Probing Students' Mathematical Difficulties in Introductory Physics

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Motivation

- Instructors often report apparent difficulties, among introductory university physics students, with mathematical skills and concepts normally taught in high school or earlier.

- The difficulties have apparently not been effectively addressed in students' previous mathematical education.

- Physics instructors therefore desire a means for addressing and correcting these difficulties within the context of physics classes *themselves.*

Our Approach

- *Investigate* difficulties with mathematical skills and reasoning among students in introductory physics courses
  - Use written diagnostic instruments, oral interviews, focus groups

- *Address* students' mathematical difficulties within the context of physics classes themselves, using both paper and on-line instructional materials
  - In-class and/or in-recitation instruction; supplementary remote instruction using written and on-line materials

Initial Work

- Administer written diagnostic to 257 students in algebra- and calculus-based physics classes during Spring 2016

- Carry out individual interviews with 12 students enrolled in those or similar courses during same period

Written Diagnostic

- Questions involving trigonometry (4 items), vectors (3 items), algebra (4 items); calculators allowed

- Administered as no-credit quiz during first week labs and/or recitation sections in four courses (first- and second-semester algebra- and calculus-based introductory physics) at Arizona State University, Polytechnic campus

Trigonometry Questions

with sample student responses
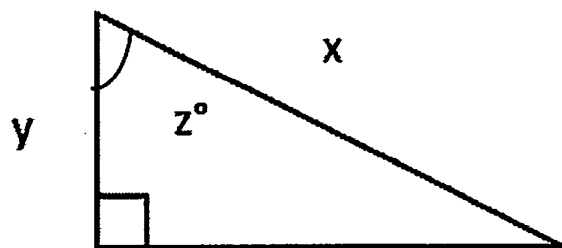
1.

10   60°   x
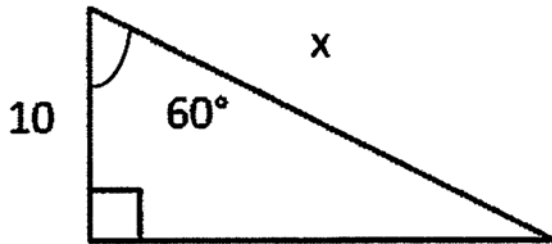
What is the value of x?

$$\cos 60 = \frac{10}{x}$$

$$x \cos 60 = 10$$
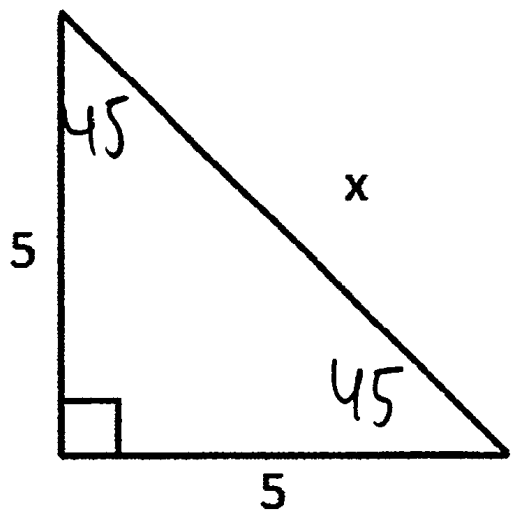
$$x = \frac{10}{\cos 60} = 20$$

2.

$$\cos z = \frac{5}{x}$$

What is the value of x?

A. ycos(z)

B. ycos(z)sin(z)

C. y/sin(z)

D. ysin(z)

E. ycos(z)/sin(z)

F. y/cos(z)

G. None of the above_____
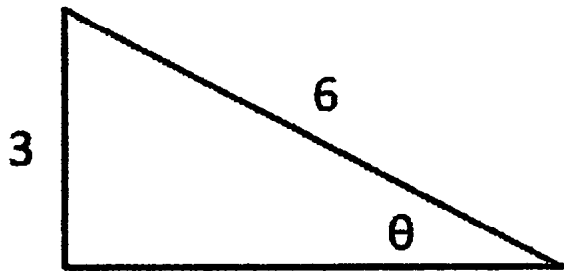
3.

45

5

x

45

5

What is the value of x?

$5\sqrt{2}$

$25 + 25 = x^2$

$50 = x^2$

$x = 5\sqrt{2}$

4.

