Evolution of Students' Ideas About Entropy

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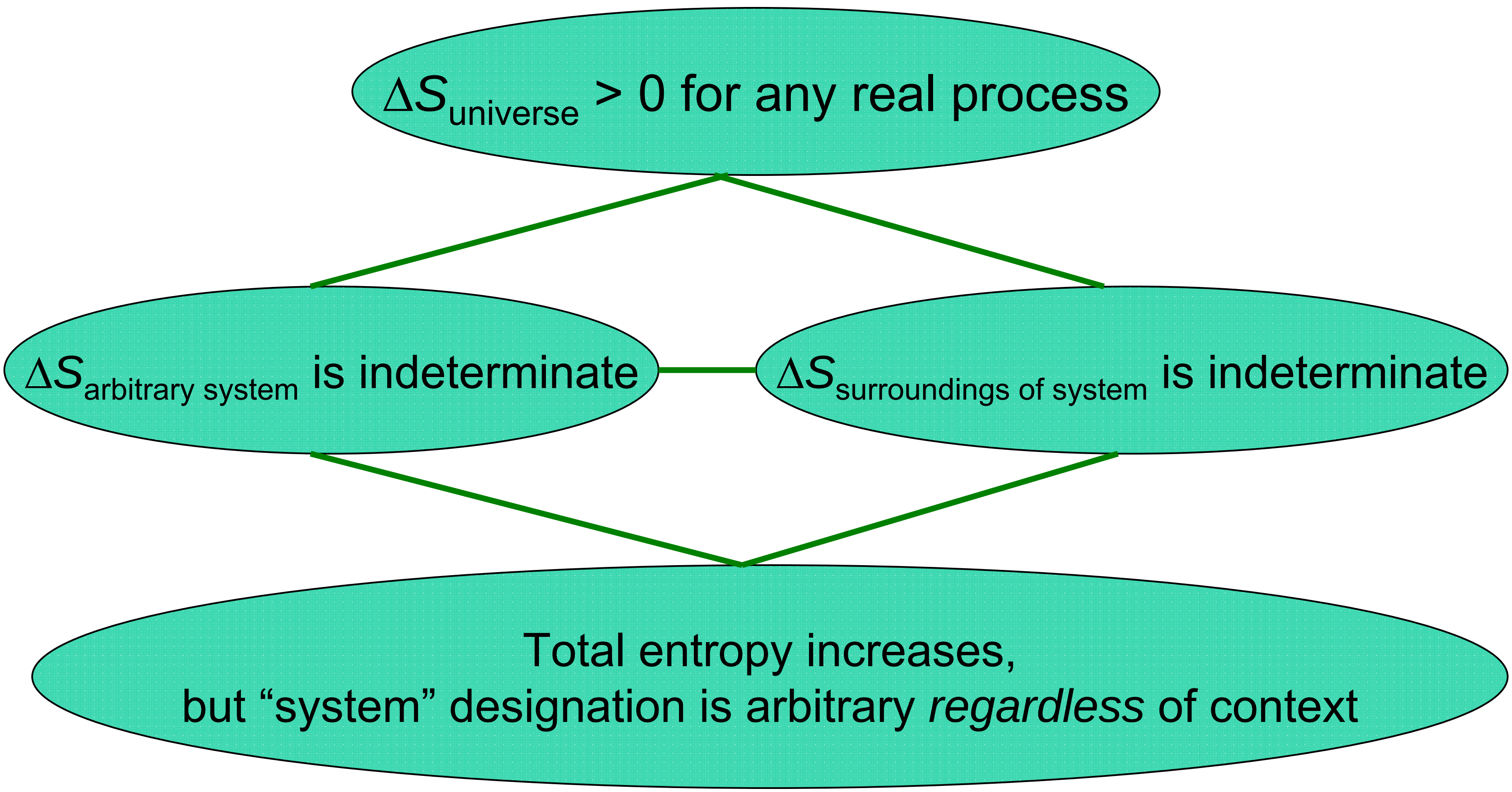
Student Ideas Regarding Entropy

- Students' thinking regarding entropy was probed in introductory calculus-based and upper-level (junior-level) physics courses.
- Here we highlight evolution of responses to questions regarding two separate ideas:
  – Total entropy increases in any real process
  – System-entropy change is zero in a cyclic process

