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Teaching algebra through physics-of-motion activities

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Preservice elementary and middle school teachers are often taught mathematics as if it were an isolated subject, a set of abstract algorithms lacking deeper meaning or reference to the real world. This project is providing preservice elementary and middle school teachers with an opportunity to deepen their understanding of mathematics content by incorporating physics-of-motion activities into an existing mathematics course during undergraduate preparation. This workshop will guide participants through a sample project lesson using simulated data of small carts rolling on low-friction tracks. The concept of a function is brought to life through tables of paired position/time values, while graphing of both linear and non-linear functions proceeds naturally through the creation of position vs. time, velocity vs. time, and acceleration vs. time graphs representing the cart's motion. The interpretation of the equation of a straight line and the meaning of slope is embodied in the relationship between the cart's velocity and the slope of the position-time graph; positive and negative slopes correspond to motions in opposite directions. Translations between different representations are accomplished by predicting and then observing the motion graphs that correspond to various motions that are first described in words and/or illustrated in diagrams. Equations that describe the motions are first written in symbolic form and then quantified more precisely through actual measurements of moving objects.

June 3, 2022

Workshop on Teaching Algebra Through Physics-of-Motion Activities

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Overview

- This project provides preservice elementary and middle school teachers opportunities to deepen their understanding of mathematics content, by incorporating physics-of-motion activities into an existing math course.
- This workshop utilizes activities that focus primarily on constant-velocity motion. These were used in a summer professional development workshop for instructors of a mathematics course for preservice elementary teachers.
- We will be using observational data based on videos of a person walking in a straight line at constant speed. Tables, graphs, and equations representing the data will be used to develop the concepts of linear function and slope.
- Worksheets employing activities involving nonlinear motion and quadratic variation will be provided, and briefly discussed.

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Instructional Strategy

- Students record observations of a person walking in a straight line at constant speed, first at a slow pace, and then at a fast pace.
- A position vs. time graph is plotted to represent the observations. The concepts of function and slope are introduced; using graphs, slopes of different linear functions are calculated and compared.
- The equation of a straight line is introduced. Other motions are used to introduce the meaning of *y*-intercept.
- Timing data from computer-linked motion sensors are used to represent nonlinear motion of a cart rolling down an inclined track.

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Abstract

Preservice elementary and middle school teachers are often taught mathematics as if it were an isolated subject, a set of abstract algorithms lacking deeper meaning or reference to the real world. This project is providing preservice elementary and middle school teachers with an opportunity to deepen their understanding of mathematics content by incorporating physics-of-motion activities into an existing mathematics course during undergraduate preparation. This workshop will guide participants through a sample project lesson using simulated data of small carts rolling on low-friction tracks. The concept of a function is brought to life through tables of paired position/time values, while graphing of both linear and non-linear functions proceeds naturally through the creation of position vs. time, velocity vs. time, and acceleration vs. time graphs representing the cart's motion. The interpretation of the equation of a straight line and the meaning of slope is embodied in the relationship between the cart's velocity and the slope of the position-time graph; positive and negative slopes correspond to motions in opposite directions. Translations between different representations are accomplished by predicting and then observing the motion graphs that correspond to various motions that are first described in words and/or illustrated in diagrams. Equations that describe the motions are first written in symbolic form and then quantified more precisely through actual measurements of moving objects.

ACTIVITY ONE: Lesson One, Part 1—Plotting Motion on a Graph

MTE 301

Groups: In the first part of this activity, we will be gathering data as a class. Students will be placed in groups of at least 15, preferably 20. There will be one “Starter,” one “Walker,” and everybody else will be “Timers.” Ideally, there should be 3 timers for each of the meter marks labeled 2 meters, 4 meters, 6 meters, 8 meters, 10 meters and 12 meters (described in more detail below).

