Response Patterns by Introductory Physics Students on Mathematics Diagnostic Tests

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Arizona State University

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Outline

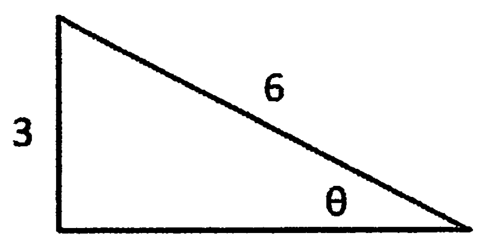
- We have explored the nature and prevalence of physics students' difficulties with elementary mathematics, using "stripped-down" problems with little or no physics context
- In collaboration with Ohio State University, we are developing and testing an online "skill-practice" tool to improve performance

Work to Date

- Administer (and analyze) written diagnostic quiz, given to > 5000 students at Tempe and Poly campuses of Arizona State University; calculators *are* allowed
- Carry out individual interviews with 75 students enrolled in those or similar courses during same period
- Comparison data: University of Colorado, algebra-based course (*N* = 384); Ohio State University, calculus- and algebra-based courses (N > 1000).*

*Thanks to Steve Pollock and Colin West (CU-Boulder), and Andrew Heckler and Beatriz Burrola Gabilondo (OSU)

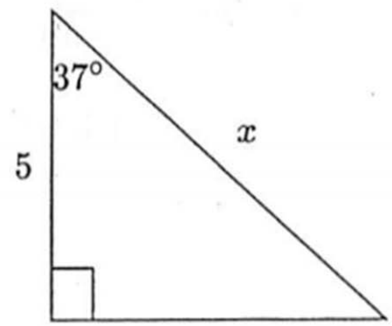
Find Unknown Angle



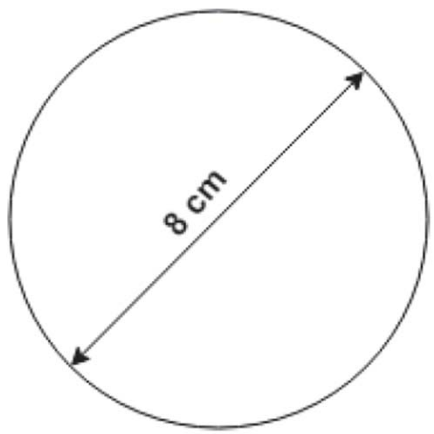
What is the value of θ?

Find Unknown Side

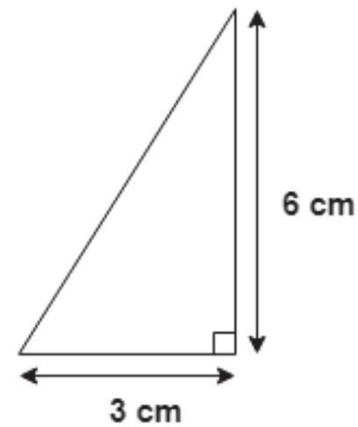
1. What is the length of side x?



Find Area



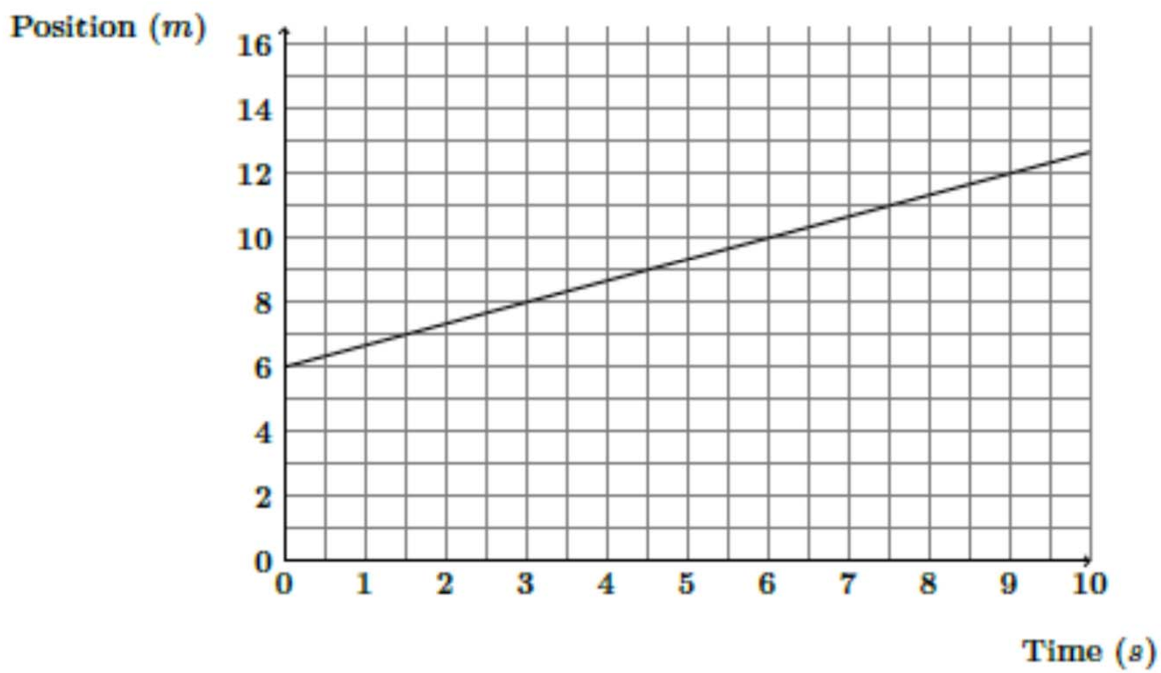
(a) **Area of the circle =**



(b) **Area of the triangle =**

Find Slope of Graph

What is the slope of the graph below?



Simultaneous Equations, Numeric Coefficients

What is the numerical value of x?

$0.5y = 2x$

$78.4 - y = 8x$

Simultaneous Equations, Symbolic Coefficients

$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

Symbolic Multiple-Choice Problem

14. $v^2 = v_0^2 + 2ad$

$$v_0 = 0$$

$$a = \frac{v_1}{t_1}$$

$$v = \frac{v_1}{2}$$

$$d = ?$$

(Please clearly *circle* your answer and show all work.)

A. $d = v_1 t_1$ B. $d = \frac{v_1 t_1}{2}$ C. $d = \frac{v_1 t_1}{4}$ D. $d = \frac{v_1 t_1}{8}$ E. $d = \frac{v_1 t_1}{16}$

(Some) Other Items

4. Find the value of each of the following.

$cos(0°) =?$

$sin(90°) =?$

$tan(0°) =?$

6. Solve for θ.

$\gamma\theta + \eta = \lambda\theta + \omega$

7. Solve for x.

$ax + b = cx + d$

10. $2\left(\frac{3}{4}\right) = ?$

11. $\frac{a/b}{c^2/d} = ?$

A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

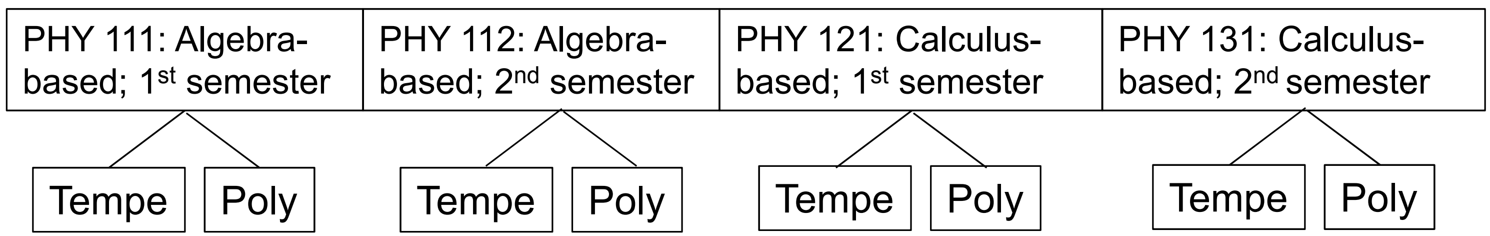
17. $ax - dx = c$

$x =?$

Our Primary Sample Populations

PHY 111: Algebra-based; 1st semester	PHY 112: Algebra-based; 2nd semester	PHY 121: Calculus-based; 1st semester	PHY 131: Calculus-based; 2nd semester

Our Primary Sample Populations



Findings

1. Difficulties with pre-college mathematical operations are widespread among students in both algebra- and calculus-based courses; average error rates range from 20-70%;
2. Results were highly consistent among four different campuses at three different state universities (ASU Tempe, ASU Poly, CU-Boulder, Ohio State U.)
3. Performance on algebraic problems using *symbols* for constant coefficients is significantly worse than on problems using numbers, for all populations;
4. Despite the great diversity of diagnostic item types, students' item responses were *highly* correlated with each other, and with *total* score on test

   → Class-average score on a *single* test item can accurately predict class-average *total* score
5. Differences between universities were observed (e.g., CU-Boulder scored 20% higher than ASU-Tempe), but *individual* item scores co-varied consistently and predictably
6. During problem-solving interviews, students self-correct approximately 50% of errors following minimal prompts, suggesting prevalence of "careless" errors.
7. The proportion of errors that could be described as "careless" ["non-operational"] was *highly* correlated with overall correct response rate. [Higher scores → more "careless"]

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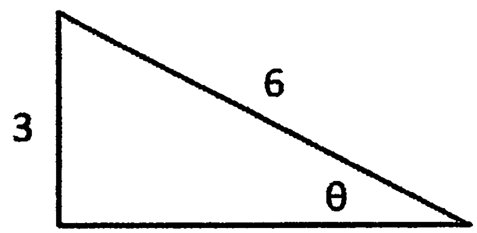
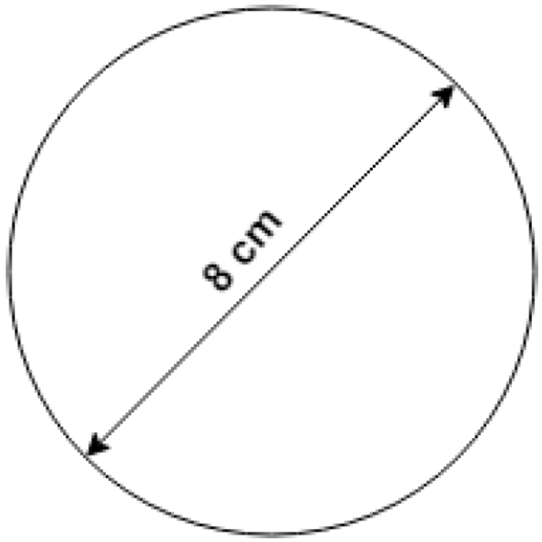
"Find Unknown Angle"



What is the value of θ?

Algebra-based course, 2018

ASU-Poly: 35% correct (*N* = 152)
ASU-Tempe: 52% correct (*N* = 533)



<u>**Algebra- and Calculus-based courses combined, 2018**</u>

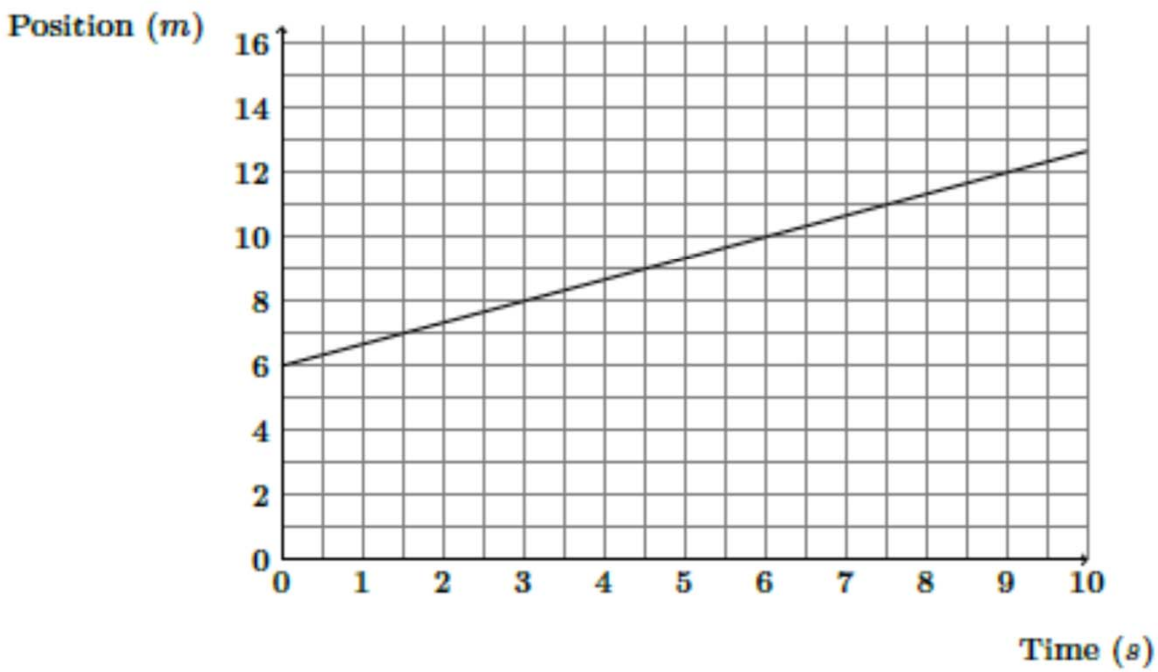
ASU-Poly: 57% correct (N = 250)
ASU-Tempe: 76% correct (N = 1086)

…*with correct units:* 29% and 45% correct, respectively

(a) Area of the circle =

All courses: 30-60% correct (*N* > 2000)

What is the slope of the graph below?