What is the value of θ?

$$\sin^{-1}(\theta) = \sin^{-1}\left(\frac{3}{6}\right)$$

$$\theta = 30^{\circ}$$

Results on Trigonometry Questions

Calculus-based physics, second semester
[N = 29]

   3 or 4 items correct: 76%

   All 4 items correct: 59%

Calculus-based physics, first semester
[N = 104]

   3 or 4 items correct: 62%

   All 4 items correct: 45%

Results on Trigonometry Questions

Algebra-based physics, second semester
   [N = 52]

   3 or 4 items correct: 38%

   All 4 items correct: 23%

Algebra-based physics, first semester
   [N = 72]

   3 or 4 items correct: 28%

   All 4 items correct: 15%
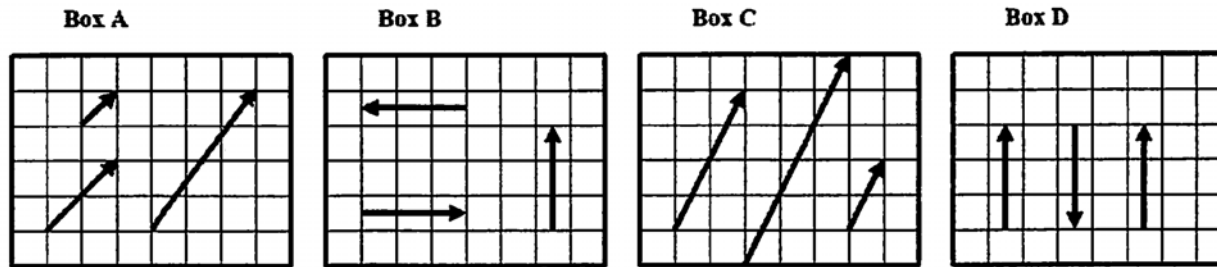
Results on Trigonometry Questions

Most common error, by far: item left blank

Other discernable errors: use of incorrect trig function (e.g., cosine instead of sine), calculator on radians, algebra errors

Interview sample (one item different): 3 of 4 algebra-based students made at least one error; only 1 of 8 calculus-based students made any errors

Vector Direction Question

7. In the four boxes below are collections of vectors on top of equally spaced grid lines. Choose the answer from the list below that most correctly describes the comparative **directions** of the vectors within each box.

Box A   Box B   Box C   Box D

Possible answers. Select the best one.

A. Box **A** has all vectors with the same direction
B. Box **B** has all vectors with the same direction
C. Box **C** has all vectors with the same direction
D. Box **D** has all vectors with the same direction
E. Both boxes **A** and **C** have vectors that all have the same direction
F. Both boxes **A** and **D** have vectors that all have the same direction
G. Both boxes **C** and **D** have vectors that all have the same direction
H. The boxes, **A, C,** and **D** have vectors that all have the same direction
I. None of the above boxes have vectors with the same direction

Vector Direction Question

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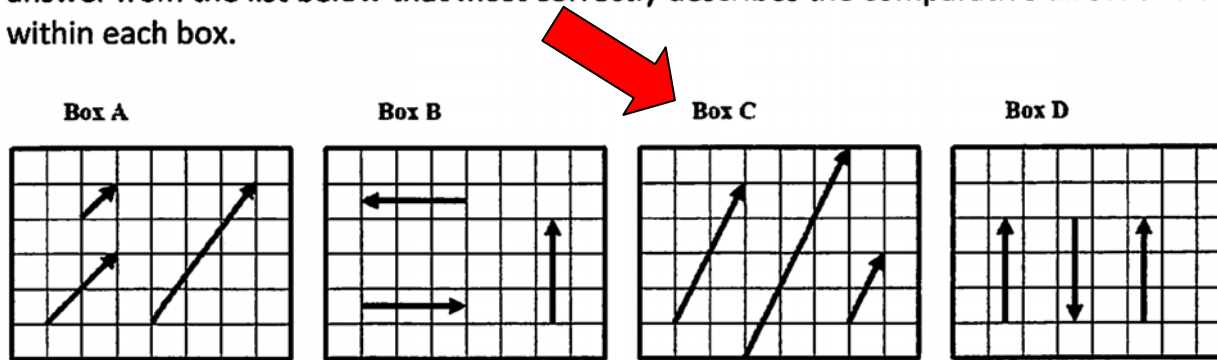

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For comparison…

A very similar question administered to students at Iowa State University in 2003:

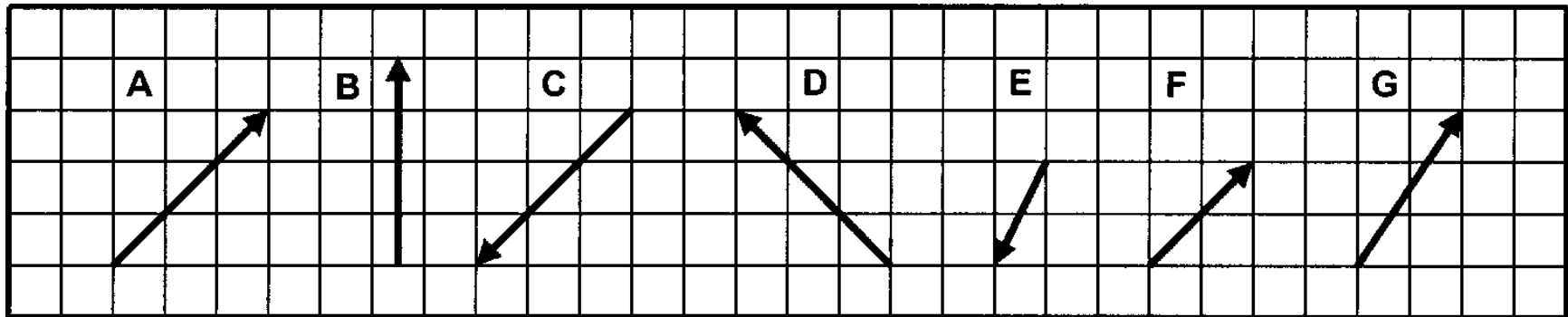# Vector Direction, ISU Diagnostic (2003)

2. List all the vectors that have the same **direction** as the first vector listed, $\vec{A}$. If there are none, please explain why.
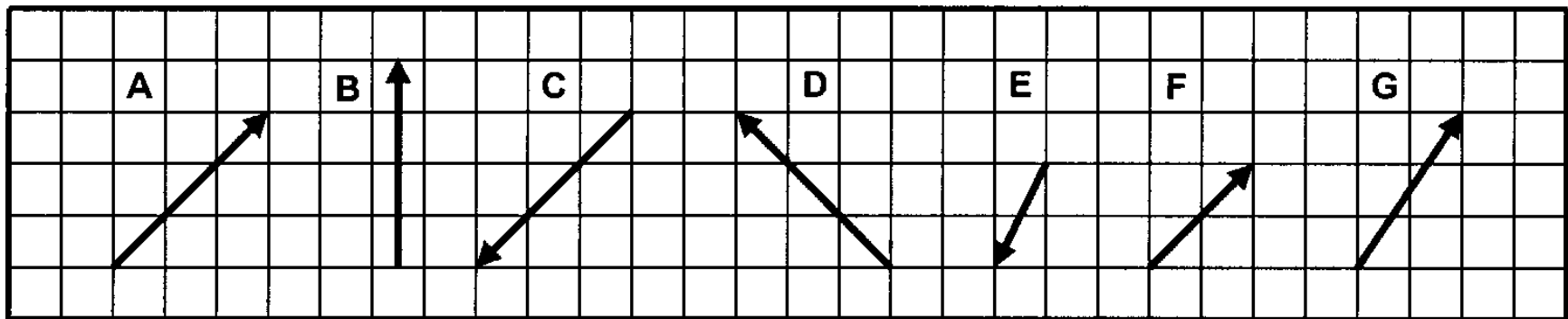
Explain _____

Vector Direction, ISU Diagnostic (2003)

2. List all the vectors that have the same **direction** as the first vector listed, $\vec{A}$. If there are none, please explain why.

Explain _____

Results on Vector Direction

Calculus-based physics, second semester
[N = 29]

Correct, ASU (2016): 66%

[Correct, Iowa State item #2 (2003): 77%]

Calculus-based physics, first semester
[N = 104]

Correct, ASU (2016): 51%

[Correct, Iowa State item #2 (2003): 71%]

Results on Vector Direction

Algebra-based physics, second semester

[N = 52]

Correct, ASU (2016): 40%

[Correct, Iowa State item #2 (2003): 64%]

