Analysis Of Shifts In Students' Reasoning Regarding Electric Field And Potential Concepts

David E. Meltzer

Department of Physics, University of Washington, Seattle, WA 98195, USA

Abstract. Students' reasoning regarding the relationships among electric fields, forces, and equipotential line patterns was explored using pre- and post-test responses to selected multiple-choice questions on the Conceptual Survey of Electricity and Magnetism. Students' written explanations of their reasoning, provided both pre- and post-instruction, allowed additional assessment of the changes in their thinking. In particular, the data indicate that although students largely abandon an initial tendency to associate stronger fields with wider equipotential line spacing, many of them persist in incorrectly associating electric field magnitude at a point with the electric potential at that point.

Keywords: physics education, electricity and magnetism.
PACS: 01.30.Cc; 01.40.Fk

INTRODUCTION

There has been extensive discussion in recent years regarding the desirability of extracting, from multiple-choice test results, information regarding student thinking that is more detailed than that provided simply by net scores of correct and incorrect responses [1, 2]. With that objective in mind, I have carried out a detailed analysis of student responses to several related items from the Conceptual Survey of Electricity and Magnetism (CSEM) [3] that were administered in an algebra-based physics course taught with interactive-engagement methods at Iowa State University from 1998-2002 [4]. The 1998-2001 sample consists of four separate classes (N_{total} = 299); students responded to the test both on a first-day pretest and on a final-day posttest. In 2002, students wrote explanations of their answers on selected test items; this sample consists of an unmatched set (pretest, $N = 72$; posttest, $N = 68$).

The CSEM test items I will discuss here are (1) Item #18, which asks students to compare electric-field magnitudes at three points shown on equi-potential-line diagrams, and (2) Item #20, which asks students to compare electric force magnitude and direction at two different points on an equipotential-line diagram, for a proton located at those points. (See Fig. 1.) The correct answer on Item #18 is *D*; response *E* is that all field magnitudes at the selected points are equal (note that $V = 40$V at all three points), while response *C* corresponds to *lower* field magnitude where line spacing is tightest. Both responses *B* and *D* on Item #20 are consistent with the correct *D* response on Item #18 (that is, larger force vectors where line spacing is tightest). Responses *A* or *C* on Item #20 could both be seen as consistent with *both* *C* and *E* responses on Item #18: with *C*, because force is larger where line spacing is wider, and with *E* because force is larger where potential is larger. However, although these responses may be consistent in the manner described, the items do *not* test exactly the same concepts; in one case field magnitude is at issue, and in the other case it is force magnitude.

ANALYSIS OF RESPONSE PATTERNS

Consistency Comparison For Correct And Incorrect Responses On Item #18

The proportion of total responses corresponding to the correct answer *D* on Item #18 increases from 46% (pre-) to 75% (post-instruction), reflecting a significant increase in correct responses. The *D* responses may be further broken down into "consistent" and "inconsistent" responses, where "consistent" corresponds to the proportion of those students who answered *D* on Item #18 who *also* answered either *B* or *D* on Item #20. For the pre-instruction results, the proportion of *D* responses that are consistent (45%) is almost that which would correspond with random guessing on Item #20, i.e., 40%. Similarly, of students who an-

CP883, *2006 Physics Education Research Conference*, edited by L. McCullough, L. Hsu, and P. Heron

swered Item #18 *incorrectly* pre-instruction, 40% also gave a *B* or *D* response on Item #20. Thus, before instruction, students who answered Item #18 correctly were not significantly more likely ($p > 0.38$) to give the preferred *B* or *D* responses on Item #20 than were students who answered Item #18 incorrectly.

Post-instruction, however, the situation is different. The proportion of *D* answers that are accompanied by *B* or *D* responses on Item #20 rises to 83%, far above the random-guessing rate. This implies that a large majority of those students who gave the correct *D* response on Item #18 after instruction were able to answer Item #20 in a manner consistent with their answer on Item #18, whereas before instruction that was not the case. The rise in consistency from 45% pre-instruction to 83% post-instruction is statistically significant ($p = 0.001$ by a paired two-sample *t*-test).