$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

Calculus-based course, 2018

ASU-Tempe: 55% correct (*N* = 862)

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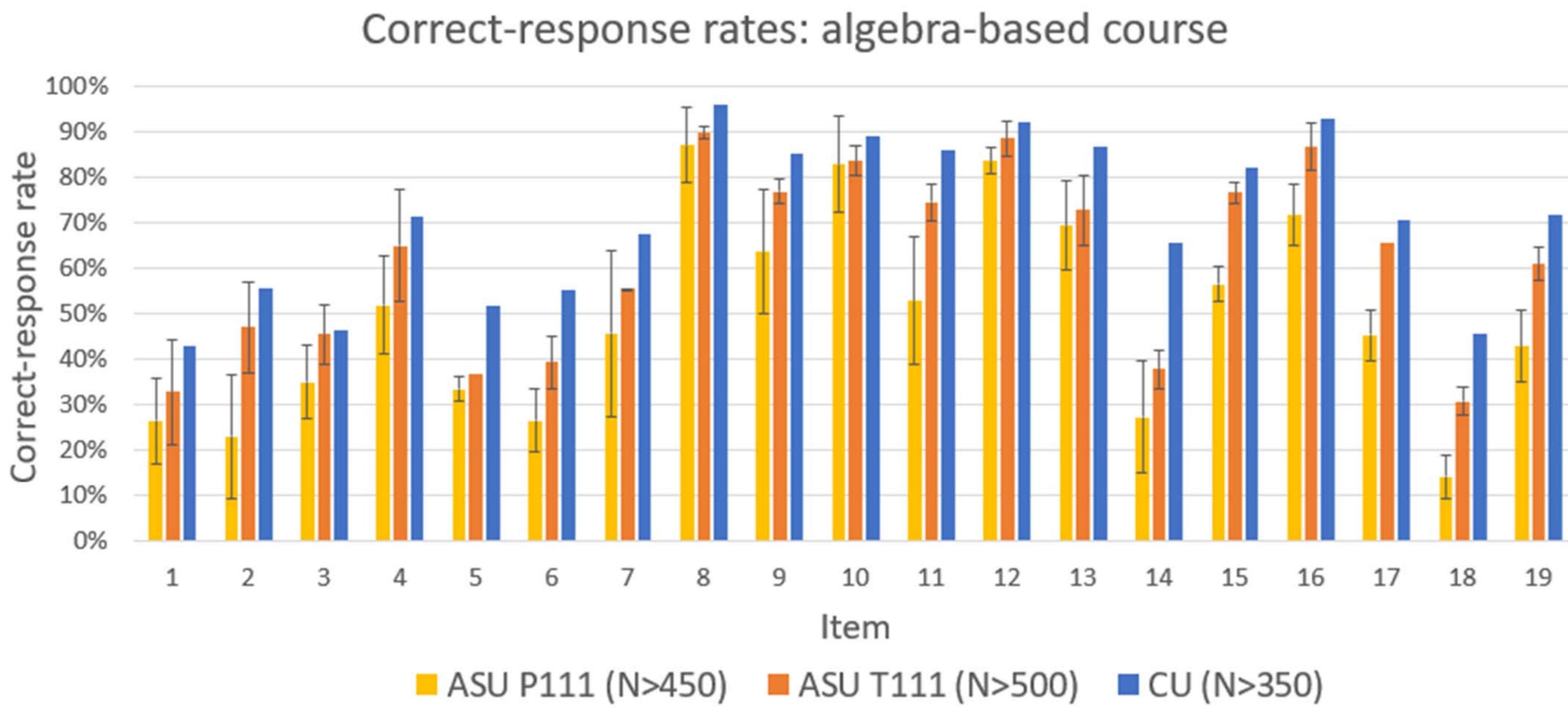
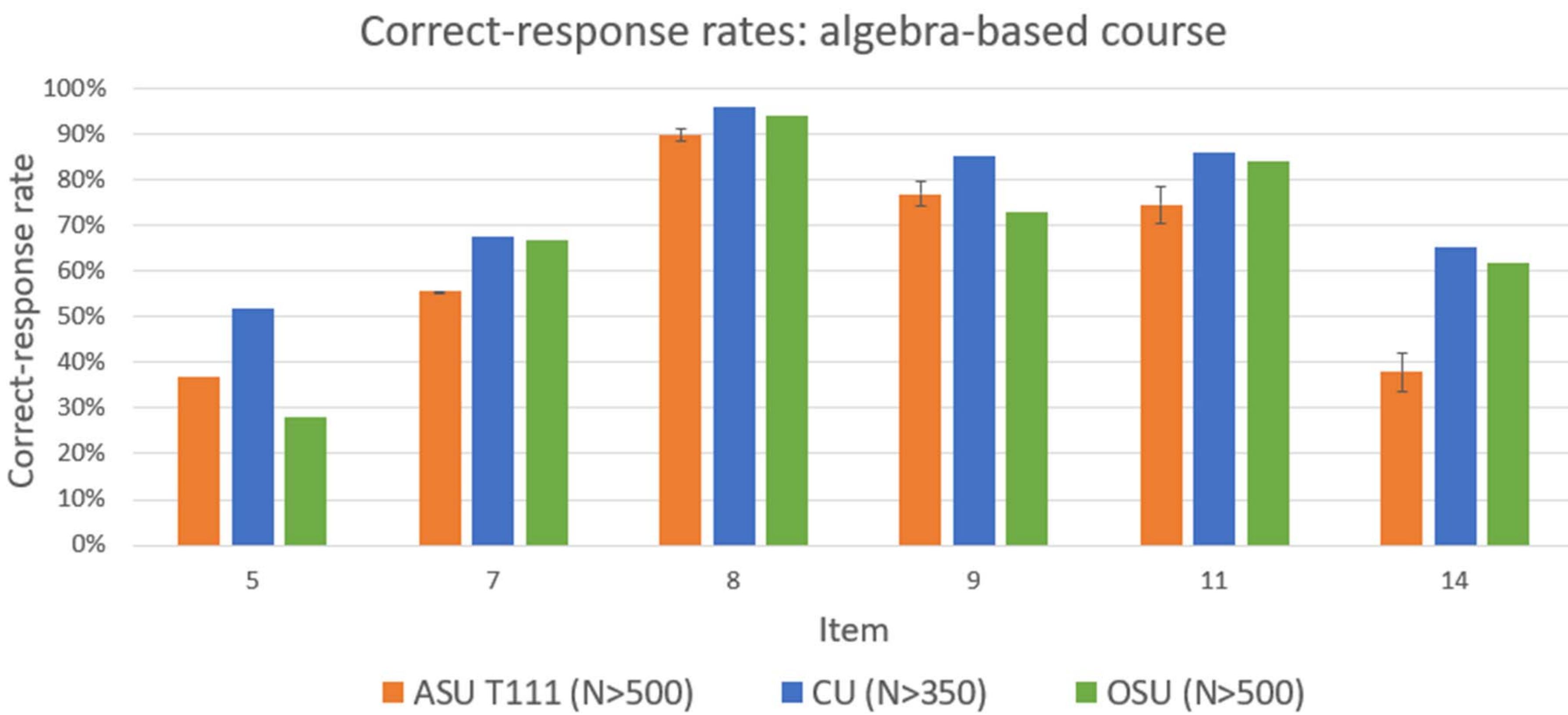
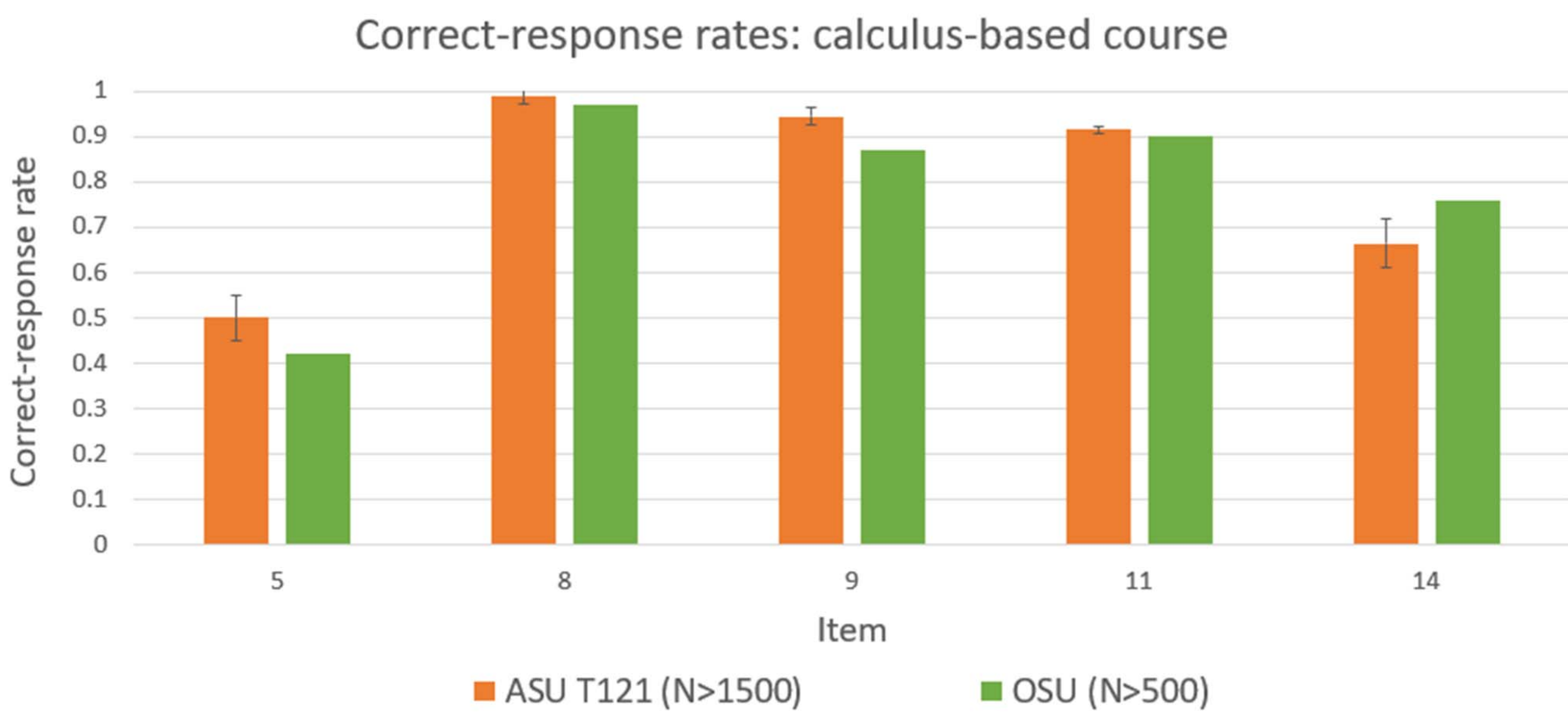

Correct-response rates: algebra-based course
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Correct-response rate
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
Item
ASU P111 (N>450)
ASU T111 (N>500)
CU (N>350)



Correct-response rates: algebra-based course
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Correct-response rate
5
7
8
9
11
14
Item
ASU T111 (N>500)
CU (N>350)
OSU (N>500)



Correct-response rates: calculus-based course
Correct-response rate
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
5
8
9
11
14
Item
ASU T121 (N>1500)
OSU (N>500)

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Algebra: Symbolic vs. Numeric Coefficients

Calculus-based course, 1st semester (% correct; ASU-Tempe, 2018 fall + spring)

What is the numerical value of x?