Algebra-based physics, first semester
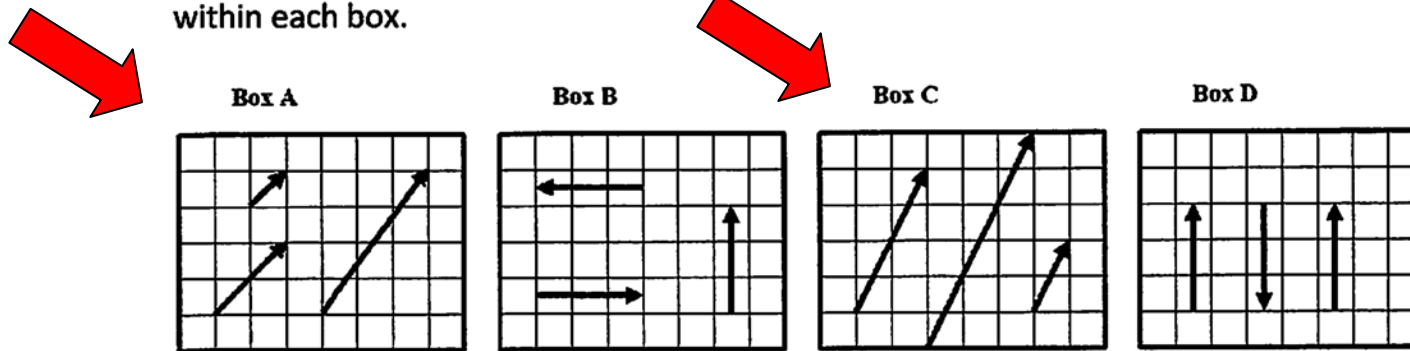
[N = 72]

Correct, ASU (2016): 40%

[Correct, Iowa State item #2 (2003): 55%]

Vector Direction, Most Common Error

7. In the four boxes below are collections of vectors on top of equally spaced grid lines. Choose the answer from the list below that most correctly describes the comparative **directions** of the vectors within each box.

Box A   Box B   Box C   Box D

Possible answers. Select the best one.

A. Box **A** has all vectors with the same direction
B. Box **B** has all vectors with the same direction
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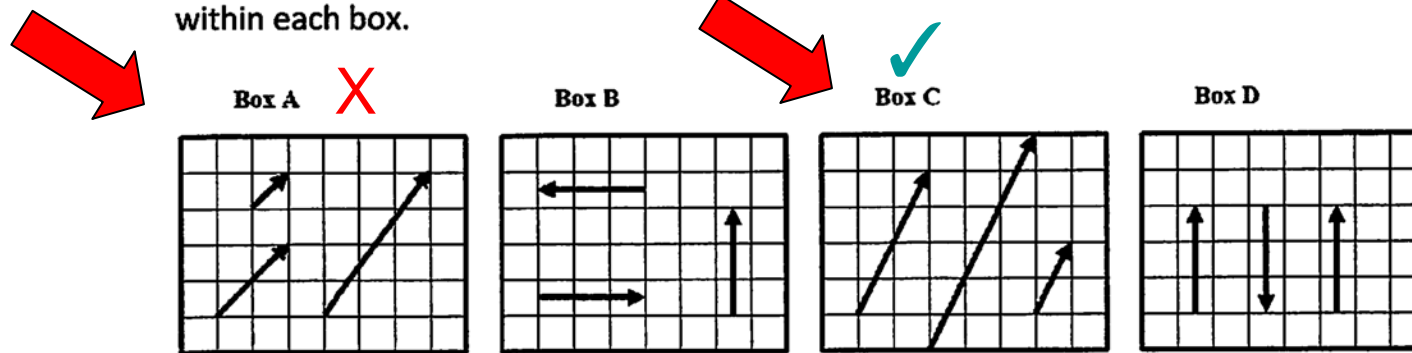
Box A X   Box B   ✓ Box C   Box D

Possible answers. Select the best one.

- A. Box **A** has all vectors with the same direction
- B. Box **B** has all vectors with the same direction
- C. Box **C** has all vectors with the same direction
- D. Box **D** has all vectors with the same direction
- E. Both boxes **A** and **C** have vectors that all have the same direction
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- I. None of the above boxes have vectors with the same direction

Algebra: Simultaneous Equations

10. Solve for y as a function of x from the following equations:

$5x - 3t = 2$

$y = 3(3t + 2)$

Algebra: Simultaneous Equations

10. Solve for y as a function of x from the following equations:

$5x - 3t = 2 \quad \Rightarrow \quad t = \dfrac{5x - 2}{3}$

$y = 3(3t + 2)$

$y = 3\left((5x - 2) + 2\right)$

$y = 15x$

Results on Algebra Question

Calculus-based physics, second semester
   [N = 29]

   Correct: 76%

Calculus-based physics, first semester
   [N = 104]

   Correct: 57%

Results on Algebra Question

Algebra-based physics, second semester
  [N = 52]

   Correct: 31%

Algebra-based physics, first semester
  [N = 72]

   Correct: 28%

Notable Observation

On all types of questions, students frequently *self-corrected* careless errors during the interviews, while such errors were common on the written instruments.

Summary

- Errors on basic math skills are common among introductory university physics students, including skills needed for many standard physics problems.

- Errors are frequently due to weakness with basic operations, and/or carelessness.

- Basic skills practice will likely be necessary as part of remedial treatment, in part to reduce prevalence of careless errors.