

Exploring the Factors Underlying Physics Students' Mathematical Difficulties

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The Problem

- Difficulties with basic math skills impact performance of introductory physics students
- The difficulties were evidently not resolved by students' previous mathematical training
- Students can't effectively grapple with physics ideas when they feel overburdened in dealing with calculational issues

Our Approach

- **Assess** nature and scope of difficulties using written diagnostic instruments and one-on-one oral interviews
- **Address** students' mathematical difficulties within the context of physics classes themselves, using in-class and out-of-class instructional materials

Work to Date

- Administer written diagnostic to 1300 students in 14 algebra- and calculus-based physics classes over three semesters at Arizona State University during 2016-2017
- Carry out individual interviews with 56 students enrolled in those or similar courses during same period

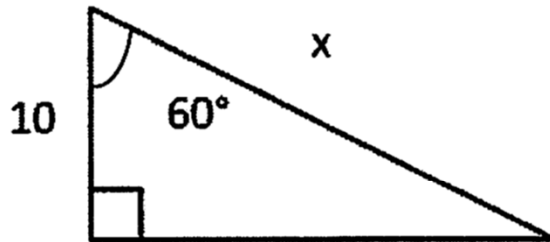
Written Diagnostic

- Questions involving trigonometry, vectors, and algebra; calculators allowed
- Administered as no-credit quiz during first week labs and/or recitation sections in four courses (first- and second-semester algebra- and calculus-based introductory physics), at two different campuses of ASU
- Total number of courses: 14; total $N > 1300$

Trigonometry Questions

with sample student responses

1.



What is the value of x?

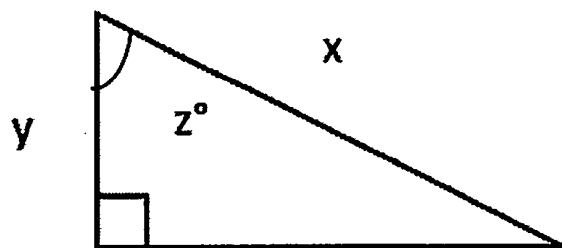
$$\cos 60 = \frac{10}{x}$$

$$x \cos 60 = 10$$

$$x = \frac{10}{\cos 60}$$

$$= 20$$

2.

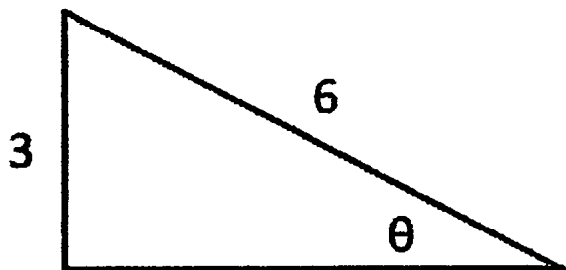


$$\cos z = \frac{y}{x}$$

What is the value of x ?

- A. $y \cos(z)$
- B. $y \cos(z) \sin(z)$
- C. $y / \sin(z)$
- D. $y \sin(z)$
- E. $y \cos(z) / \sin(z)$
- F. $y / \cos(z)$
- G. None of the above _____

3.



What is the value of θ ?

$$\sin^{-1}(\theta) = \sin^{-1}\left(\frac{3}{6}\right)$$

$$\theta = 30^\circ$$

Factors Affecting Results

Differences in results observed between:

- Spring- and fall-semester offerings of same course
- Offerings of same course on different campuses (Polytechnic campus in Mesa, main campus in Tempe)
- Different type of courses (algebra- and calculus-based) and different semesters of same course (first-semester and second-semester)
- Multiple-choice and non-multiple-choice versions of same questions

Trigonometry Questions:

Spring/Fall Semester Difference

Error Rate (% incorrect responses)

Algebra-based course, first semester; #1-3 combined:

ASU Polytechnic campus, Spring ($N = 72$): 67%

ASU Polytechnic campus, Fall ($N = 44$): 58%

Algebra-based course, second semester; #1-3 combined:

ASU Polytechnic campus, Spring ($N = 52$): 59%

ASU Polytechnic campus, Fall ($N = 27$): 44%

Calculus-based course, first semester; #1 only:

ASU Polytechnic campus, Spring ($N = 104$): 40%

ASU Polytechnic campus, Fall ($N = 98$): 56%

Trigonometry Questions:

Spring/Fall Semester Difference

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Calculus-based course, first semester; #1 only:

ASU Polytechnic campus, Spring ($N = 104$): 40%

ASU Polytechnic campus, Fall ($N = 98$): 56% ←

Trigonometry Questions: Polytechnic/Tempe Campus Difference

Error Rate (% incorrect responses)

Algebra-based course, second semester; #1-3 combined:

ASU Polytechnic campus, Spring ($N = 52$): **59%**

ASU Tempe campus, Spring ($N = 61$): **35%**

Trigonometry Questions: Course Difference

Error Rate (% incorrect responses), #1 only

ASU Polytechnic campus, Spring + Fall average:

Algebra-based course, 1st semester, ($N = 166$): 59%

Algebra-based course, 2nd semester, ($N = 106$): 53%

Calculus-based course, 1st semester, ($N = 202$): 48%

Trigonometry Questions: Multiple-Choice vs. Non-Multiple-Choice

(Higher Error Rate on Non-Multiple-Choice [Non-MC])

Error Rate Difference (% incorrect responses), Non-MC–MC

Course #1, Problem #2: +15

Course #1, Problem #3: +18

Course #2, Problem #2: +9

Course #2, Problem #3: +9

Course #3, Problem #2: +5

Course #3, Problem #3: +34

Course #4, Problem #2: +10

Course #4, Problem #3: +5

Algebra: Simultaneous Equations

$$3x = 2y$$

$$5x + y = 26$$

What are the values of x and y ? Show all your steps. For example, $x = 2, y = 5$ (These are NOT the correct answers).

Error Rate (% incorrect responses)

Algebra-based course, second semester ($N = 123$): 30%

Algebra: Simultaneous Equations

10. Solve for y as a function of x from the following equations:

$$5x - 3t = 2$$

$$y = 3(3t + 2)$$

Algebra: Simultaneous Equations

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$$5x - 3t = 2$$

$$y = 3(3t + 2)$$

Error Rate (% incorrect responses)

Algebra-based course, second semester ($N = 52$): 69%

Algebra: Simultaneous Equations

$$x \cdot \cos(20^\circ) = y \cdot \cos(70^\circ)$$

$$x \cdot \cos(70^\circ) + y \cdot \cos(20^\circ) = 10$$

What are the values of x and y ? Show all your steps. Note: The value for x should NOT include y , and the value for y should NOT include x .

Error Rate (% incorrect responses)

Algebra-based course, second semester ($N=150$): 70-80%
(different campuses, slightly different versions)

Algebra: Simultaneous Equations

$$a \cdot x = b \cdot y$$

$$b \cdot x + a \cdot y = c$$

a , b , and c are constants.

What are the values of x and y in terms of a , b , and c ? Show all your steps. Note: The value for x should NOT include y , and the value for y should NOT include x .

Algebra: Simultaneous Equations

$$a \cdot x = b \cdot y$$

$$b \cdot x + a \cdot y = c$$

a, b, and c are constants.

What are the values of x and y in terms of a, b, and c? Show all your steps. Note: The value for x should NOT include y, and the value for y should NOT include x.

$$x = \frac{by}{a}$$

$$b\left(\frac{by}{a}\right) + ay = c$$

$$\frac{b^2y}{a} + ay = c$$

$$y\left(\frac{b^2}{a} + a\right) = c$$

$$y = \frac{c}{\left(\frac{b^2}{a} + a\right)}$$

$$x = \frac{b\left(\frac{c}{\left(\frac{b^2}{a} + a\right)}\right)}{a}$$

Algebra: Simultaneous Equations

$$a \cdot x = b \cdot y$$

$$b \cdot x + a \cdot y = c$$

a , b , and c are constants.