$78.4 - y = 8x$

$0.5y = 2x$

Numeric version

79% correct (N = 1043)

$cy = dx$

$a - y = bx$

$x = ?$

Symbolic version

55% correct (N = 862)

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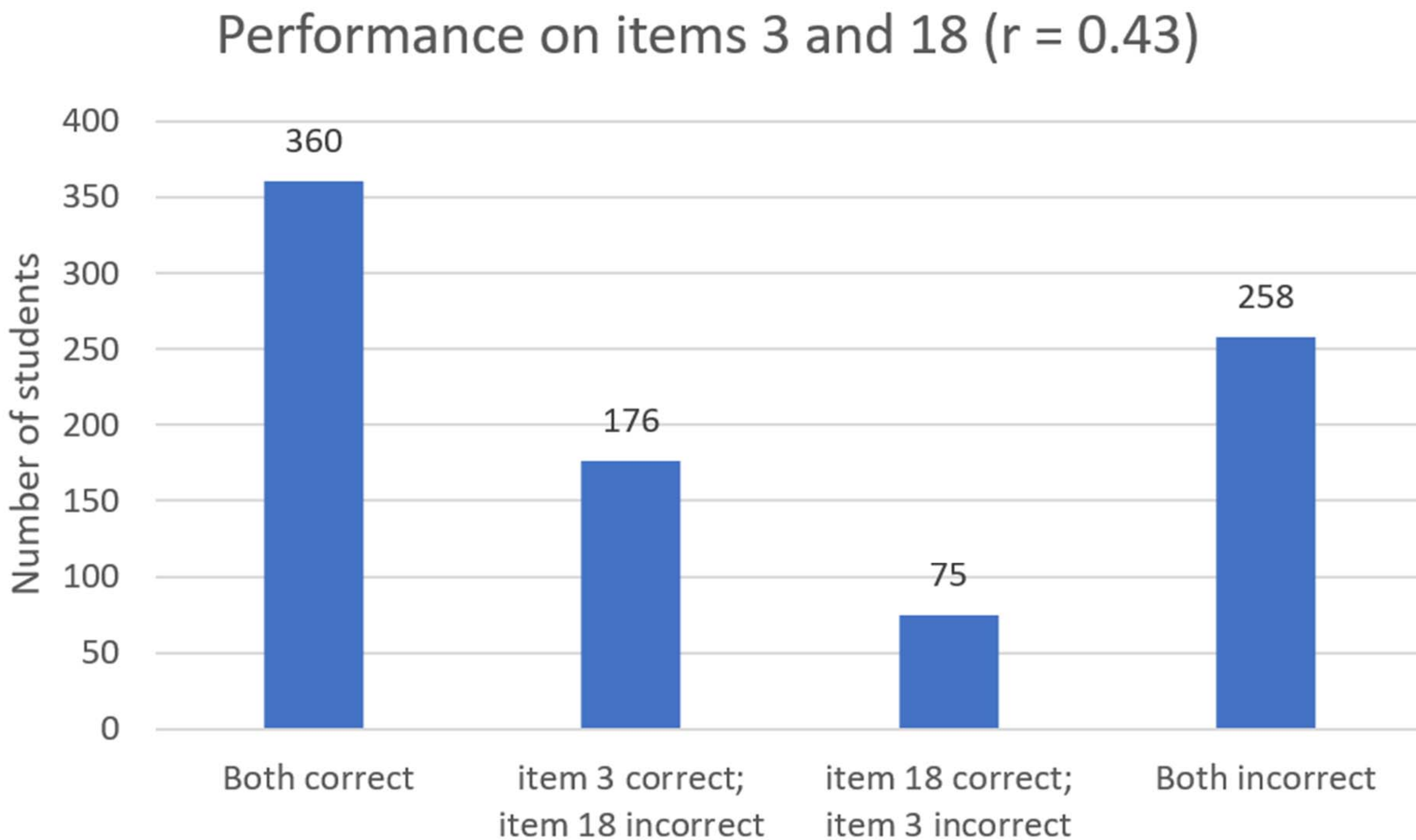
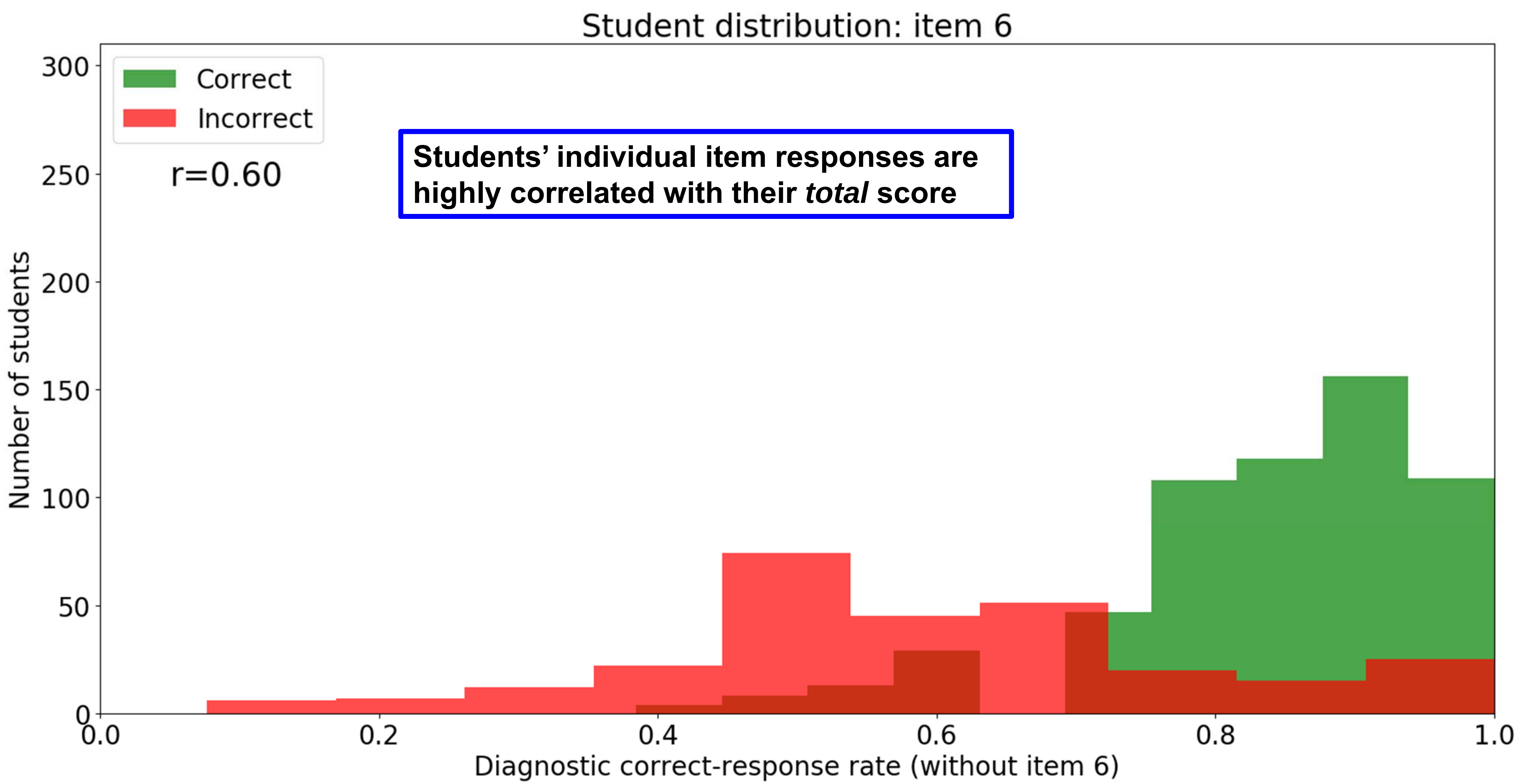

Performance on items 3 and 18 (r = 0.43)
Number of students
400
350
300
250
200
150
100
50
0
360
176
75
258
Both correct
item 3 correct; item 18 incorrect
item 18 correct; item 3 incorrect
Both incorrect



Student distribution: item 6
Correct
Incorrect
r=0.60
Students' individual item responses are highly correlated with their *total* score
Number of students
300
250
200
150
100
50
0
0.0
0.2
0.4
0.6
0.8
1.0
Diagnostic correct-response rate (without item 6)

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   ➢ **Class-average score on a *single* test item can accurately predict class-average *total* score**

