| < |\Delta T_B|$

C. $Q_{to\ A} = Q_{away\ from\ B}$; $|\Delta T_A| > |\Delta T_B|$

D. $Q_{to\ A} = Q_{away\ from\ B}$; $|\Delta T_A| = |\Delta T_B|$

E. $Q_{to\ A} = Q_{away\ from\ B}$; $|\Delta T_A| < |\Delta T_B|$

Fig. 4. Object-in-liquid question, multiple-choice versions: (a) text-based; (b) symbol-based.

gained from the written free-response diagnostics. Student thinking on calorimetry was, in many cases, much more tentative, insecure, and based on "rules of thumb" (rather than on reasoning from specific principles, correct or incorrect) than was evident from the written data. On the one hand, approximately half of the students had a good understanding

Table IV. Comparison of three versions of object-in-liquid question: free response, text multiple-choice, and symbol multiple-choice. The free-response version was given after lecture instruction only, and the multiple-choice versions were given after all instruction was complete. "Greater c, smaller ΔT" corresponds to the sum of answers B and E on the multiple-choice versions, "Equal ΔT" corresponds to answer D, and "Greater c, greater ΔT" corresponds to the sum of answers A and C. "Heat transfers are not equal" corresponds to the sum of answers A and B, and "heat transfers are equal" corresponds the sum of answers C, D, and E (see Ref. 21, Appendix V).

	Object in liquid free response (three samples) $N = 1036$	Object in liquid text MC (two samples) $N = 760$	Object in liquid symbol MC (two samples) $N = 731$
Greater c, smaller ΔT	63% ± 4%	66%	70%
Equal ΔT	22% ± 7%	13%	8%
Greater c, greater ΔT	15% ± 2%	22%	23%
Heat transfers are not equal	—	*21%*	*16%*
Heat transfers are equal	—	*79%*	*85%*

of the relevant principles and were able to quickly and confidently solve problems in diverse contexts. On the other hand, a substantial proportion of students did not seem able, and often did not attempt, to solve the problems by reasoning from basic principles. Instead, they would either begin immediately to try and solve problems algebraically, or would give quick responses based on intuitive "rules" or impressions such as "it would depend on the initial temperatures," "larger specific heat means greater temperature change," or "larger specific heat means greater energy dissipation." They frequently were led astray by lengthy, unproductive calculations resulting in errors that they could not resolve. Most of them went into long explanations of their thinking that led them to change their answers, sometimes to more correct, but often to less correct responses. They often had great difficulty in employing consistent reasoning in different contexts and resolving discrepancies, even when prompted to do so by the interviewer.

Approximately 85% of the interview subjects were successful with the two-liquid question, although the relative slopes of their graphs were often quantitatively inaccurate. They realized the liquids would heat at different rates and knew or guessed that the one with lower specific heat would have the most rapid rise in temperature. In contrast, on the object-in-liquid question, only around 60% of the interview subjects gave a correct response accompanied by an acceptable explanation. There were two distinct tendencies associated with incorrect answers, and neither matched common patterns observed on the written responses. About 15% of all interviewees argued that the initial temperature would affect the magnitude of the temperature change, a higher percentage than the 5% or less observed in the free-response data. The other notable student difficulty observed during the interviews was the number of mathematical errors: about 20% of the interview subjects made algebraic errors that prevented them from obtaining a correct answer. For instance, while answering the object-in-liquid (free-response) question, some students would set up a correct algebraic expression equating heat transfers between the object and the liquid. After obtaining a correct expression that related the magnitudes of the temperature changes to the specific heats, students would incorrectly interpret the proportional relation as implying that higher specific heat was associated with a larger change in temperature.

Aside from the two errors we have already identified, no other type of incorrect reasoning had more than a single adherent among the students in the interview sample. For example, only one student said that the temperature changes would be equal on the object-in-liquid question, and this error was the outcome of a string of algebraic mistakes.

Only 1%–3% of all written responses contained clear evidence of algebraic errors. Algebraic arguments were not often employed even to justify correct responses: only around 15% of all written responses relied on explicit calculations. In contrast, during the interviews it was common for students to begin a correct explanation by writing equations; half or more of the students giving correct responses followed such a procedure. However, all of them proceeded to provide a correct qualitative argument linking temperature change to specific heat. It seems that the greater algebra use among the interviewees may be related to the extra care they were taking in justifying their responses, in comparison to students responding to the written questions. Ironically, it might be that this extra care led some students in the interview sample astray, drawing them into reliance on equation-based reasoning that proceeded to go off track due to algebraic errors. This intention to be "extra-careful" might be why mathematical errors were observed more frequently among the interview sample.