Entropy-Increase Target Concepts



General-Context Question

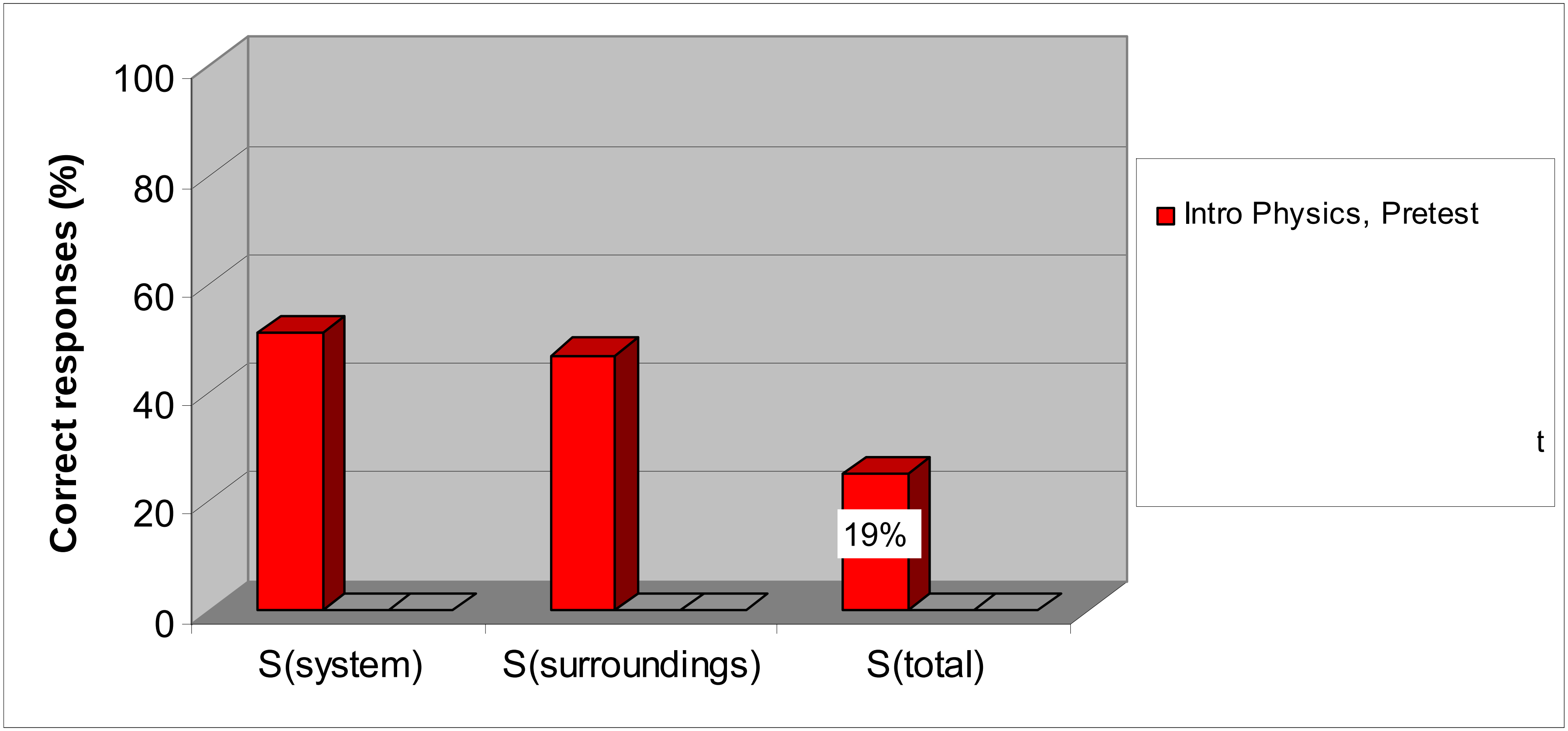
For each of the following questions consider a system undergoing a naturally occurring ("spontaneous") process. The system can exchange energy with its surroundings.

A. During this process, does the entropy of the **system** [S_{system}] *increase*, *decrease*, or *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

B. During this process, does the entropy of the **surroundings** [$S_{surroundings}$] *increase*, *decrease*, or *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

C. During this process, does the entropy of the system *plus* the entropy of the surroundings [$S_{system} + S_{surroundings}$] *increase*, *decrease*, or *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