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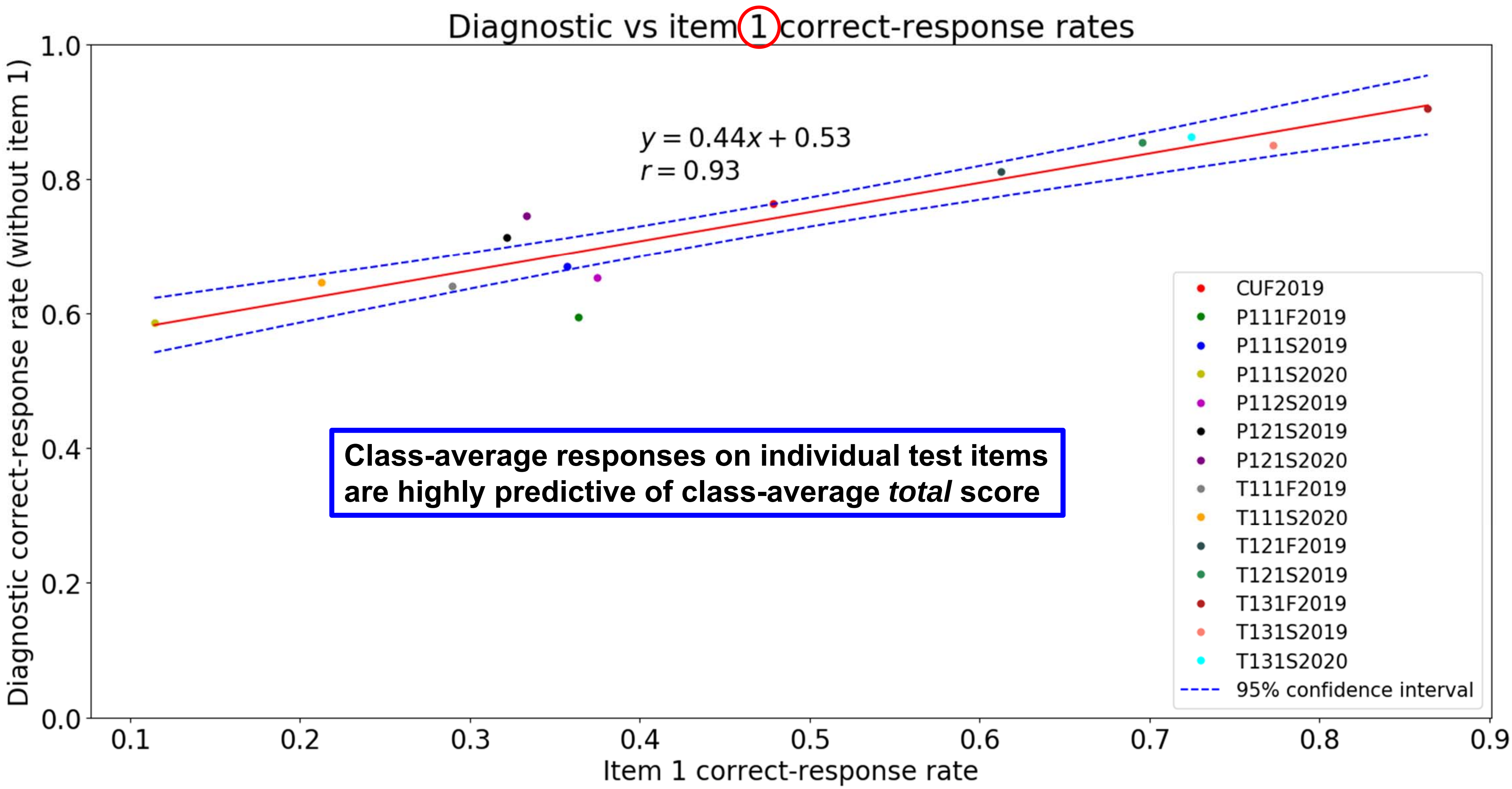
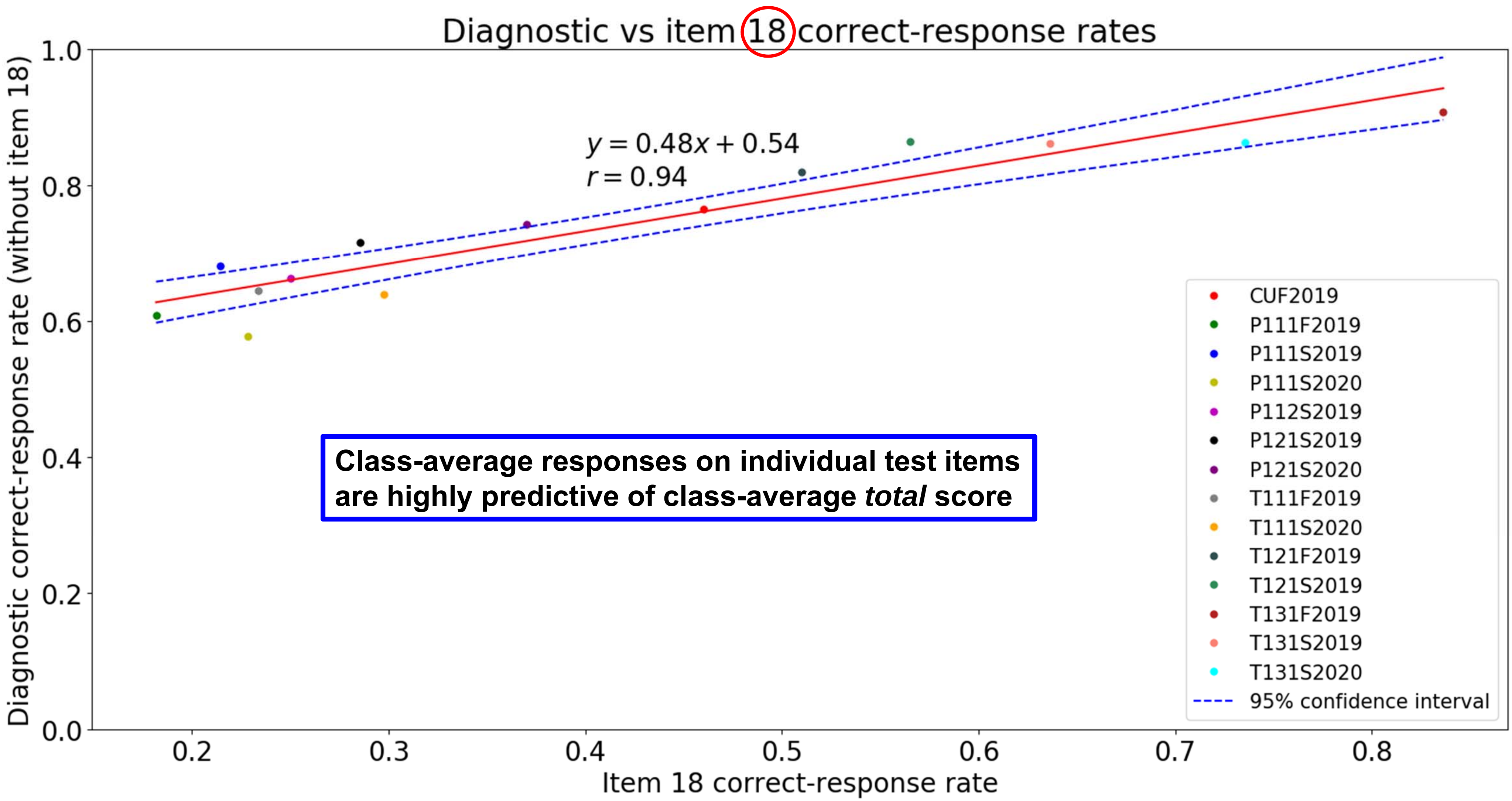

Diagnostic vs item 1 correct-response rates
Diagnostic correct-response rate (without item 1)
Item 1 correct-response rate
y = 0.44x + 0.53
r = 0.93
CUF2019
P111F2019
P111S2019
P111S2020
P112S2019
P121S2019
P121S2020
T111F2019
T111S2020
T121F2019
T121S2019
T131F2019
T131S2019
T131S2020
95% confidence interval
Class-average responses on individual test items are highly predictive of class-average total score


Diagnostic vs item 18 correct-response rates
y = 0.48x + 0.54
r = 0.94
Class-average responses on individual test items are highly predictive of class-average total score
Diagnostic correct-response rate (without item 18)
Item 18 correct-response rate
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
CUF2019
P111F2019
P111S2019
P111S2020
P112S2019
P121S2019
P121S2020
T111F2019
T111S2020
T121F2019
T121S2019
T131F2019
T131S2019
T131S2020
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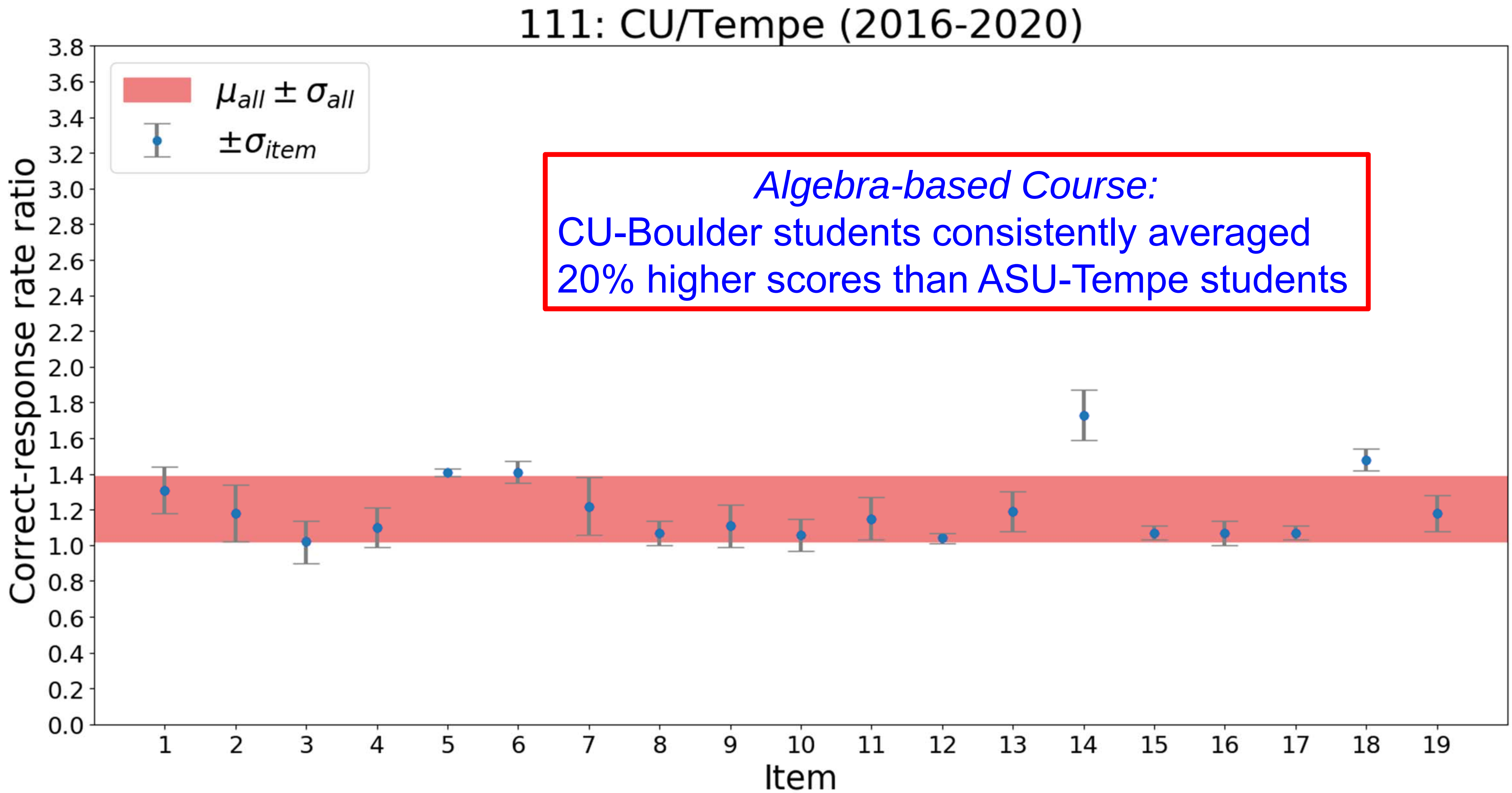
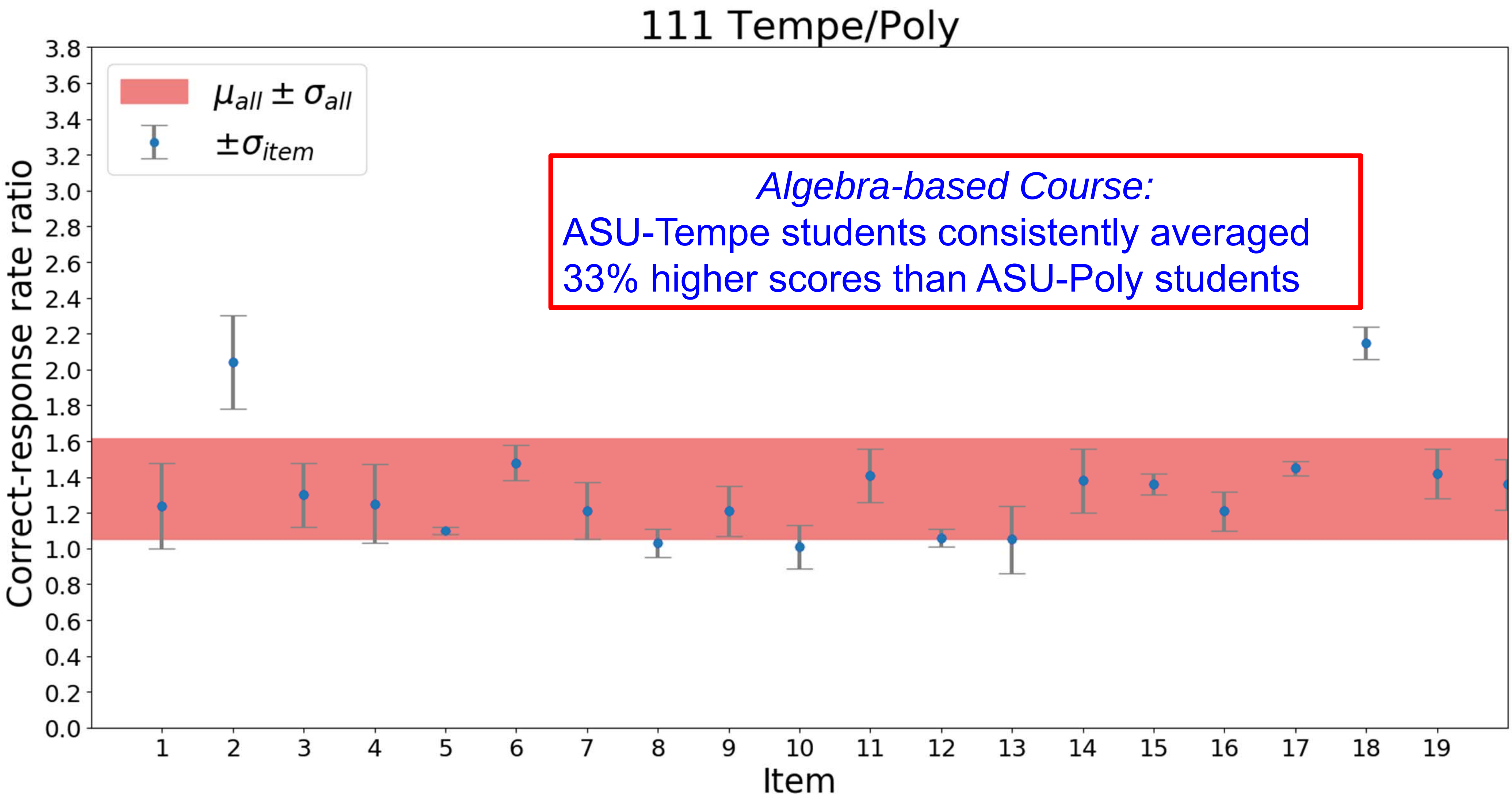