By comparison, of those students who had *incorrect* answers on Item #18 after instruction, only 57% gave *B* or *D* responses on Item #20. Thus, students who had correct answers on Item #18 were significantly more likely ($p < 0.001$) to give *B* or *D* responses on Item #20 than students who had incorrect answers on Item #18, but *only* after instruction. Before instruction, correct answers on Item #18 implied *no* increased probability of the favored *B* or *D* response on Item #20, despite the similarities between the two items. The implication is that *D* responses on the posttest corresponded to significantly better student understanding of the targeted concept than did *D* responses on the pretest, even though both would ordinarily be scored simply as "correct" responses.

Pre- To Post-Instruction Consistency Shift For "E" Responses On Item #18

The proportion of total responses on Item #18 corresponding to answer *E* barely changes from pre- to post-instruction. Before instruction, 18% of all students give answer *E*, while after instruction that rate is virtually unchanged at 20%; the difference is negligible and not statistically significant. Evidently (and disappointingly), the net popularity of this incorrect answer was essentially unaltered by instruction. (It is interesting that the data reported by Maloney et al. [3] also show increases in the popularity of this response on the posttest, compared to the pretest.)

In order to explore the consistency of thinking reflected by *E* responses on Item #18, we examine how frequently students who gave that response also gave the related responses *A* or *C* on Item #20. Before instruction, we find that 51% of students who answered *E* on Item #18 also gave an *A* or *C* response on Item #20. Although this is slightly above the random-guessing rate of 40%, the difference is not statistically

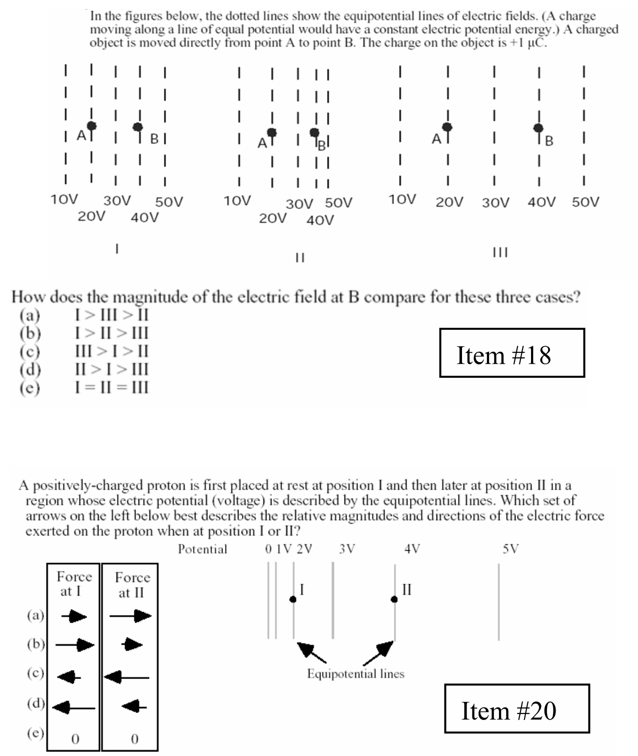

FIGURE 1. CSEM Items #18 and #20 discussed in text.

significant; it is virtually identical to the 50% rate found among students who did *not* answer *E* on Item #20. That is, an *E* pretest response on Item #18 was associated with a marginally greater-than-random probability of giving an *A* or *C* response on Item #20, but no more so than shown by students who did *not* answer *E* on Item #18. After instruction, two changes in these results are noted. First, the proportion of students giving answer *E* on Item #18 who answered *A* or *C* on Item #20 is down to 38% (compared to 51% pre-instruction); this drop is statistically significant ($p = 0.01$) according to a paired two-sample (pre- and post-instruction) *t*-test where the four independent subsamples correspond to the years 1998-2001. This contrasts sharply with the *increase* in consistency that was clearly shown for the correct response *D* (as discussed in the section immediately above). This implies that in this case, the incorrect response *E* when given on the posttest is less closely associated with *consistent* incorrect thinking, than it is when given on the pretest.