Equipment Needed (per group):

- Stopwatches—Every person but two people in the group will need one (sometimes available on smart phones, or as a free downloadable app)
- One-meter stick or long tape measure with metric units
- 1 stick of sidewalk chalk or large stick of chalk;

Procedures:

1) Find an open space; it should be about 12 meters long and 2 or 3 meters wide.
2) With a tape measure or meter sticks, lay out a 10-to-15-meter straight-line path, with chalk markings at every meter, starting from a position marked “0 meters” at one end. A tick mark should be drawn every two meters and labeled with that distance. It should look like this:

12 Meters --

10 Meters --

8 Meters --

6 Meters --

4 Meters --

2 Meters --

0 Meters --

3) One person—the “Walker”—will walk the distance while all of the others time him/her. Decide who will be the Walker. You will also need someone to start the data collection. Decide who will be the Starter. All other group members will stand near one of the labeled meter distances. For example, you might have 2 to 3 students with stopwatches at each of the labeled distances (except for 0 meters).
4) Working in your large group of 20 or so, you will first carry out a few practice runs so that you are confident in how to gather the data. The Starter will begin the data collection by loudly counting down so everyone can hear. He/she will say “3, 2, 1, Go!” At the exact moment the Starter says the word “Go!,” every Timer will start their stopwatch. The Walker will walk at a slow, steady pace—neither speeding up nor slowing down—starting from the position marked “0 meters” and continuing until he/she passes the line marked “12 meters.” As he/she passes the students with the stopwatch who are standing at the meter marks, the students with the stopwatches will stop their watches. For example, if you are standing at “6 meters”

and the Walker walks by and crosses the chalk line, stop your stopwatch at the exact moment the Walker crosses your line. After the trial, compare your findings with other people at the same meter mark. Were your times similar? The times recorded by Timers at your meter mark should be really close to each other. Run at least 3 practice trials to make sure you are consistent in gathering your data.

5) Once you have finished at least 3 practice runs and you have consistency in gathering your data, you are ready to collect formal data. The Starter will begin the data collection by loudly counting down so everyone can hear. He/she will say "3, 2, 1, Go!" The Walker will walk at a slow, steady pace—neither speeding up nor slowing down—starting from the position marked "0 meters" and continuing until he/she passes the line marked "12 meters." The Walker needs to try to walk as consistently as possible. Record the times in the table to the nearest tenth of a second and then calculate the arithmetic mean (the average) for each of the three timers.

Data Collection

TABLE FOR EACH GROUP							
Position (meters)	**0**	**2**	**4**	**6**	**8**	**10**	**12**
Time (seconds), timer #1	0.0						
Time (seconds), timer #2	0.0						
Time (seconds), timer #3	0.0						
Time (average of 3 timers, in seconds)	0.0						

6) Answer the following questions in a few sentences:
Question 1: What was the purpose of running trials before data collection?

ANSWER:

Question 2: Why are times from several Timers being averaged rather than just collecting data from one Timer at each meter mark?

ANSWER:

7) Repeat the previous activity, beginning at step 5. This time, the Walker should walk at a steady *but faster* pace than was used in the first activity. A new table to represent data for this motion should be created. ***Note: This activity omitted in the Workshop.***

8) Make sure each student has a copy of the data from this lesson. Each student should have his/her own copy of the completed tables. The data will be used in the next part of the lesson. The data used from the next section is from the table entitled "Data Collection." Sample data are shown below.

Data Collection

TABLE FOR EACH GROUP							
Position (meters)	**0 m**	**2 m**	**4 m**	**6 m**	**8 m**	**10 m**	**12 m**
Time (seconds), timer #1	0.0	1.51	2.87	4.46	6.08	7.52	9.08
Time (seconds), timer #2	0.0	1.67	3.05	4.48	6.03	7.56	9.06
Time (seconds), timer #3	0.0	1.50	2.95	4.50	6.03	7.54	9.09
Average of three timers (in seconds)	0.0	1.6	3.0	4.5	6.0	7.5	9.1

ACTIVITY TWO: Lesson One, Part 2—Plotting Motion on a Graph

MTE 301

Groups: Working with someone from your group from Part 1, get into groups of 2 or 3.
Equipment Needed (per group):

- Completed table with all data, from Part 1
- Ruler or straight edge

Procedures:

1) In Part 2 of this lesson, you will work in pairs to analyze the data you collected in Part 1. The data used from this section is from the row labeled "Time (average of 3 timers, in seconds)." For your convenience, a new table is below, but only the relevant data to be used in this activity are included in this new table. Copy your data into this table.