111: CU/Tempe (2016-2020)
$\mu_{all} \pm \sigma_{all}$
$\pm\sigma_{item}$
Algebra-based Course:
CU-Boulder students consistently averaged
20% higher scores than ASU-Tempe students
Correct-response rate ratio
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
Item



111 Tempe/Poly
$\mu_{all} \pm \sigma_{all}$
$\pm\sigma_{item}$
Algebra-based Course:
ASU-Tempe students consistently averaged 33% higher scores than ASU-Poly students
Correct-response rate ratio
Item

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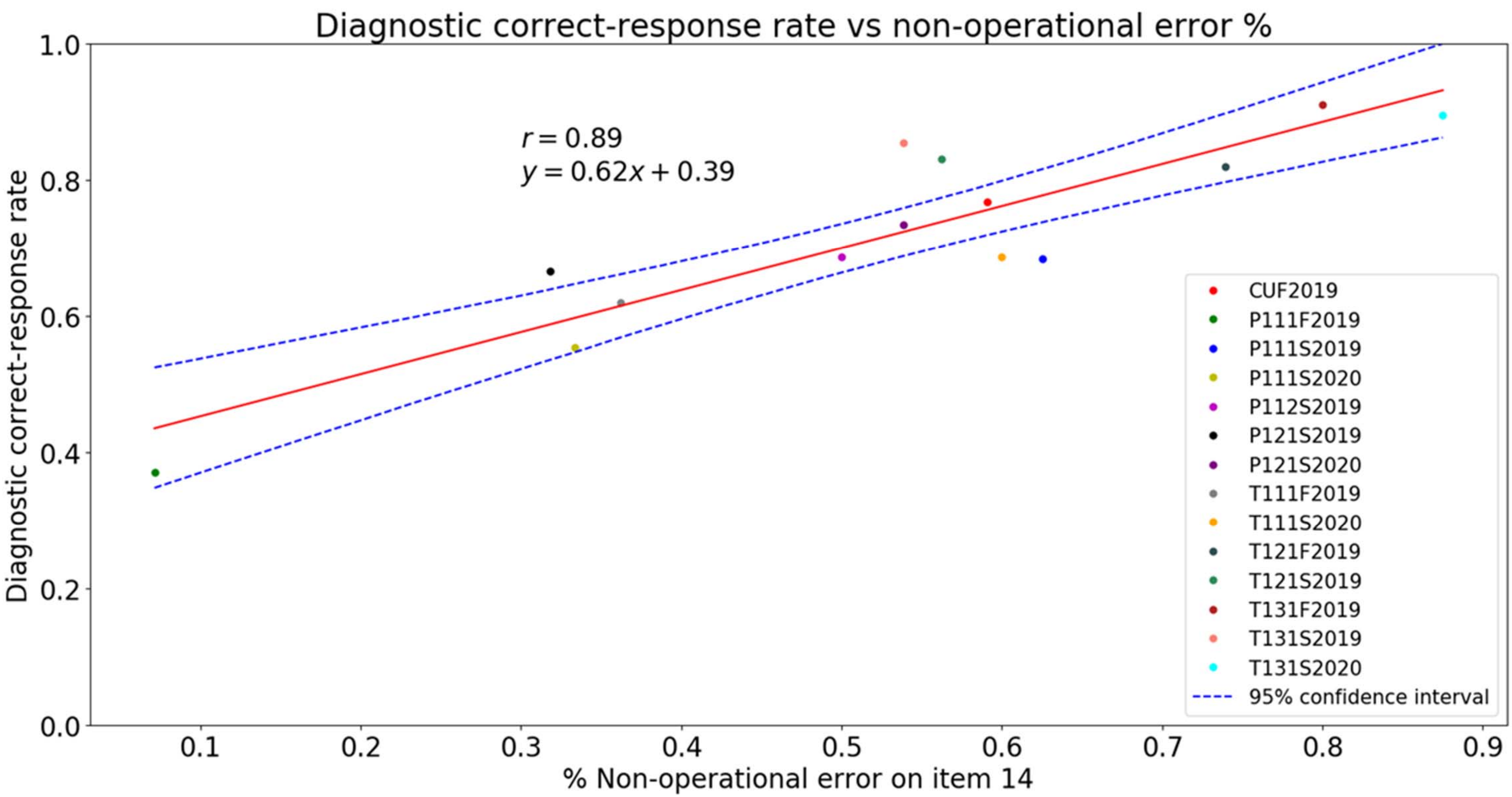
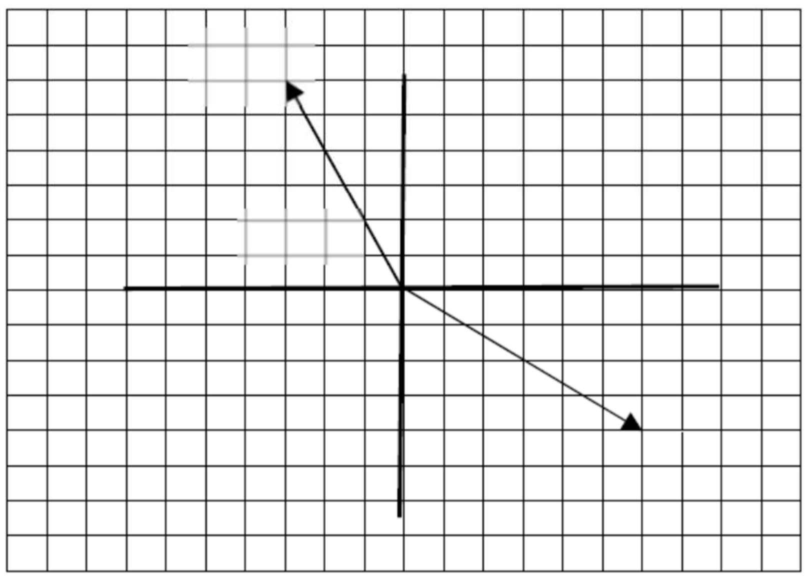

Diagnostic correct-response rate vs non-operational error %
r = 0.89
y = 0.62x + 0.39
Diagnostic correct-response rate
% Non-operational error on item 14
0.0
0.2
0.4
0.6
0.8
1.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
CUF2019
P111F2019
P111S2019
P111S2020
P112S2019
P121S2019
P121S2020
T111F2019
T111S2020
T121F2019
T121S2019
T131F2019
T131S2019
T131S2020
95% confidence interval

Possible Instructional Strategies

- Difficulties due to **skill-practice deficits** might be addressed by short-term, in- and out-of-class tutorials and assignments, designed to refresh students' previously learned knowledge and skills (e.g., Mikula and Heckler, 2017)
  - Current project, OSU + ASU, NSF DUE #1914709/1914712
  - Regular low-stakes on-line homework assignments requiring multiple consecutive correct answers
- Inclusion of multi-step contexts in these assignments ***may*** reduce the prevalence of non-operational errors as well.

Pretest (February 13)

2. A charge at the origin experiences electrical forces from two separate source charges (source charges are not shown; the arrows represent those two forces. Draw an arrow to represent the net electrical force acting on the charge at the origin.



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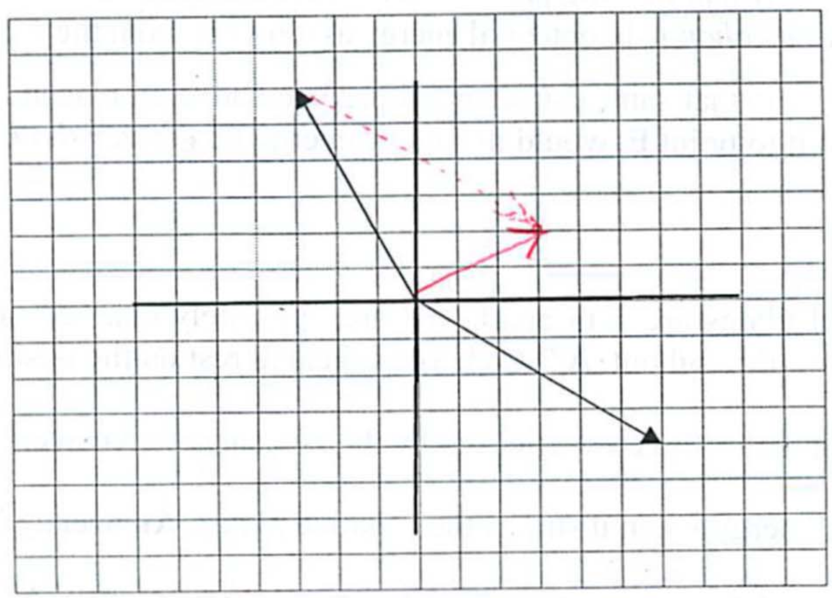
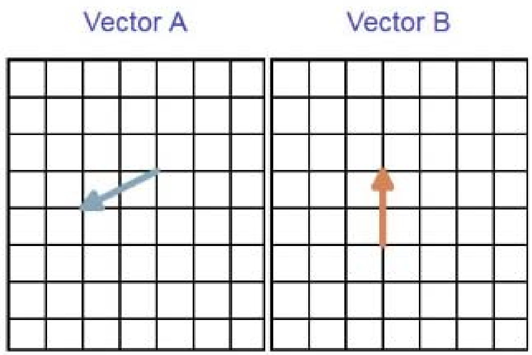


x : 6 − 3 = +3

y : 6 − 4 = +2

Instructional Intervention

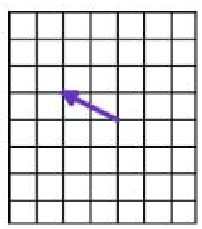
- Initial instruction and group practice in class *before* pretest was administered
- Virtually *no* in-class review after pretest
- Three rounds (one every two weeks) of online “Essential Skills” [from Ohio State University] practice assignments (homework points awarded for completion of four correct items in a row) *before* post-test

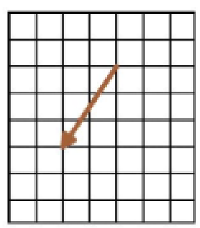


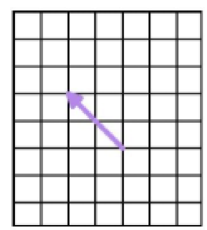
“Essential Skills” practice, example #1

Vector $\vec{A}$ and Vector $\vec{B}$ are shown above.

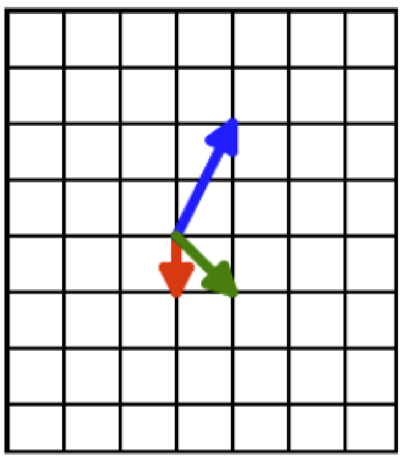
Which of the following options represents the vector sum, $\vec{A}+\vec{B}$?