Responses to General-Context Questions



Only 19% correct on "total entropy" question on pretest

"Concrete-Context" Question

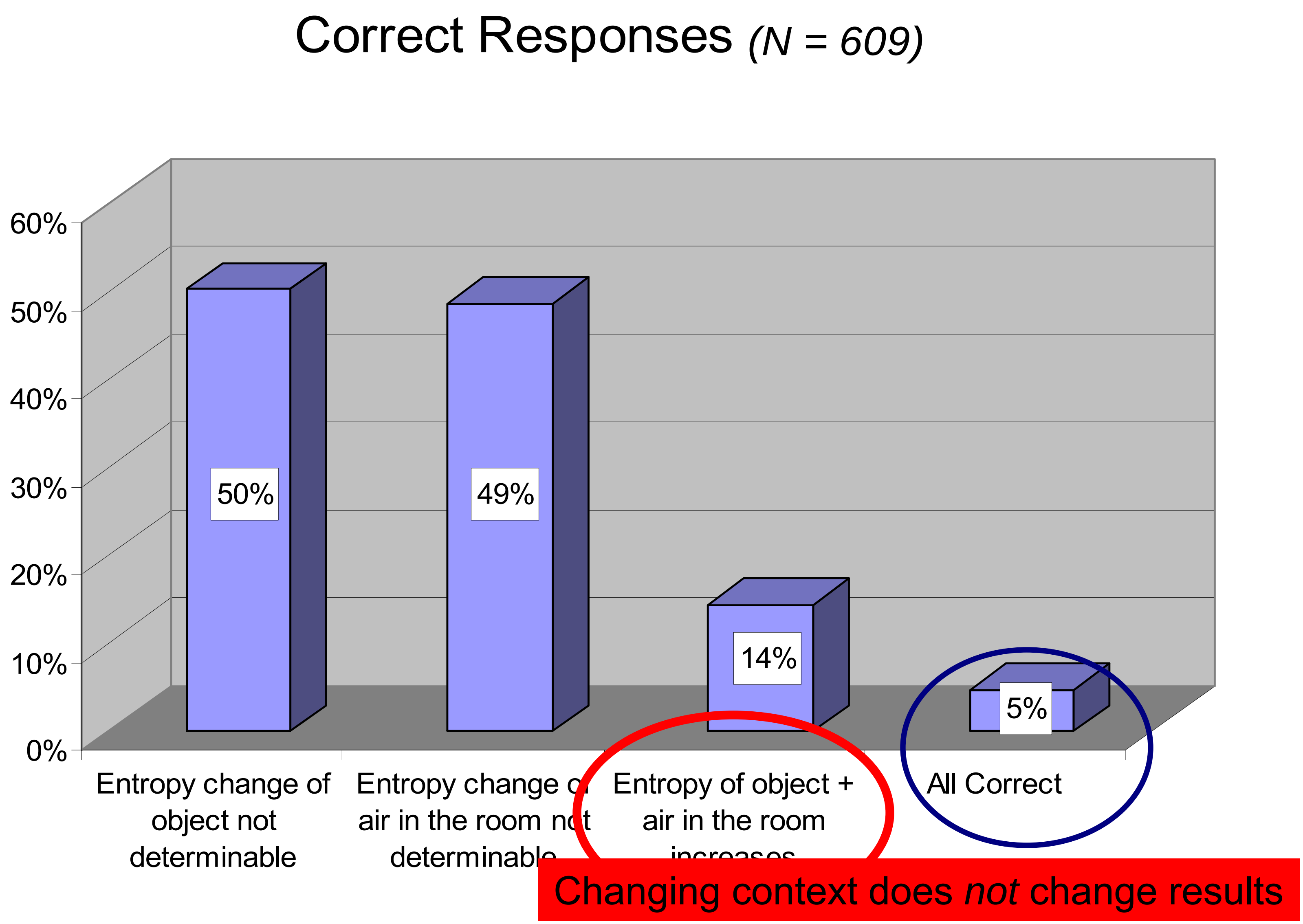
An object is placed in a thermally insulated room that contains air. The object and the air in the room are initially at different temperatures. The object and the air in the room are allowed to exchange energy with each other, but the air in the room does not exchange energy with the rest of the world or with the insulating walls.

A. During this process, does the entropy of the **object** [S_{object}] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

B. During this process, does the entropy of the **air in the room** [S_{air}] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