It is plausible that algebraic difficulties might also have been reflected, in part, in the incorrect "greater c, greater ΔT" responses given by students on the written diagnostic. Such an outcome would be consistent with research on the relation between mathematics skill and physics performance[31] which suggests that weak algebra skills might be associated with student difficulties on questions requiring simple proportional reasoning, such as those in calorimetry. The mathematical errors that arose during the interviews consistently interfered with students' ability to solve the problems correctly even when other intuitive reasoning approaches eventually allowed them to arrive at a correct answer. Previous work has examined apparently analogous correlations between students' algebraic skills and their performance on qualitative physics questions.[31] These and other results suggest that errors on simple algebraic operations can decrease the performance of a substantial fraction of students, even on a topic involving simple mathematics.[32] However, we have no direct evidence for algebraic difficulties playing a significant role in the written sample data, in contrast to the interview sample.

V. DISCUSSION

A. Student reasoning about calorimetry

The two multiple-choice versions of the object-in-liquid question explicitly probed student thinking about the concept that magnitude of heat transfer from one object is equal to the magnitude of heat transfer to the other object. Responses from over 1000 students across three semesters indicate that between 12% and 25% believed that the magnitude of heat transfer to the colder substance was not equal to the magnitude of heat transfer away from the hotter substance (see Table IV).[29] Explanations of student reasoning were not required on these questions, and the issue was not explicitly probed during the interviews, so no further information is available.

We probed students' thinking about the relation of the specific heat to the amount of energy required to change the temperature of an object by asking them to compare temperature changes for two substances that have identical masses but different specific heats; the objects are assumed to be in thermal contact with each other or with a common energy source. About half of the students correctly stated that the substance with the larger specific heat would have a smaller temperature change and also provided adequate explanations for their responses. However, substantial numbers of students either failed to provide adequate explanations, or responded incorrectly regarding the temperature changes.

Some students answered that the temperature changes of the two substances would be equal. On all free-response versions of the object-in-liquid question, 20%–25% gave this response both after lecture instruction and after all instruction. Between 7% and 12% of students gave that response on the multiple-choice versions of the question after all instruction.[33] The most common justification for this response was either that the substances ended up with the same final temperature or that they reached "equilibrium," with the specific response depending on how the question was worded. Other popular explanations hinged on the equality of the substances' masses or that of the heat transfers.

Despite the fact that all versions of the object-in-liquid question yielded substantial numbers of "equal temperature change" answers, responses to the alternative-context two-liquid question yielded no answers that were consistent with an "equal temperature change" idea. This difference is a striking example of the well-known fact that questions based on identical physical principles but posed in different contexts can lead to very diverse student responses.[34–38]

The other common incorrect student response was to claim that the substance with higher specific heat would have the larger temperature change. The proportion of students who gave this response after instruction (out of nearly 1500 students) was in the range of 14%–28%, with only one low-enrollment summer class falling below this range. Responses were in this range regardless of whether the questions were administered after lecture instruction only, or after all instruction, and on both of our diagnostic questions (including both free-response and multiple-choice versions of these questions). Student explanations were straightforward assertions, with no supporting arguments, that larger specific heat implied a larger temperature change. The implication seemed to be that students had made this assumption either without critical examination or on the basis of persuasive, albeit faulty, intuitive thinking.

B. Relation to previous work

Our findings are consistent with the only other detailed study on related concepts.[15] For example, on a question similar to our object-in-liquid question, it was found that 70% of the students gave a correct answer, 20% responded that the temperature changes would be equal, and 10% stated that the temperature change of the substance with higher specific heat (the water) would be greater.[15] These results are similar to our own as reported in Table I.[39] Similarly, it was found that 10% of the students responded to this question by claiming that the heat transfer magnitudes would not be equal;[15] this response is similar to our findings on a similar question in which 12%–25% of students made a similar incorrect assertion.[25]

C. Rule-based reasoning

Various reports suggest that "rule-based reasoning" may originate in part from the perceptions that students have regarding their role in the classroom. "Rule-learners" have been described as students who view their primary task as memorizing rules and algorithms which are then practiced until they can be applied flawlessly.[40] Successful problem solvers may utilize a similar procedure, but more often include a step where they check the validity of their answer or evaluation method before reporting a final answer.[40]