○ 

○ 

○ 

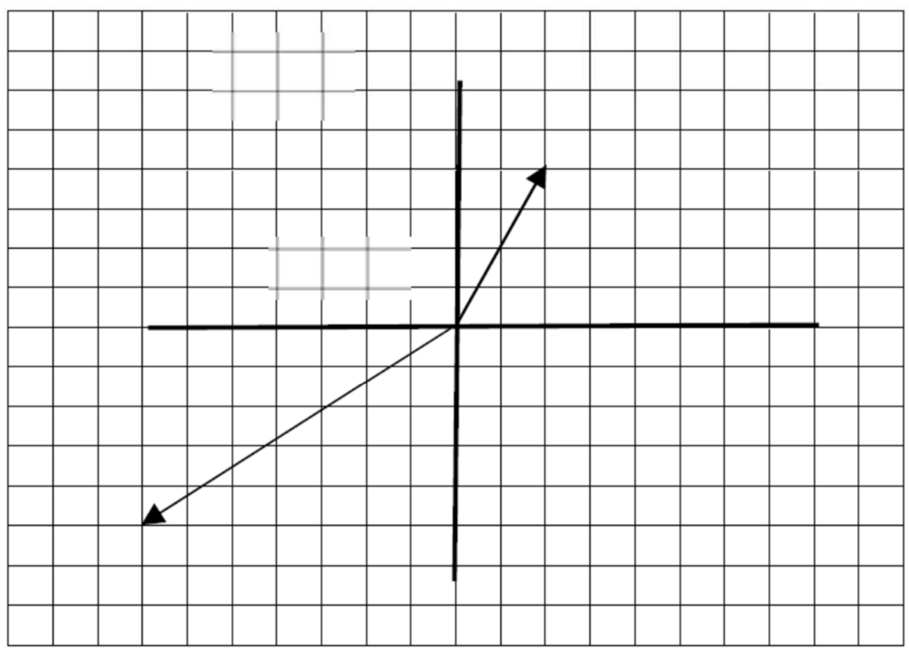
“Essential Skills” practice, example #2



Three forces (vector A, vector B, and vector C) are shown above in free-body diagram form. Which of the options below represents the net force?

Posttest (April 2)

1. A charge at the origin experiences electrical forces from two separate source charges (source charges are not shown; the arrows represent those two forces. Draw an arrow to represent the net electrical force acting on the charge at the origin.



Results on Vector-Diagram Problem

N = 39 (Matched, pre- and post)

Class Average Pretest Score: 44%

Class Average Posttest Score: 79%

(Difference in means significant, $p < 0.001$)

Summary

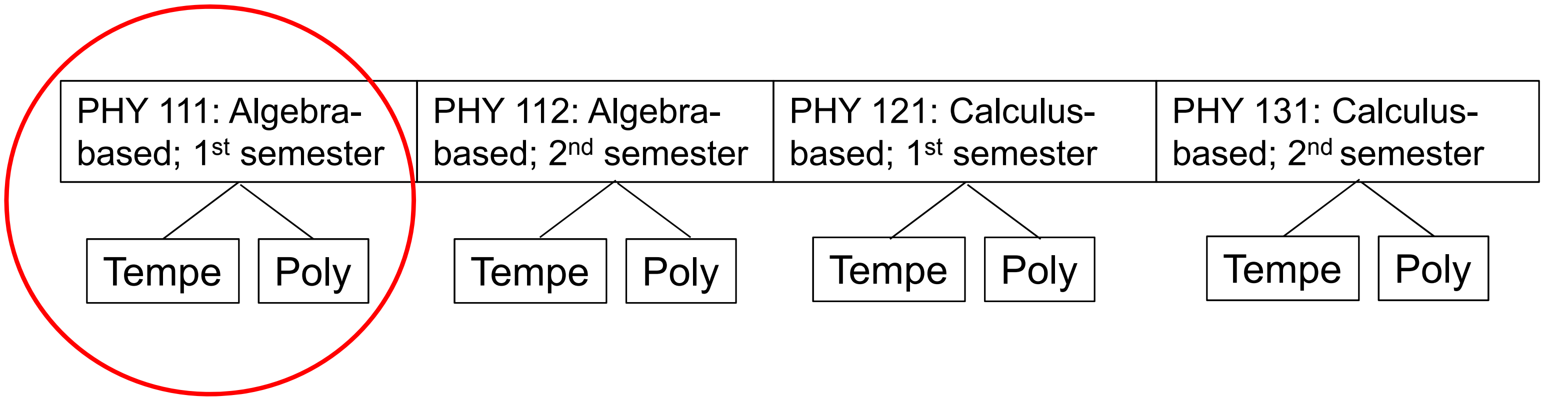
- Physics students’ mathematical errors have a variety of causes
- Errors due to skill-practice deficits with “unfamiliar” operations (such as vectors) may be addressable through regular, brief online assignments
- Errors due to deeply-ingrained difficulties carried over from K-12 instruction may be harder to address

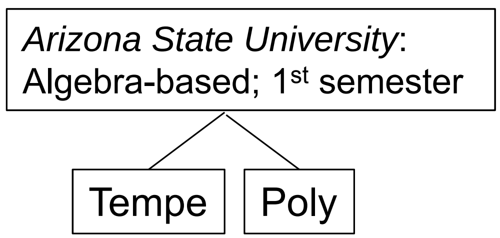
Possible Instructional Strategies

- Difficulties due to **"carelessness"** might be addressed by guiding students to (1) carefully check and re-check key steps in their calculation; (2) slow down, review problem statements, and re-solve when possible

- Other studies (e.g., G. White) have shown that much practice and repetition is needed to induce students to adopt consistent error-checking habits

Our 8 Sample Populations



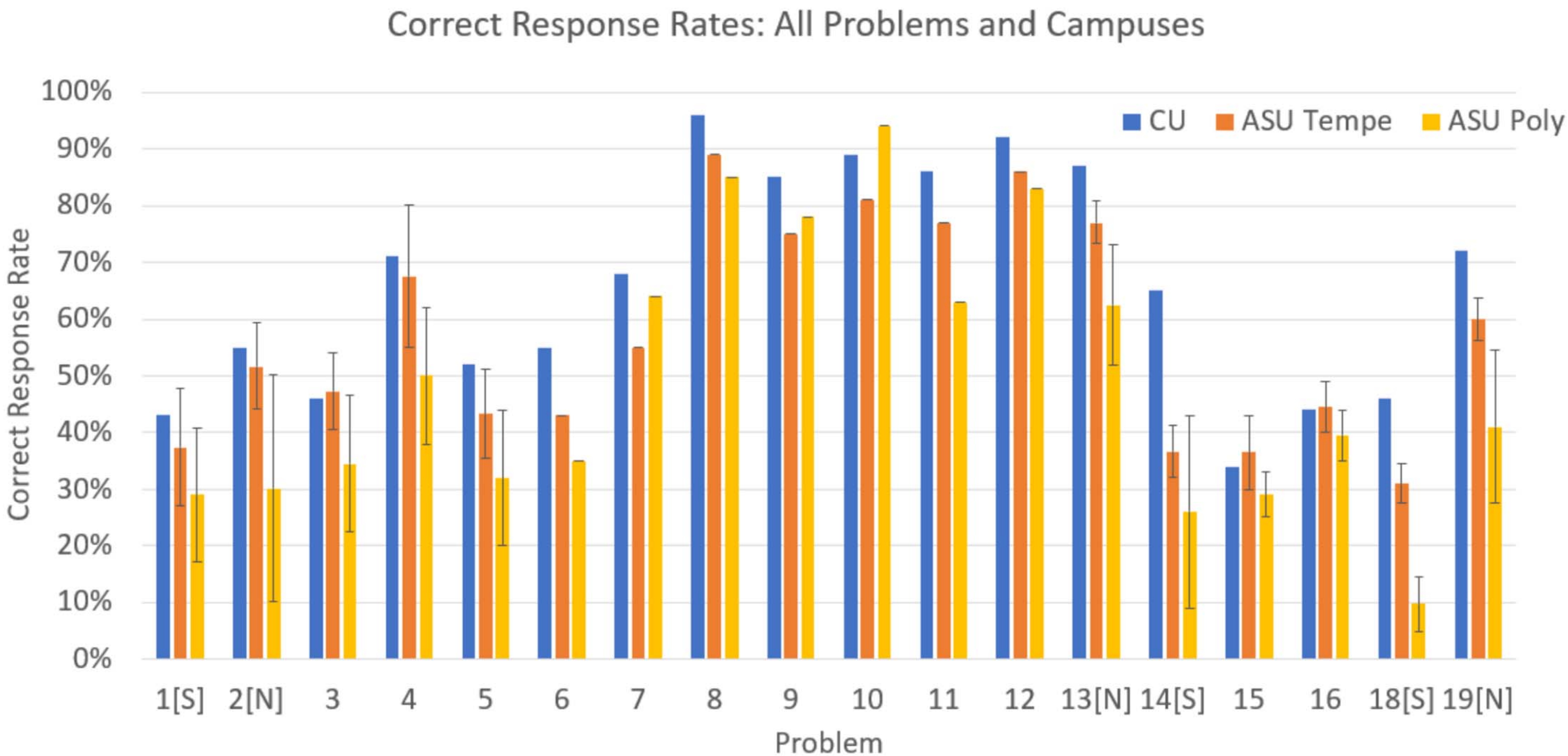

Arizona State University:
Algebra-based; 1st semester
Tempe
Poly

Our 3 Sample Populations

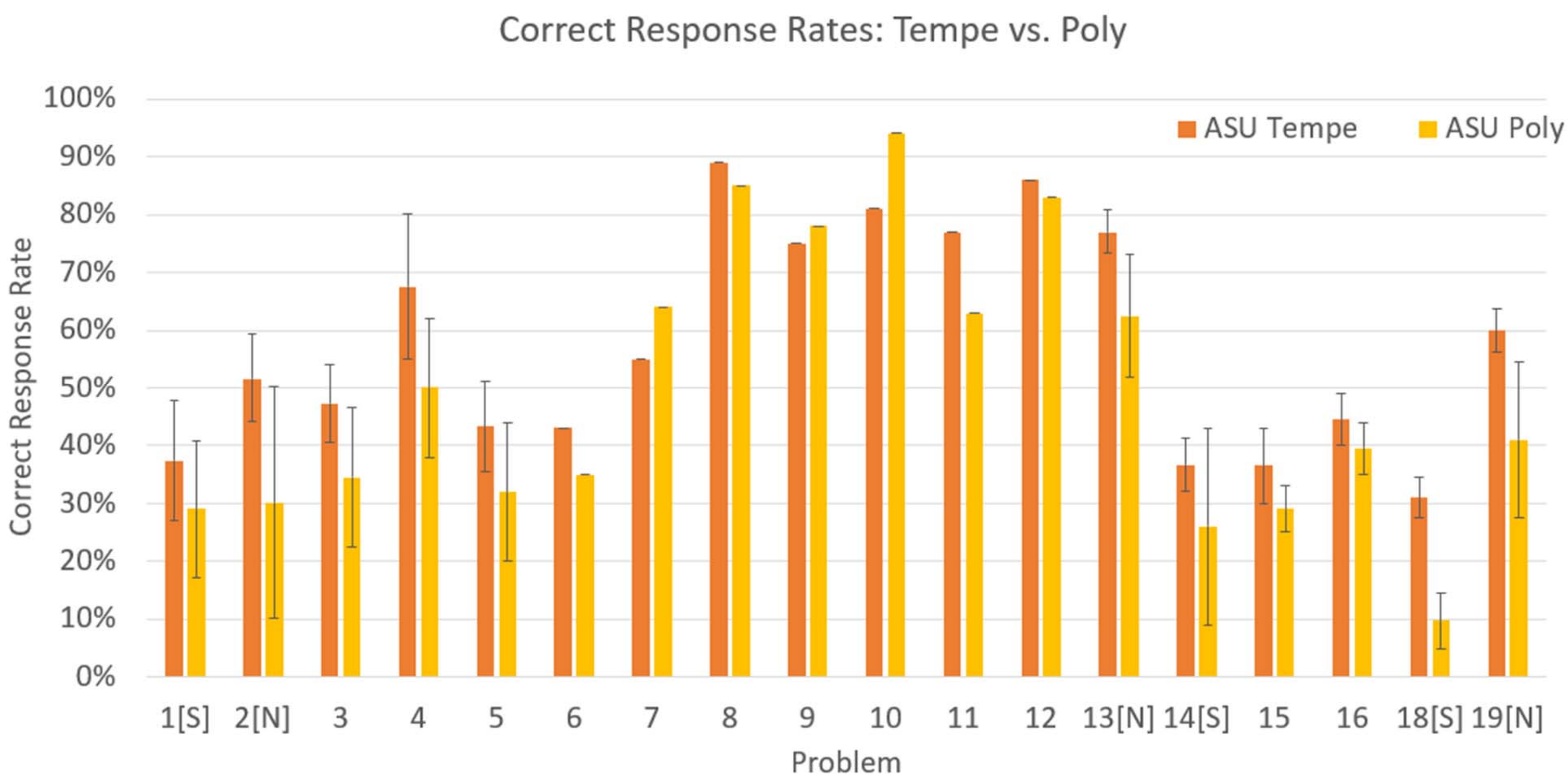
Arizona State University: Algebra-based; 1st semester