Arithmetic mean (the average) of the three timers

TABLE FOR EACH GROUP							
Position (meters)	**0**	**2**	**4**	**6**	**8**	**10**	**12**
Time (average of 3 timers, in seconds)	0						

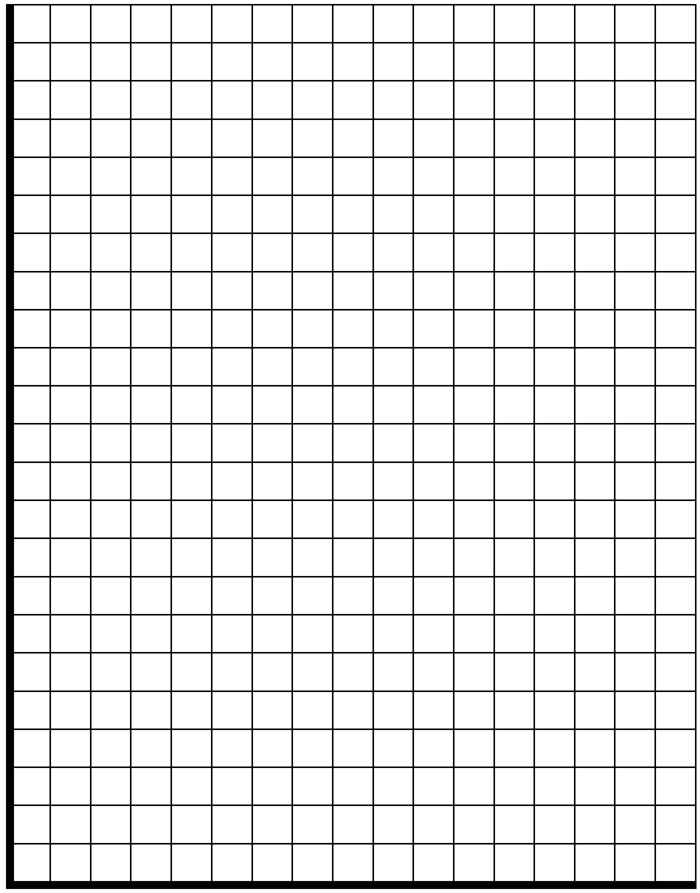
Here we have an example of a variable quantity—the position—whose value depends on the value of another variable quantity—the time. Corresponding to each of the values in the "Time" row, the "Position" has a specific value. We describe this by saying that that both time and position are **variables**, and that position is a **function** of time. A table of values, such as the one above, is one way to represent a function. Another way is to use a **line graph**.

2) Can you think of another quantity that can be described as a function of something else?

ANSWER:

3) Use graph paper (below); label it with "Position (meters)" along the vertical line and "Time (seconds)" along the horizontal line, with numbers along each line beginning with 0 (meters) and 0 (seconds) at the place where the lines meet. The vertical and horizontal lines are called **axes**, and the place where they meet is called the **origin**. The numbers are called the **scale**. Numbers should run from 0 to 20 along the vertical "position" axis, and from 0 to 15 along the horizontal "time" axis.

Place a large dot at each grid point that matches up with a position-time pair from the table of values above; this process is called **plotting** the points representing our data. Since we only had a limited number of timing teams, we were not able to tabulate times for every possible position of the student Walker, such as 3 meters, 7 meters, or 9 meters. However, we know that the Walker did pass by all of those positions.



Try to estimate the time at which the Walker passed the 7-meter mark. Explain your reasoning.

ANSWER:

4) After you have made your conjecture (educated guess) about the time when the Walker passed the 7-meter mark, follow these next steps. Take a clear plastic ruler and lay it down on the graph so that its edge goes through the origin, and lies as close as possible to the points that were plotted; some points should lie slightly to one side of the edge, and other dots on the other side of the edge. With a pencil, draw a straight line along the ruler's edge. We say that this is a **line-graph** representation of position as a function of time, and we call the line that we have drawn a **best-fit line** for the data points.

5) Using your best-fit line, estimate the time at which the Walker passed by the positions 3 meters, 7 meters, and 9 meters. Write your estimates below:

- time at 3 meters:______
- time at 7 meters: ______
- time at 9 meters: ______

How close is the new estimate of the time for 7 meters to your previous estimate from #3 above?