What are the values of x and y in terms of a , b , and c ? Show all your steps. Note: The value for x should NOT include y , and the value for y should NOT include x .

Algebra: Simultaneous Equations

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a , b , and c are constants.

What are the values of x and y in terms of a , b , and c ? Show all your steps. Note: The value for x should NOT include y , and the value for y should NOT include x .

Error Rate (% incorrect responses)

Algebra-based course, second semester ($N=150$): 80-90%
(different campuses, slightly different versions)

Sources of Difficulties

- Carelessness
 - Students *very frequently* self-correct errors during interviews
 - Evidence of carelessness on written diagnostic
- Skill practice deficit: Insufficient repetitive practice with learned skills
 - E.g., applying definitions of sine and cosine
- Conceptual confusion
 - E.g., not realizing that sides and angles of right triangle are related by trigonometric functions

Example of Carelessness?

(Algebra-based Course, 2nd semester)

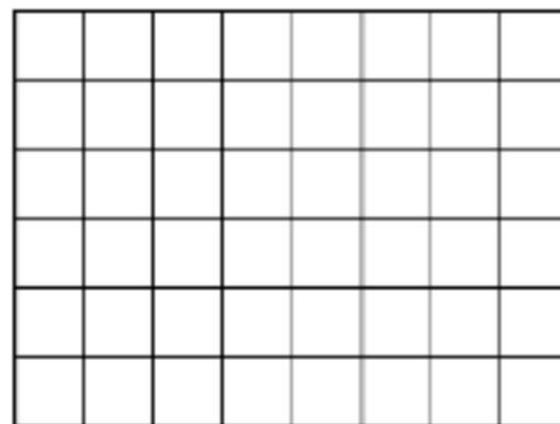
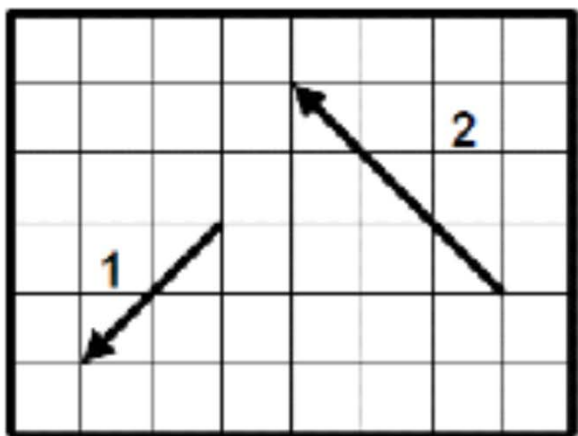
Error Rate, Multiple-Choice Version (N = 62): 73%

Error Rate, Non-Multiple-Choice Version (N = 61): 64%

6)

In the figure below there are two vectors $\vec{1}$ and $\vec{2}$. In the empty grid, draw the sum or vector addition \vec{R} of the two (i.e., $\vec{R} = \vec{1} + \vec{2}$).

Note: You can draw other vectors in the empty grid, but be sure to label \vec{R} clearly.



Example of Carelessness?

(Algebra-based Course, 2nd semester)

Error Rate, Multiple-Choice Version (N = 62): 73%

Error Rate, Non-Multiple-Choice Version (N = 61): 64%

Box A

Box B

Box C

Box D

Box E

Box F

Possible answers. Select the best one.

X

✓

How to Address Difficulties?

- Carelessness: (1) review and check steps; (2) find alternative solutions; (3) habitual use of estimation; (4) apply dimensional analysis (for physical problems)
- Skill deficit: Practice and repetition
- Conceptual confusion: Review and study of basic ideas

Summary:

What Options do Physics Instructors Have?

- Test to assess scope of problem
- Take time to review basic math
- Assign or suggest out-of-class math review practice
[We will be developing appropriate instructional materials]
- Reduce mathematical burden of syllabus
 - more qualitative problems, fewer problems requiring algebraic manipulation
- Nothing: Leave it up to the students