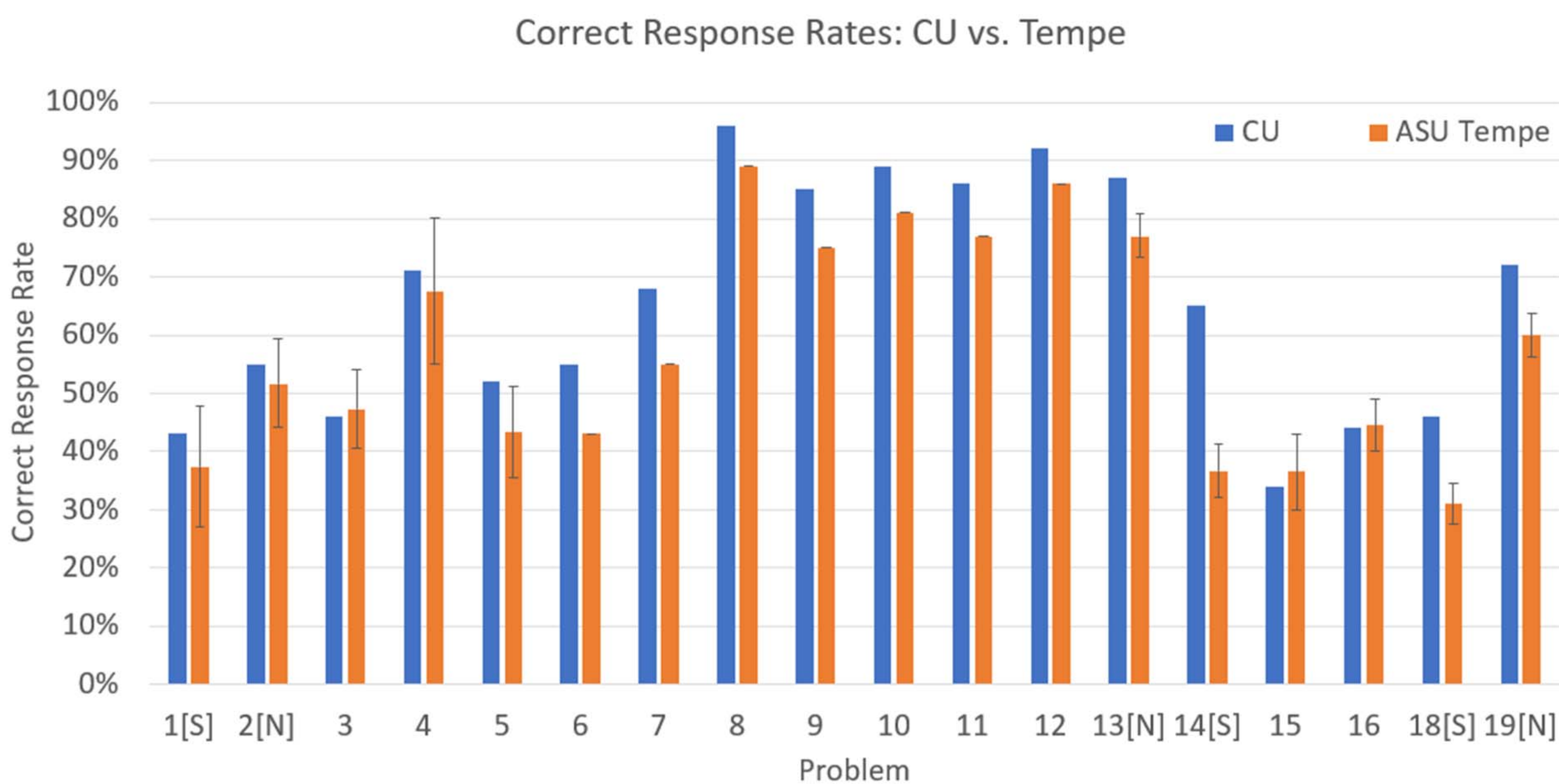
- Tempe
- Poly

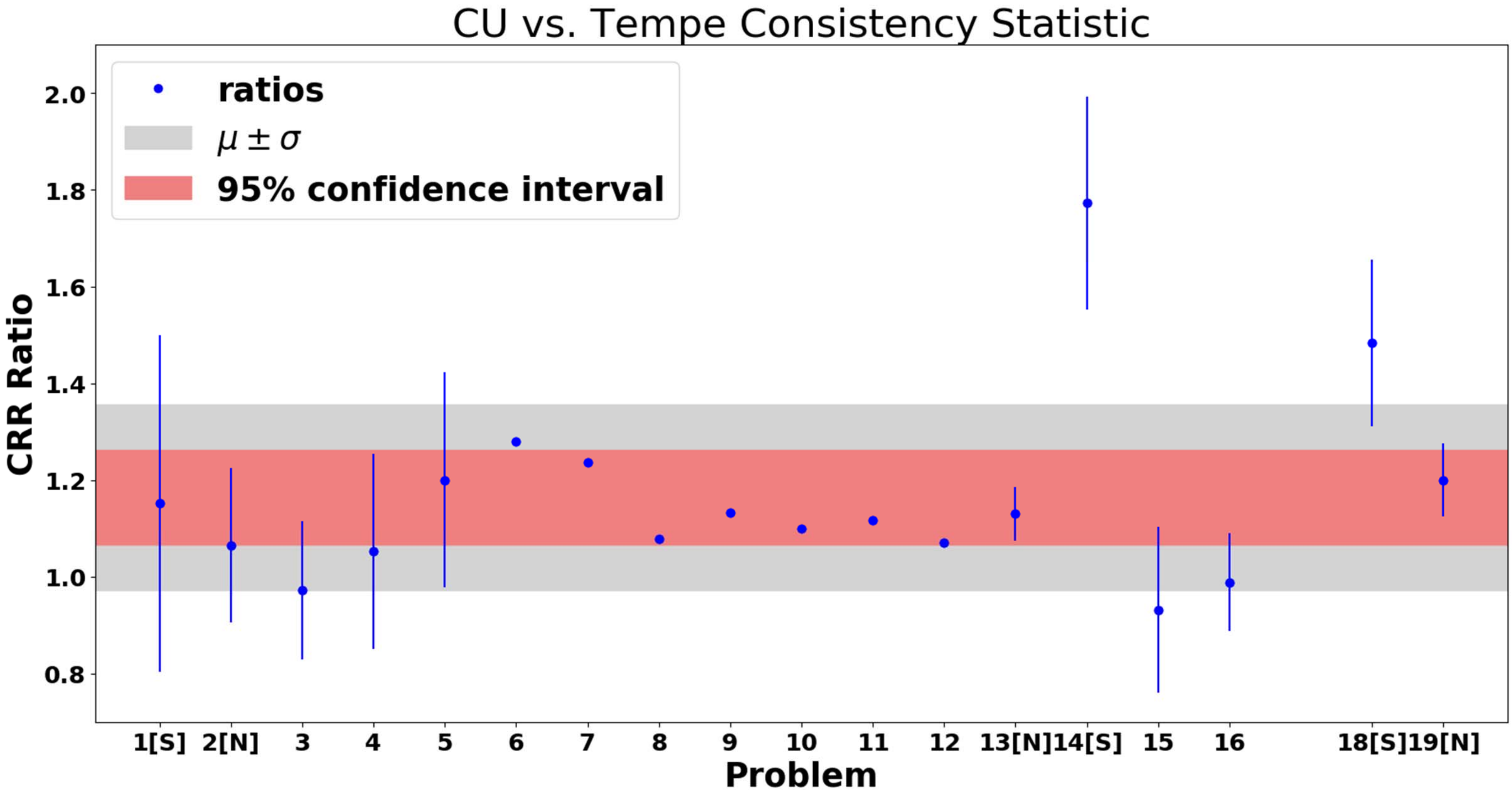
University of Colorado: Algebra-based, 1st semester



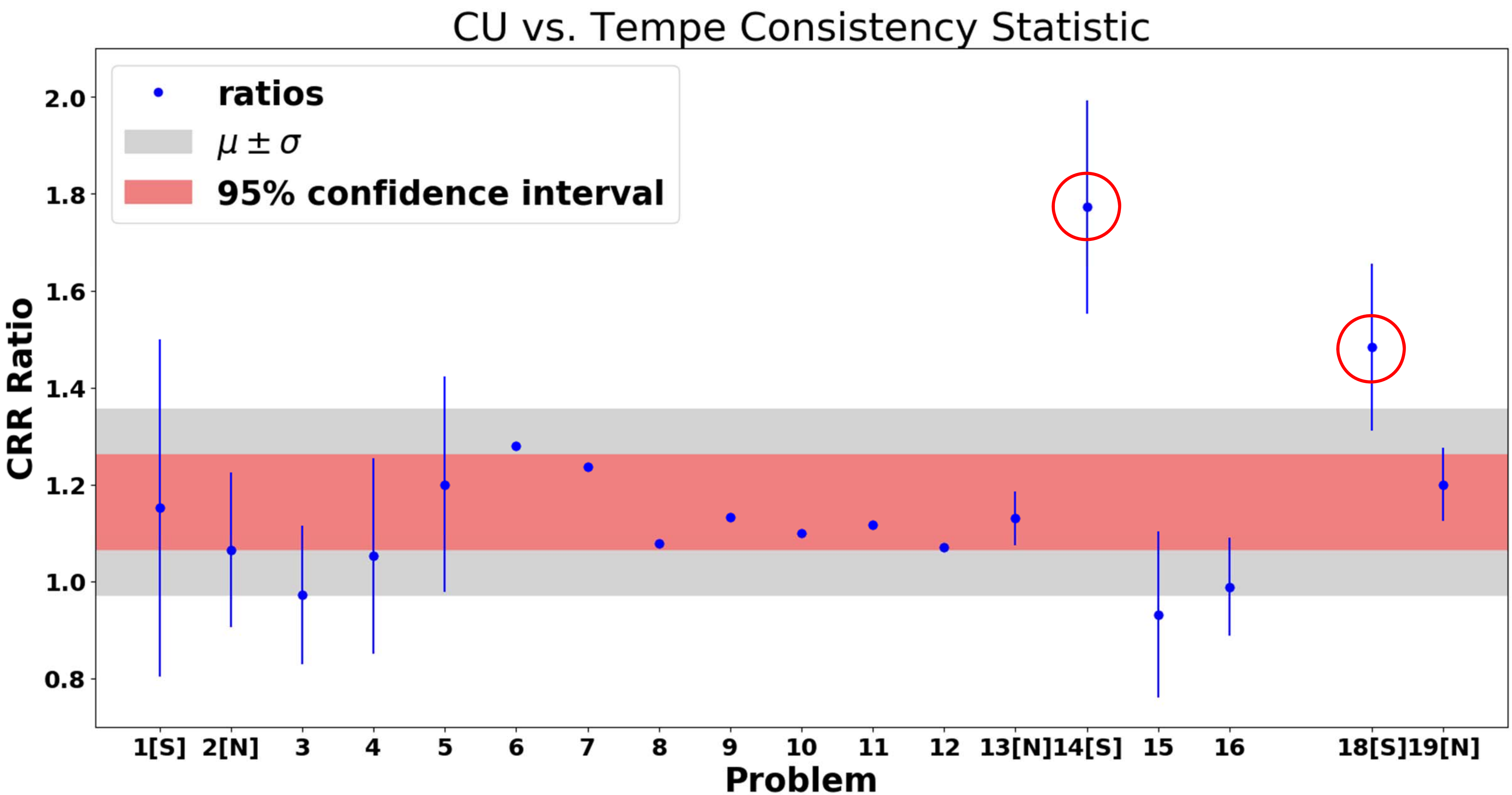
Correct Response Rates: All Problems and Campuses
CU
ASU Tempe
ASU Poly
Correct Response Rate
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
1[S]
2[N]
3
4
5
6
7
8
9
10
11
12
13[N]
14[S]
15
16
18[S]
19[N]
Problem