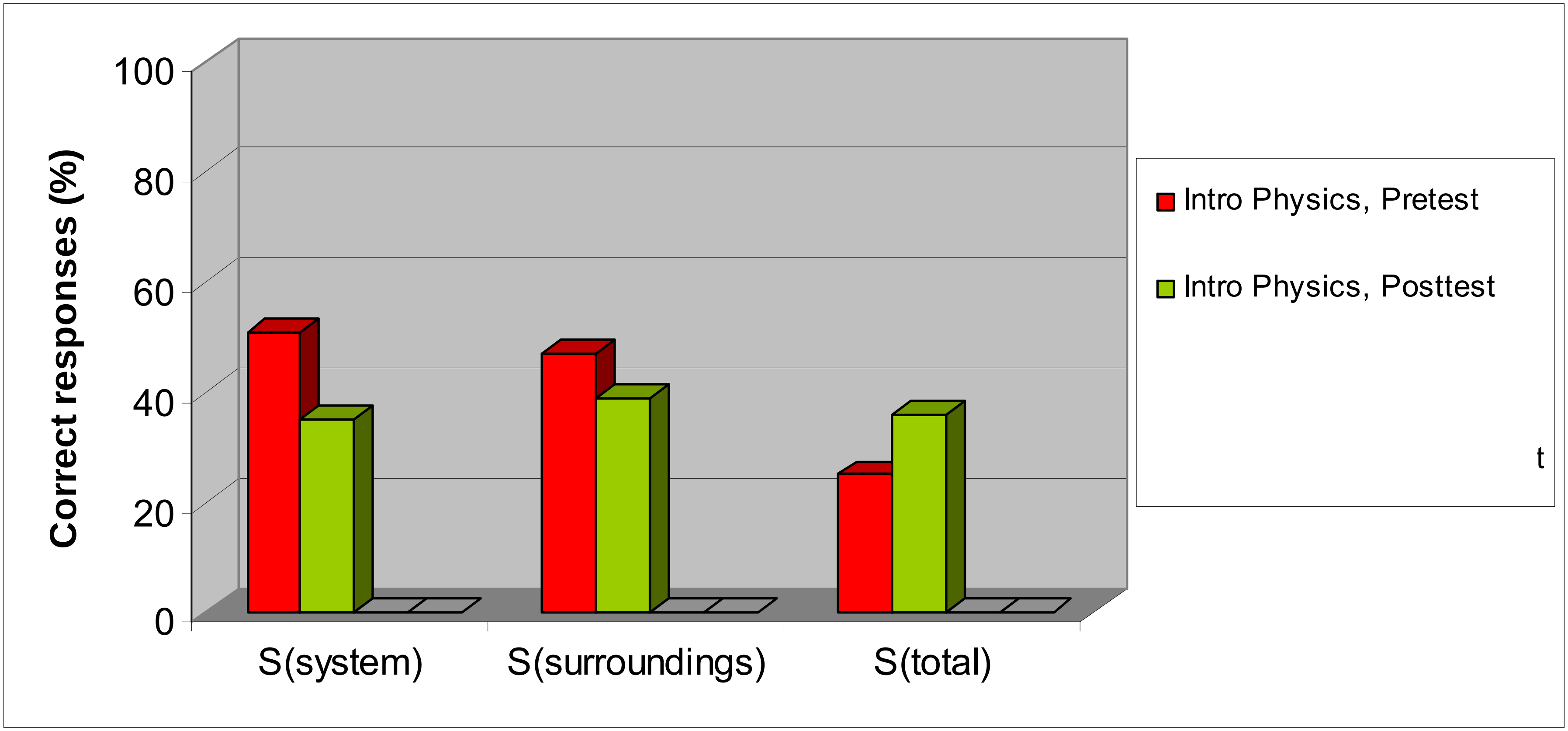
C. During this process, does the entropy of the object *plus* the entropy of the air in the room [$S_{object} + S_{air}$] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

Responses to Concrete-Context Questions



Responses to General-Context Questions

before … and *after* instruction…



Little change on post-test

“Total entropy” responses

- Nearly two-thirds of all students responded that the “total entropy” (“system plus surroundings” or “object plus air”) remains the same.
- We can further categorize these responses according to the ways in which the other two parts were answered
- 90% of these responses fall into one of two specific conservation arguments:

Conservation Arguments

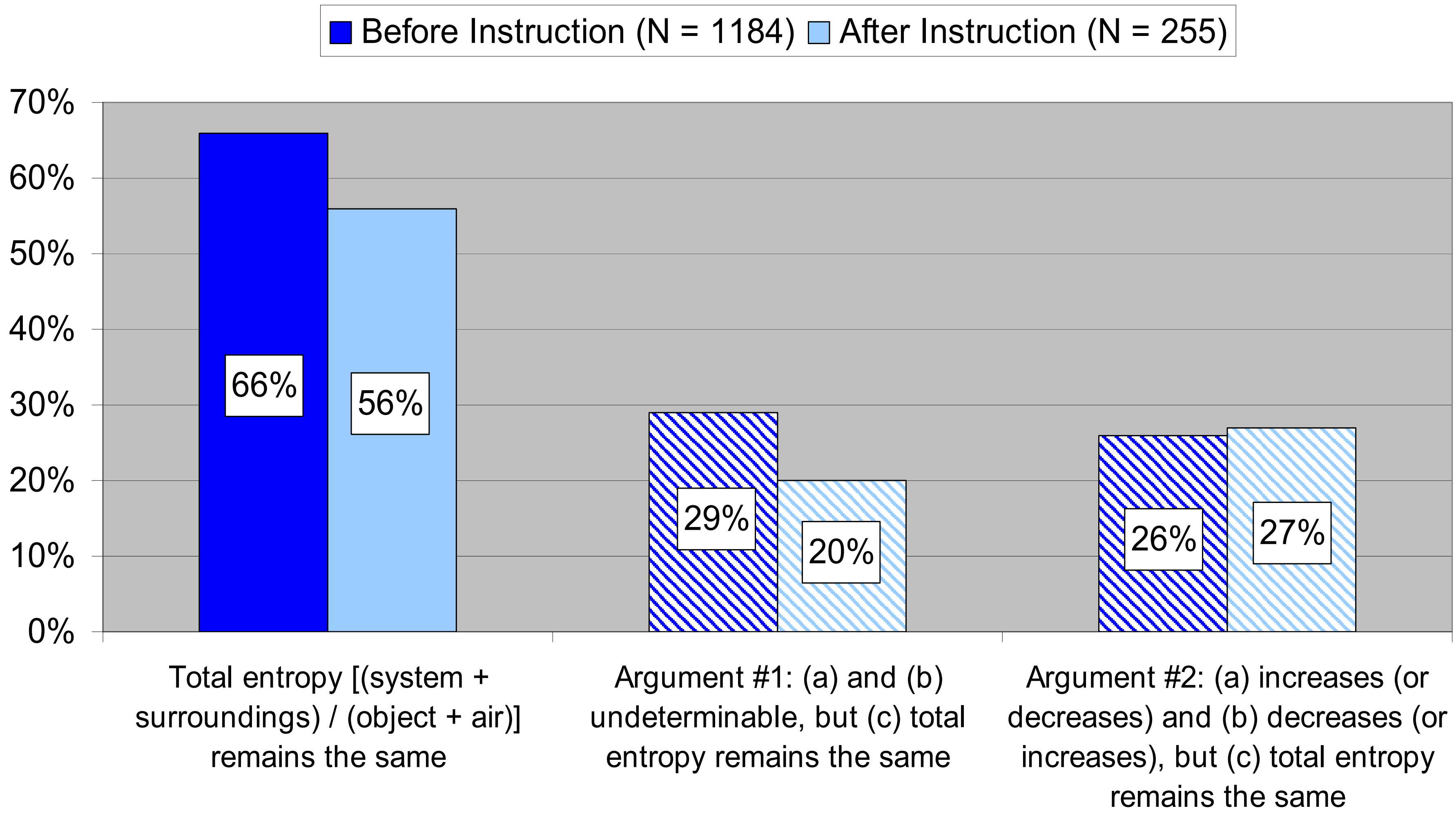
Conservation Argument #1

ΔS_{System} not determinable,
$\Delta S_{Surroundings}$ not determinable, and
$S_{System} + S_{Surroundings}$ stays the same