The second change noted was among students who did *not* answer *E* on Item #18; the proportion of these students giving an *A* or *C* response on Item #20 is now significantly lower (at 18%) than it was before instruction (i.e., 50%). The difference in *A*/*C* response rate on Item #20 for the *E*-responders on Item #18

(38%) compared to the non-*E*-responders (18%) is now statistically significant ($p < 0.03$), and so even though the *A* and *C* responses on Item #20 become less popular with *E*-responders than before, they are still more often given by this group than by those who do *not* answer *E* on Item #18.

Pre- To Post-Instruction Shift For "C" Responses On Item #18

Besides choices *D* and *E*, the one other answer option on Item #18 to garner a substantial proportion of pretest responses was answer *C*. Before instruction, 26% of all students gave response *C*. On the posttest, however, the response rate for choice *C* dropped sharply to only 3%; that is, support for this option essentially disappeared. Apparently, instruction was effective in eliminating the attractiveness of this answer.

Response *C* on Item #18 can be considered consistent with responses *A* and *C* on Item #20. Of all students who gave the *C* response on Item #18 pre-instruction, 53% also gave an *A* or *C* response on Item #20, not significantly different than random guessing ($p > 0.10$). This suggests relatively low consistency in students' thinking reflected by this *C* response.

ANALYSIS OF EXPLANATIONS

Students' pre- and post-instruction written explanations provided in 2002 offer considerable additional insight into student thinking.

Response "D": Correct Answer, Incorrect Explanation

Nearly half of all students—46%—gave the correct response *D* for Item #18 on the pretest in 1998-2001. In the 2002 offering of the same course, students were specifically asked whether or not they had previously studied electricity and magnetism, and in particular whether they had studied the material represented on the questions dealt with here. Of the students who gave the correct answer *D* (53% of all students in 2002), almost 60% said that they had not previously studied electricity and/or magnetism, and less than 20% said that they had studied the material represented on these questions. How, then, did so many of them get the correct answer?

Their explanations indicate that many students either based their correct answer on some "intuitive" reasoning, or they simply guessed. A sample of some of the explanations offered for answer *D* bears this out:

"Chose them in the order of closest lines"
"Magnitude decreases with increasing distance"
"Greatest because 50 [V] is so close"
"More force where fields are closest"
"Because charges are closer together"
"Guessed"

Although it is difficult to categorize many of the responses on the pretest, it is clear that many of the students who chose answer *D* justified their answer with vague or inconsistent arguments based on the tighter spacing of equipotential lines. However, for most of this group, inconsistent answers on Item #20 (i.e., choosing *A*, *C,* or *E* on that item), ambiguous explanations of what "closer spacing" implied, or explicit admissions that they were guessing, all implied that most of students' correct answers on this pretest item were not indicative of a clear understanding of the conceptual principles involved.

On the posttest, by contrast, most students (83%) who answered *D* on Item #18 gave consistent answers on Item #20 (*B* or *D;* see section above), and most (63%) of the group with this *D/B-or-D* answer pattern gave explanations on *both* items that were consistent with an adequate understanding of the concept involved.

Response "C": Wider Spacing ⇒ Stronger Field

On the pretest, the second most popular response for Item #18 was *C*, consistent with wider equipotential line spacing being associated with larger-magnitude electric fields. Similarly, in 2002, 24% of pretest responses on this item were *C*. About half of the explanations offered made explicit reference to the line spacing, although few of these specifically related wider spacing to larger field magnitude. Most of the explanations in this category indicated that the students were unsure of their reasoning, or indeed that they were making outright guesses.

Here is a sampling of students' pretest explanations for response *C*:

"III is the farthest apart, then 1 and then 2"
"The equipotential lines are farther apart so a greater magnitude is needed to maintain an electrical field"
"I guessed"

On the posttest, as discussed above, *C* responses on Item #18 fell to an insignificant 3% of all responses.