Correct Response Rates: Tempe vs. Poly
ASU Tempe
ASU Poly
Correct Response Rate
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
1[S]
2[N]
3
4
5
6
7
8
9
10
11
12
13[N]
14[S]
15
16
18[S]
19[N]
Problem

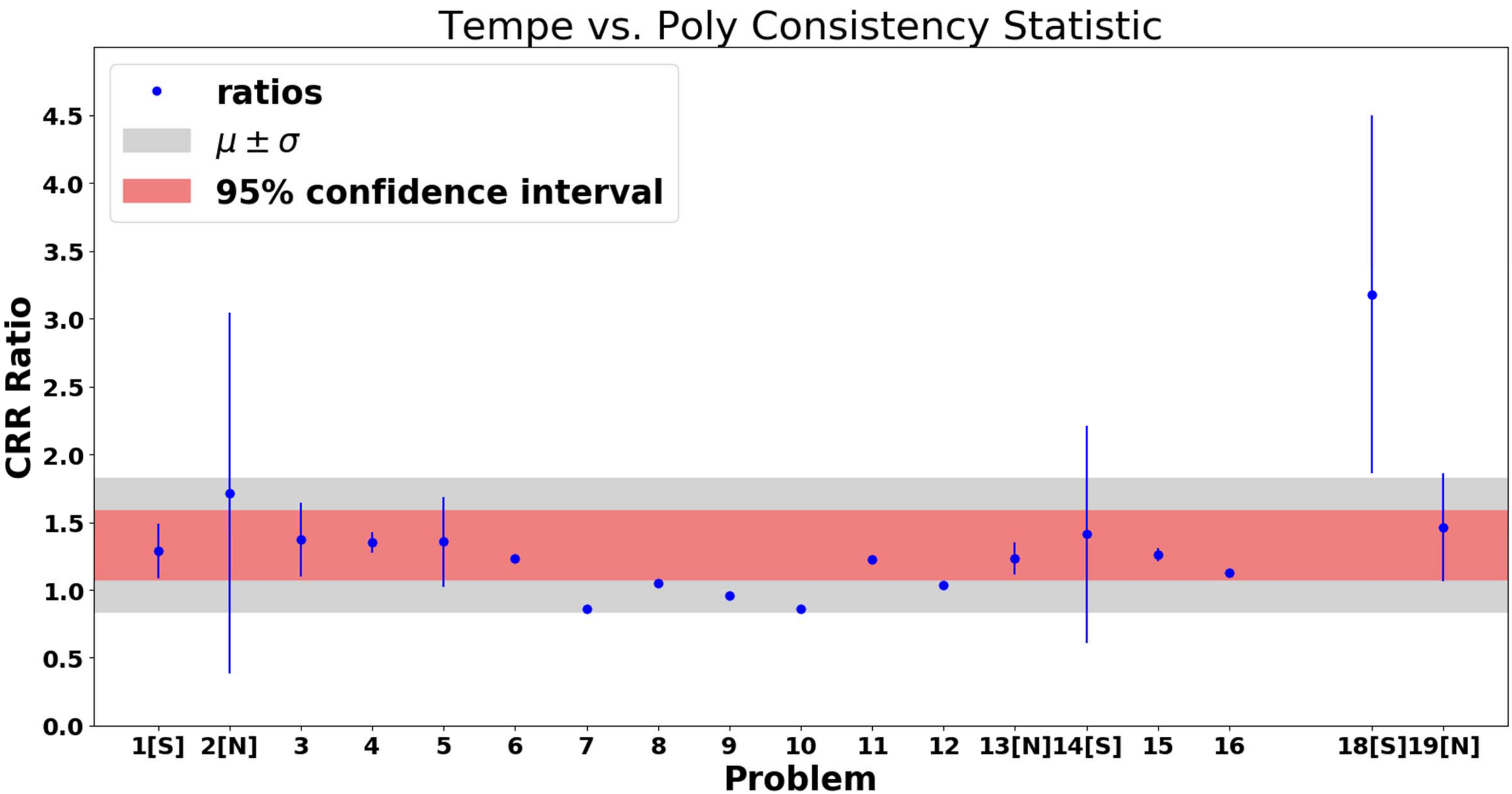

Correct Response Rates: CU vs. Tempe
CU
ASU Tempe
Correct Response Rate
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
1[S]
2[N]
3
4
5
6
7
8
9
10
11
12
13[N]
14[S]
15
16
18[S]
19[N]
Problem



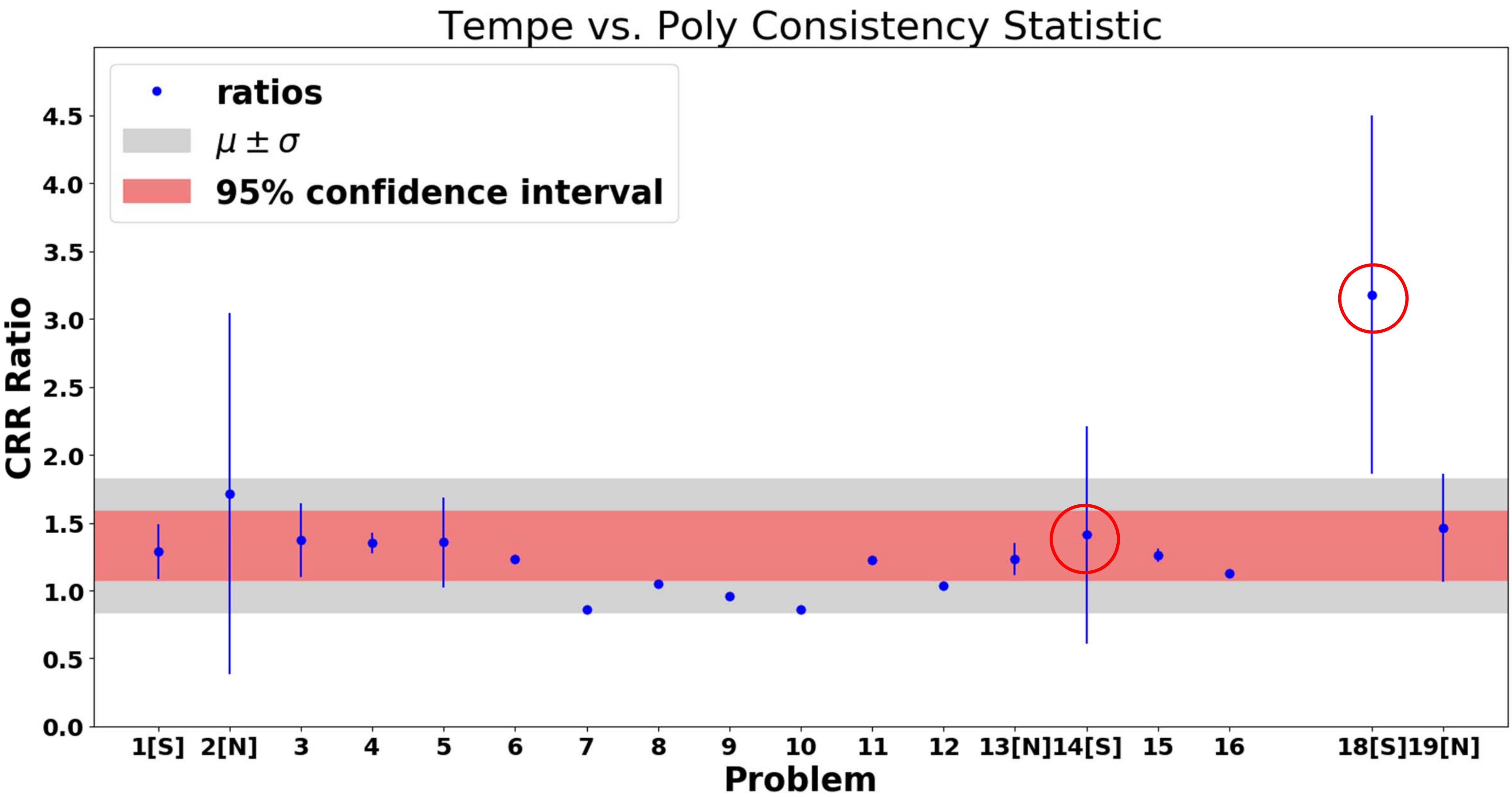
CU vs. Tempe Consistency Statistic
ratios
$\mu \pm \sigma$
95% confidence interval
CRR Ratio
2.0
1.8
1.6
1.4
1.2
1.0
0.8
1[S]
2[N]
3
4
5
6
7
8
9
10
11
12
13[N]
14[S]
15
16
18[S]
19[N]
Problem



CU vs. Tempe Consistency Statistic
ratios
$\mu \pm \sigma$
95% confidence interval
CRR Ratio
2.0
1.8
1.6
1.4
1.2
1.0
0.8
1[S]
2[N]
3
4
5
6
7
8
9
10
11
12
13[N]
14[S]
15
16
18[S]
19[N]
Problem



Tempe vs. Poly Consistency Statistic
ratios
μ ± σ
95% confidence interval
CRR Ratio
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
1[S]
2[N]
3
4
5
6
7
8
9
10
11
12
13[N]
14[S]
15
16
18[S]
19[N]
Problem



Tempe vs. Poly Consistency Statistic
ratios
$\mu \pm \sigma$
95% confidence interval
CRR Ratio
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
1[S]
2[N]
3
4
5
6
7
8
9
10
11
12
13[N]
14[S]
15
16
18[S]
19[N]
Problem

Item Responses Reflect Institutional Differences

- The correct-response rate (CRR) for CU on the 19 test items averages 16% higher than those at ASU-Tempe, while Tempe averages 33% higher than Poly, with ratios of all but two test items falling within fairly narrow bands (mean +/− 1 sd).
- Conjecture #1: The differences in mean CRRs reflect differences among the institutions' student populations
- Conjecture #2: Most of the (otherwise diverse) test items probe operational ability to similar "degrees"
- Conjecture #3: Another "level" of operational-ability difference is probed by the multi-step symbolic test items

Why the Difficulties with Symbols?

Some Suggestions Arising from the Interviews

- In elementary math courses, “simplified forms” of equations are emphasized (i.e., few messy symbols and functions).
- Many students get “overloaded” by seeing all the variables, and are unable to carry out procedures that they do successfully with numbers.
- Many students have had *insufficient practice* with algebraic operations to avoid being overwhelmed by standard algebraic manipulations.
  - Students tend to become *careless*

Error Types

- "Operational" Errors: Inadequate learning or expertise with fundamental operations
  - ***Conceptual*** confusion, e.g., What is an inverse sine? What is slope?
  - Weak ***skills*** with numerical and/or algebraic operations (e.g., factoring)
  - Inadequate ***practice*** with symbolic operations (e.g., dividing fractions)
- "Non-operational" Errors: Difficulties connecting context of problem to context in which operations were learned
  - Physics context, e.g., position-time graph with appropriate units
  - Problems involving multiple linked steps, each involving basic operations

Weak Operational Skills, or Carelessness?

- We define “non-operational errors” as errors that occur when the student apparently demonstrates knowledge of the mathematical operations needed to solve individual steps of a multi-step problem, yet fails to solve the problem correctly
  - causes for error might include not accessing previously learned skills, or not exercising sufficient care.
- With certain assumptions, we can estimate the percentage of students that solved certain problems incorrectly because of “non-operational errors”

Possible Origins of Errors

- We assume several different possible sources for students' errors:
  - Difficulty with operations: Inadequate learning or expertise with fundamental operations, including symbolic operations
  - Difficulty accessing knowledge: Students don't connect context of problem to context in which operations were learned, e.g., "multi-step" context
  - "Careless" errors, due to simple inattention, lack of checking, etc.; can be corrected (in principle) by greater attentiveness.
    - (*Note:* ≈50% of errors were "self-corrected" during interviews)