The Maryland Physics Expectations Survey[41] and the Colorado Learning Attitudes about Science Survey[42] probed student expectations and attitudes about science and science learning. Both surveys found that a substantial number of students both before and after instruction believe that they must memorize all the information, and then simply find the right equation to solve a problem. This notion of needing to "determine the rule" often leads students to try to learn the material without bothering to search for any underlying conceptual framework or unifying ideas. Analogous behavior has been found in a variety of contexts.[43]

We have interpreted our findings within the framework of students' specific learning difficulties in a particular physical context, a model that has proven effective in improving physics instruction.[44] Alternative interpretations (for instance, involving student "resources") could be employed as well.[45] Some of the behaviors we observed can be interpreted as students making use of "phenomenological primitives,"[46] such as "more means more" or "more A-more B."[43] As plausible as such an interpretation might be, we have no direct evidence that students were employing a context-independent mode of reasoning that led to their incorrect responses.

Despite the fact that explanations of both correct and incorrect responses followed a similar pattern of reliance on brief, qualitative rules instead of on detailed reasoning, a majority of the students had an adequate grasp of the basic physics concepts. This conclusion is supported by consistently correct responses by a majority of students on written questions in diverse contexts, and by interviews that indicated that most students giving correct responses were capable of providing well-reasoned arguments when pressed to provide one.

VI. IMPLICATIONS FOR INSTRUCTION

Following a well-known model of physics instruction,[47] we sought to develop curricular materials that would explicitly address student difficulties in calorimetry. It would seem that exercises that guide students to recognize the interrelations among mass, specific heat, temperature change, and heat transfer would be essential. We sought to guide students to resolve inconsistencies in their answers, especially when using representations that often elicit inconsistent responses (such as the object-in-liquid and two-liquid questions); our goal was to prevent reliance on intuitive but faulty rule-based reasoning. It is also possible that efforts to directly improve students' facility with algebraic manipulations might result in improved performance on the calorimetry assessments, but we did not investigate this question directly. We are unable to report any consistent successes in improving student performance on the calorimetry questions discussed here. Apparent improvements observed among some of the experimental groups were not reproduced consistently among others, and the question of how best to improve student learning in calorimetry awaits resolution.[48]

ACKNOWLEDGMENTS

Major contributions to initiating, formulating, and executing this investigation were made by Ngoc-Loan Nguyen, who also wrote initial drafts of the manuscript and developed preliminary versions of research-based curricular materials before his untimely death in 2005. C. A. Ogilvie provided substantial guidance and support during the latter stages of this work. The authors acknowledge significant assistance from faculty members and teaching assistants in the Department of Physics and Astronomy at Iowa State University, and are grateful for the support of members of the Physics Education Research Laboratory at the University of Maine. We are also grateful for the detailed and thoughtful comments of two anonymous referees; they led to significant improvements in this paper. This work was supported in part by the National Science Foundation under DUE 9981140, PHY 0406724, PHY 0604703, and DUE 0817282.

[a]Electronic mail: warren.christensen@ndsu.edu
[b]Electronic mail: david.meltzer@asu.edu
[c]Deceased, 2005.

[1]R. L. Reese, *University Physics* (Brooks-Cole, Pacific Grove, CA 2000), p. 602.