Response "E": Field Magnitudes All Equal

As discussed above, this response is consistent with the idea that electric-field magnitude scales with the value of the electric potential at a particular point. About half of the explanations offered for this response, both pre- and post-instruction, explicitly argued along these lines. On the posttest, another popular explanation was based on the (incorrect) claim that the diagram portrayed a uniform electric field, thus ensuring equal field magnitudes at all points. This particular explanation—which ignores the line spacing issue—was not present on the pretest. Otherwise, pre- and posttest explanations were similar. For example,

Pretest explanations:

"They are all at the same voltage"
"The magnitude is 40 V on all three examples"
"The voltage is the same for all 3 at B"
"The change in voltage is equal in all three cases"

Posttest explanations:

"The potential at B is the same for all three cases"
"They are all from 20 V–40 V"
"The equipotential lines all give 40 V"
"They all have the same potentials"

As discussed in the sections above and below, response *E* retained its popularity after instruction as it accounted for 20% of all posttest responses on this item, compared to 18% on the pretest. It seems that the addition of new justifications for this response (e.g., uniform electric field) helped sustain its attractiveness even after instruction.

AN ALTERNATIVE PERSPECTIVE ON CONSISTENCY SHIFTS

Another way to analyze pretest-to-posttest shifts in consistency is to probe for possible changes in the popularity of various student "models," in a sense analogous to that of Bao and Redish [1]. Here a model will be defined as a specific set of answers to two or more related questions. Certainly, using responses on only two questions to attempt to specify a student model is far from optimal, if only because the probability that random guessing could produce the specified response patterns is relatively high. Moreover, as discussed above, the questions on this test were not designed to probe exactly identical ideas regarding field and potential. For these reasons, the present discussion regarding student models is meant to be illustrative only.

We may for instance presume that one student model corresponds to the idea that "*electric-field magnitude is larger at a point where electric potential is larger*," and define this model as corresponding to an *E* response on Item #18 when accompanied by an *A* or *C* response on Item #20. With that definition we find that 9% of all responses (1998-2001) on the pretest were consistent with this model, along with 8% of responses on the posttest. Analogously, a model corresponding to "*electric-field magnitude is larger at a point where equipotential lines have wider separation*" could correspond to a *C* response on Item #18, and an *A* or *C* response on Item #20. This model dropped from 14% of the responses on the pretest to only 2% on the posttest. Meanwhile, the "correct" model (*D* on Item #18, *B* or *D* on Item #20) rose from 20% on the pretest to 63% on the posttest.

Although this method of analysis is potentially useful and informative, it is worth probing in more detail to try and determine the extent to which the specified models actually correspond to distinct patterns of student thinking. We can look at the students' 2002 written explanations to shed light on this issue. For example, on the posttest, we find 6% of all responses following the *E*/*A-or-C* pattern, while only half of those (3%) offered explanations consistent with the idea that "*electric-field magnitude is larger at a point where electric potential is larger*." On the pretest, the corresponding figures were 7% of all responses, with 3% offering consistent explanations for this model. Similarly, although 17% of pretest responses corresponded to the pattern *C*/*A-or-C*, only one-quarter of those (4% overall) provided explanations consistent with "*electric-field magnitude is larger at a point where equipotential lines have wider separation.*" For the "correct" model (*D*/*B-or-D*) the proportion of consistent explanations was higher, but still only about two-thirds of pretest explanations accompanying that pattern were consistent with a correct explanation.

Although these results are illustrative rather than definitive, they do suggest the need for caution in interpreting patterns of multiple-choice responses as corresponding to well-defined student models.

ACKNOWLEDGMENTS

The original motivation for this work was a series of discussions with Lei Bao regarding his work on Concentration Analysis. This work was supported in part by NSF REC-#0206683 (T. J. Greenbowe, Co-Principal Investigator), DUE-#0243258, and DUE-#0311450.

REFERENCES

1. L. Bao and E. F. Redish, *Am. J. Phys*. **69**, S49-S53 (2001).
2. R. J. Dufresne, W. J. Leonard, and W. J. Gerace, *Phys. Teach.* **40**, 174-180 (2002).
3. D. P. Maloney, T. L. O'Kuma, C. J. Hieggelke, and A. Van Heuvelen, *Am. J. Phys*. **69**, S12-S23 (2001).
4. D. E. Meltzer and K. Manivannan, *Am. J. Phys*. **70**, 639-654 (2002).