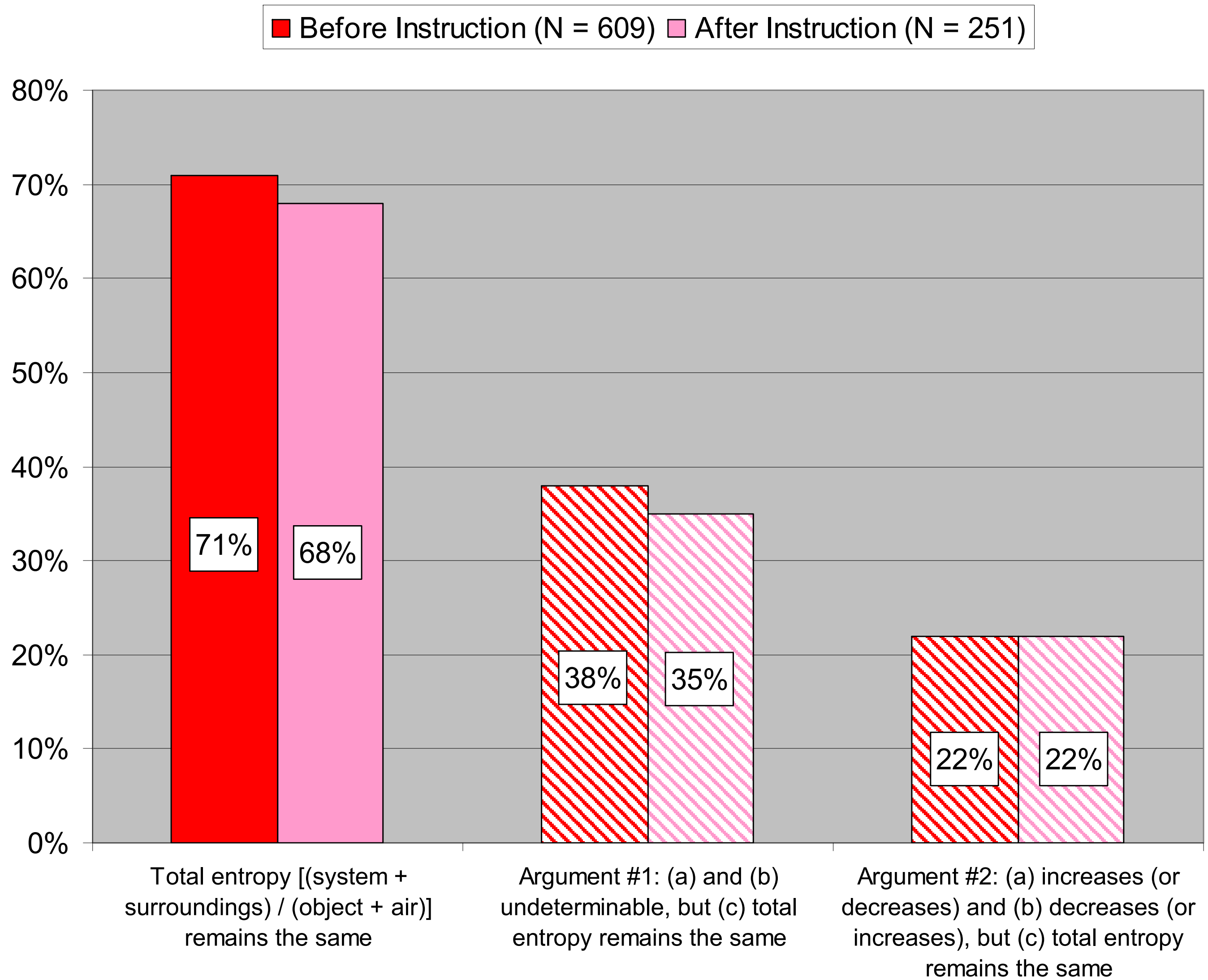
Conservation Argument #2

S_{System} increases [*decreases*],
$S_{Surroundings}$ decreases [*increases*], and
$S_{System} + S_{Surroundings}$ stays the same