[2]G. L. Erickson, "Children's conceptions of heat and temperature," Sci. Educ. **63**, 221–230 (1979); G. L. Erickson, "Children's viewpoints of heat: A second look," *ibid.* **64**, 323–336 (1980); G. Erickson and A. Tiberghien, "Heat and temperature" [G. Erickson, "Part A: An overview of pupils' ideas," and A. Tiberghien, "Part B: The development of ideas with teaching"], in *Children's Ideas in Science*, edited by R. Driver, E. Guesne, and A. Tiberghien (Open U.P., Maidenhead, UK, 1985), pp. 52–84.
[3]A. Tiberghien "Critical review on the research aimed at elucidating the sense that the notions of *temperature and heat* have for students aged 10 to 16 years," in *Research on Physics Education, Proceedings of the First International Workshop, La Londe Les Maures, France*, directed by G. Delacôte, A. Tiberghien, and J. Schwartz (Éditions du CNRS, Paris, 1984), pp. 75–90.
[4]S. Kesidou, R. Duit, and S. M. Glynn, "Conceptual development in physics: Students' understanding of heat," in *Learning Science in the Schools: Research Reforming Practice*, edited by S. M. Glynn and R. Duit (Lawrence Erlbaum, Mahwah, NJ, 1995), pp. 179–198.
[5]I. Cohen and R. Ben-Zvi, "Improving student achievement in the topic of chemical energy by implementing new learning materials and strategies," Int. J. Sci. Educ. **14**, 147–156 (1992).
[6]M. C. Linn and N. B. Songer, "Teaching thermodynamics to middle school students: What are appropriate cognitive demands?," J. Res. Sci. Teach. **28**, 885–918 (1991).
[7]A. H. Johnstone, J. J. Macdonald, and G. Webb, "Misconceptions in school thermodynamics," Phys. Educ. **12**, 248–251 (1977).
[8]S. Novick and J. Nussbaum, "Junior high school pupils' understanding of the particulate nature of matter: An interview study," Sci. Educ. **62**, 273–281 (1978).
[9]P. L. Thomas and R. W. Schwenz, "College physical chemistry students' conceptions of equilibrium and fundamental thermodynamics," J. Res. Sci. Teach. **35**, 1151–1160 (1998).
[10]W. de Vos and A. H. Verdonk, "A new road to reactions: Part III. Teaching the heat effect of reactions," J. Chem. Educ. **63**, 972–974 (1986).
[11]H. K. Boo, "Students' understandings of chemical bonds and the energetics of chemical reactions," J. Res. Sci. Teach. **35**, 569–581 (1998).
[12]V. Barker and R. Millar, "Students' reasoning about basic chemical thermodynamics and chemical bonding: What changes occur during a context-based post-16 chemistry course?," Int. J. Sci. Educ. **22**, 1171–1200 (2000).
[13]T. J. Greenbowe and D. E. Meltzer, "Student learning of thermochemical concepts in the context of solution calorimetry," Int. J. Sci. Educ. **25**, 779–800 (2003).
[14]P. G. Jasien and G. E. Oberem, "Understanding of elementary concepts in heat and temperature among college students and K–12 teachers," J. Chem. Educ. **79**, 889–895 (2002).
[15]M. J. Cochran, "Student understanding of the second law of thermodynamics and the underlying concepts of heat, temperature, and thermal equilibrium," Ph.D. dissertation, University of Washington, 2005; UMI No. 3198778.
[16]C. H. Kautz, "Identifying and addressing student difficulties with the ideal gas law," Ph.D. dissertation, University of Washington, 1999; UMI No. 9944136.
[17]D. L. Gabel and D. M. Bunce, "Research on problem solving: Chemistry," in *Handbook of Research on Science Teaching and Learning*, edited by Dorothy L. Gabel (Macmillan, New York, 1994), pp. 301–326.
[18]This inappropriate focus on the rates of heating or temperature change is reminiscent of the confusion regarding relative temperatures that is due to physiological sensations caused by differing thermal conductivities; see Ref. 2.
[19]Four versions of the object-in-liquid question were used. The different versions featured different substances so that in one case the liquid had a higher specific heat, while in the other case the object had the higher specific heat. In addition, the identity of the substance with the higher initial temperature was varied (see Fig. 1). Similarly, three versions of the two-liquid question were used in which the ratio of c_A/c_B was 2, 3, or 4 (see Fig. 2). Six separate tests were administered in which different versions of the two questions were paired on the same question sheet. No significant difference in student performance was measured for the different versions of the diagnostic questions.
[20]A free-response version and a multiple-choice version were administered (see Figs. 2 and 3).
[21]See supplementary material at http://dx.doi.org/10.1119/1.3630936 for appendices. Appendix I contains responses given in 2005 before all instruction.
[22]See the detailed comparison in Ref. 21, Appendix II.
[23]Data tables are in Ref. 21, Appendices III and IV.
[24]For the breakdown of each response frequency, see Ref. 21, Appendix IV.
[25]Compare Table III with Appendix IV in Ref. 21
[26]A few students during the Spring 2004 midterm complained about the extensive wordiness of this question, and there were concerns that this wordiness might lead to student confusion. As a result, we gave a nearly identical question with a more compact formulation. This version relied on symbolic notation (for example, "ΔT_{liquid}" for "change in temperature of the liquid"). The text multiple-choice question was given on a midterm exam during Spring 2004, and the symbol multiple-choice question was given on a final exam in the same course. Responses in each category were similar, with a discrepancy of $\leq 6\%$ on each of the five categories (Ref. 21, Appendix VI).
[27]There are option-by-option differences in responses on the text-based question in comparison to the symbol-based question (see Ref. 21, Appendix V). When the multiple-choice responses are combined according to the categories described in the caption to Table IV, we find a very similar pattern on the two versions (see Ref. 21, Appendix VI). This pattern is also consistent with, but not identical to, that obtained on the free-response question (see Table IV).
[28]See Ref. 21, Appendix VI.
[29]For a breakdown of the responses by semester see Ref. 21, Appendix VI.
[30]D. E. Meltzer, "Investigation of students' reasoning regarding heat, work, and the first law of thermodynamics in an introductory calculus-based general physics course," Am. J. Phys. **72**, 1432–1446 (2004).
[31]D. E. Meltzer, "The relationship between mathematics preparation and conceptual learning gains in physics: A possible 'hidden variable' in diagnostic pretest scores," Am. J. Phys. **70**, 1259–1268 (2002).
[32]See, for example, E. Torigoe and G. Gladding, "Same to us, different to them: Numeric computation versus symbolic representation," in *2006 Physics Education Research Conference [Syracuse, New York, 26-27 July 2006]*, edited by Laura McCullough, Leon Hsu, and Paula Heron, AIP Conference Proceedings **883**, (AIP, Melville, NY, 2007), pp. 153–156.
[33]On the multiple-choice versions of this question, the response option that corresponded to equal temperature changes also stated that heat transfers would be equal, potentially excluding students who might have believed that equal temperature changes were linked to unequal heat transfers.
[34]L. Bao and E. F. Redish, "Concentration analysis: A quantitative assessment of student states," Am. J. Phys. **69** (S1), S45–S53 (2001).
[35]L. Bao and E. F. Redish, "Model analysis: Representing and assessing the dynamics of student learning," Phys. Rev. ST Phys. Educ. Res. **2**, 010103-1–16 (2006).
[36]P. B. Kohl and N. D. Finkelstein, "Student representational competence and self-assessment when solving physics problems," Phys. Rev. ST Phys. Educ. Res. **1**, 010104-1–11 (2005).
[37]P. B. Kohl and N. D. Finkelstein, "Effects of representation on students solving physics problems: A fine-grained characterization," Phys. Rev. ST Phys. Educ. Res. **2**, 010106-1–12 (2006).
[38]P. D. Kohl and N. D. Finkelstein, "Patterns of multiple representation use by experts and novices during physics problem solving," Phys. Rev. ST Phys. Educ. Res. **4**, 010111-1–13 (2008).
[39]See also Ref. 21, Appendix III.
[40]J. D. Herron and T. J. Greenbowe, "What can we do about Sue: A case study of competence," J. Chem. Educ. **63**, 528–531 (1986).
[41]E. F. Redish, J. M. Saul, and R. N. Steinberg, "Student expectations in introductory physics," Am. J. Phys. **66**, 212–224 (1998).
[42]W. K. Adams, K. K. Perkins, N. S. Podolefsky, M. Dubson, N. D. Finkelstein, and C. E. Wieman, "New instrument for measuring student beliefs about physics and learning physics: The Colorado Learning Attitudes about Science Survey," Phys. Rev. ST Phys. Educ. Res. **2**, 010101-1–14 (2006).
[43]R. Stavy and D. Tirosh, *How Students (Mis-)Understand Science and Mathematics: Intuitive Rules* (Teachers College Press, New York, 2000).
[44]P. R. L. Heron, "Empirical investigations of learning and teaching, part I: Examining and interpreting student thinking," in *Research on Physics Education: Proceedings of the International School of Physics "Enrico Fermi" Course CLVI*, edited by E. F. Redish and M. Vicentini (IOS Press, Amsterdam, 2004), pp. 341–350.
[45]D. Hammer, "Student resources for learning introductory physics," Am. J. Phys. **68** (S1), S52–S59 (2000).
[46]A. A. diSessa, "Toward an epistemology of physics," Cogn. Instruct. **10**, 105–225 (1993).
[47]L. C. McDermott, "Guest comment: How we teach and how students learn—A mismatch?," Am. J. Phys. **61**, 295–298 (1993).
[48]W. M. Christensen "An investigation of student thinking regarding calorimetry, entropy and the second law of thermodynamics," Ph.D. dissertation, Iowa State University, 2007; UMI No. 3274888, Chap. 4.