General-Context Question Pre-Instruction vs. Post-Instruction



Concrete-Context Question, Pre-Instruction vs. Post-Instruction

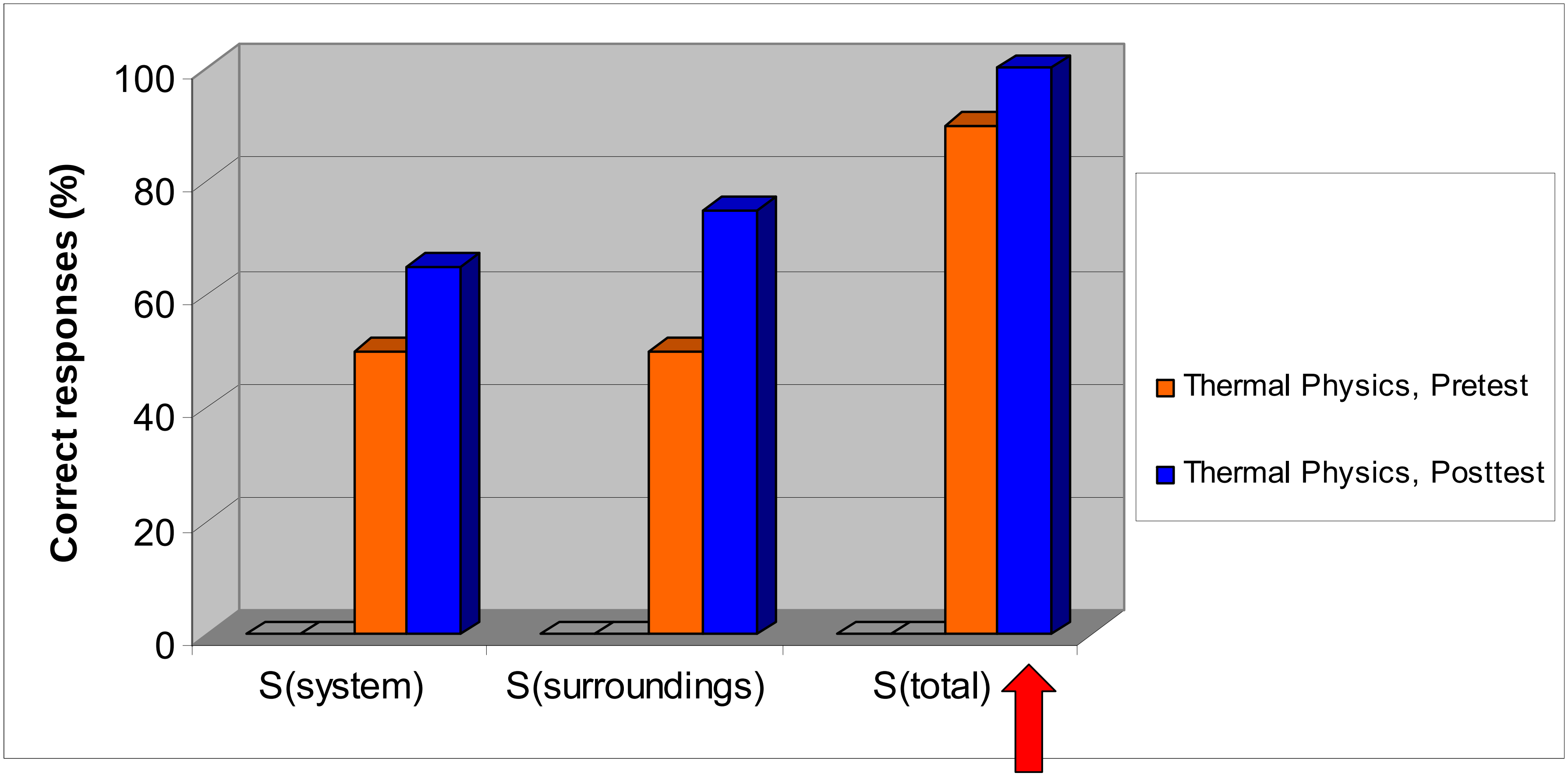


Introductory Physics Students' Thinking on Spontaneous Processes

- Tendency to assume that "system entropy" must *always* increase
- Do not accept the idea that entropy of system plus surroundings ***increases***
  - *Strong implied belief in "conservation" of total entropy*
  - *Little change after standard instruction*

Advanced Students [Junior-level Thermal Physics Course]

Responses to General-Context Question

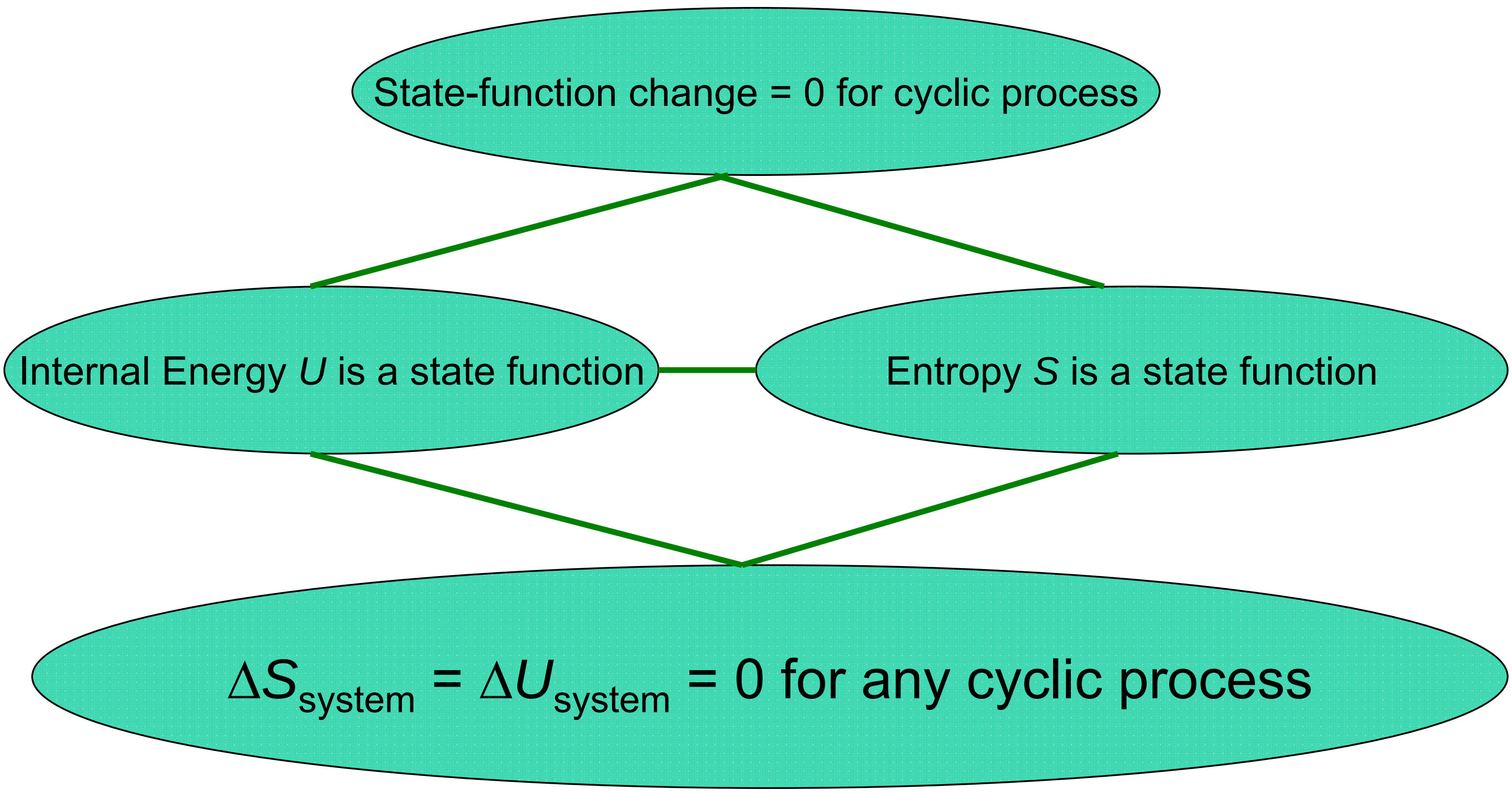


> 80% correct on “total entropy”

Thermal Physics Students’ Thinking on Spontaneous Processes

- Readily accept that “entropy of system ***plus*** surroundings increases”
  - in contrast to introductory students
- Tendency to assume that “system entropy” must *always* increase
  - similar to thinking of introductory students

Entropy-State-Function Target Concepts



Advanced Course, Pretest on Entropy

Consider a system consisting of a fixed quantity of ideal gas that undergoes a cyclic process. Consider one complete cycle (that is, the system begins in a certain state and returns to that *same* state).

(a) Is the *change* in internal energy ΔU (where $\Delta U = U_{\text{final state}} - U_{\text{initial state}}$) of the gas during one complete cycle *always equal to zero for any cyclic process* or ***not*** *always equal to zero for any cyclic process*? Explain.

(b) Is the *change* in entropy ΔS (where $\Delta S = S_{\text{final state}} - S_{\text{initial state}}$) of the gas during one complete cycle *always equal to zero for any cyclic process* or ***not*** *always equal to zero for any cyclic process*? Explain.

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Answer: $\Delta U_{\text{system}} = 0$ since process is cyclic, and U is a state function.

92% correct responses; N = *13*

(b) Is the *change* in entropy ΔS (where $\Delta S = S_{\text{final state}} - S_{\text{initial state}}$) of the gas during one complete cycle *always equal to zero for any cyclic process* or ***not*** *always equal to zero for any cyclic process*?

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54% correct responses; N = *13*

Heat Engines: Post-Instruction

- Following extensive instruction on second-law and implications regarding heat engines, graded quiz given as post-test

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For each process, there is heat transfer *to* the system at T = 400 K, and heat transfer *away from* the system at T = 100 K. There is no heat transfer at any other temperatures.

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For each process, there is heat transfer *to* the system at T = 400 K, and heat transfer *away from* the system at T = 100 K. There is no heat transfer at any other temperatures.

For each cyclic process, answer the following questions: Is the process a *reversible* process, a process that is *possible but irreversible*, or a process that is *impossible*? Explain. (You might want to consider efficiencies.)

$$\Rightarrow \eta_{Carnot} = 1 - \frac{T_{low}}{T_{high}} = 1 - \frac{100}{400} = 0.75 = \eta_{reversible} = \eta_{max}$$

← *Not* given

Cycle 1:
heat transfer at high temperature is 300 J;
heat transfer at low temperature is 100 J

Cycle 2:
heat transfer at high temperature is 300 J;
heat transfer at low temperature is 60 J

Cycle 3:
heat transfer at high temperature is 200 J;
heat transfer at low temperature is 50 J

Cycle 1:
heat transfer at high temperature is 300 J;
heat transfer at low temperature is 100 J

Cycle 2:
heat transfer at high temperature is 300 J;
heat transfer at low temperature is 60 J

Cycle 3:
heat transfer at high temperature is 200 J;
heat transfer at low temperature is 50 J

Cycle 1:
heat transfer at high temperature is 300 J;
heat transfer at low temperature is 100 J

$$\Rightarrow \eta_{process} = 1 - \frac{|Q_{low-T}|}{Q_{high-T}} = 1 - \frac{100}{300} = 0.67 < \eta_{reversible} = \eta_{max}$$

Alternative Method:

$$\Delta S_{total} = \Delta S_{surroundings} = \frac{100\,\mathrm{J}}{100\,\mathrm{K}} - \frac{300\,\mathrm{J}}{400\,\mathrm{K}} > 0$$

Process is *possible but irreversible*

55% correct with correct explanation (*N* = 15)

Cycle 1:

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40% correct with correct explanation ($N = 15$)

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At the *end* of the process, is the entropy of the system *larger than*, *smaller than*, or *equal to* its value at the *beginning* of the process?

Most common error: Assume $\Delta S_{system} = \sum_i \frac{Q_i}{T_i}$

(forgetting that this equation requires $Q_{\text{reversible}}$ and this is *not* a reversible process)

Cycle 1:
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At the *end* of the process, is the entropy of the system *larger than*, *smaller than*, or *equal to* its value at the *beginning* of the process?

Student performance on posttest was *worse* than pretest performance on similar question.

Summary

- Idea that “system” entropy must increase is persistent
- “Conservation of entropy” idea seems to fade
- Non-acceptance of state-function property of entropy remains resistant to change