ANSWER:

6) We see now that using the line graph, we can associate one—and only one—value of position with each value of time; we say that we have defined **a function *P(t)*,** or **"position *P* as a function of time *t*."** The set of time *t* values is called the **domain** of the function *P(t)*, and the set of position *P* values is called the **range** of *P(t)*.

Another way to represent a function is with a **symbolic expression**, a compact mathematical rule that specifies, for example, which position value corresponds to a particular time value. An example of such a rule would be $P = 3t$, meaning that the position value (in meters) would be exactly equal to the time value (in seconds) multiplied by the number 3.

Complete a table of position-time values using the rule $P = 3t$ for t values from 0 seconds to 6 seconds.

Time (seconds)	0	1	2	3	4	5	6
Position (meters)							

Plot the points corresponding to these values on your graph from Question 3 and connect them with a best-fit straight line on that graph.

(*Note*: The new line should lie on the same axes that you used to plot the best-fit line of your timing observations, so that you can compare the new line and the old line. You may want to use a different color pencil for the new line, or instead draw a "dashed" line instead of a solid line, to help distinguish it from the best-fit line that you drew for your timing observations.)

7) Most likely, you will find that the two lines on your graph diverge from each other; they are *not* the same line, which means that the rule $P = 3t$ does *not* accurately describe the values you measured with the stopwatches. How would you describe the differences between the $P = 3t$ line and the best-fit line you drew for your observations? Can you think of a word or phrase that describes a property that applies to both lines, yet differs between them in some respect?

ANSWER:

8) If you think of these lines as resembling a small hill or mountain, we could say that one of them is "steeper" than the other; that is, it makes a larger angle with the horizontal axis. The mathematical term for measure of steepness is called **slope**.

Which of the two lines on your graph has a steeper slope? How can you tell? Try to describe a method that could be used for comparing the slopes of two lines.

ANSWER:

9) Using the second set of timing data from the *fast*-moving Walker, plot the data points and the best-fit straight line corresponding to that motion. How does the slope of this graph compare to the slopes of the two graphs plotted previously? Specifically, do you observe any relationship between slope and walking speed? Explain.

ANSWER:

[Note: This activity is omitted in the Workshop.]

10) It is possible to calculate a precise numerical value for the slope of a line, using the line graph. To do this, we must first choose two points that lie on the best-fit line; each point corresponds to a pair of values: a value of time and a value of position. We can designate these pairs of values as (t_1, P_1) and (t_2, P_2), where $t_1 < t_2$. The quantity $(P_2 - P_1)$ is equal to the *change* of position P between time t_1 and time t_2; on the graph, it corresponds to a "rise" along the vertical axis from a smaller value to a larger value. Similarly, the quantity $(t_2 - t_1)$ is equal to the change of time values, corresponding to an interval along the horizontal axis; this interval is called the "run." The slope is sometimes described as the ratio of $\frac{rise}{run}$ or, using symbols from our graph, the ratio $\frac{P2 - P1}{t2 - t1}$. (In our case, the slope has units of meters/seconds, corresponding to the vertical-axis unit divided by the horizontal-axis unit.) It is common to use the letter m to represent slope, so we can write $m = \frac{P2 - P1}{t2 - t1}$.

11) We can rewrite the definition of slope by re-arranging the symbols according to the rules of algebra; this way, we arrive at $P_2 - P_1 = m(t_2 - t_1)$. We could have chosen *any* two pairs of values along a particular straight line, since any two pairs would yield the same value of slope. Since this is so, we can write a more general equation using P and t instead of the specific values P_2 and t_2, and so we get:

$$P - P_1 = m(t - t_1).$$

This is called the **point-slope form** of the equation of a line that goes through both point (t_1, P_1) and *any other* pair designated (t, P). (Any appropriate variables could be substituted for t and P.) With this equation, we specify the one "fixed" point (t_1, P_1), but allow P and t to take on any value; however, a specific value of t will always be linked to a specific value of P, and we can calculate that linked value of P if we know the value of m that corresponds to this line.

Calculate the slopes of the best-fit lines you plotted on your graph, following these steps: First, choose any two pairs of points for each line and record them on the table below; Second, use the slope formula to calculate the slopes.

[Note: In the actual activity, Line #3 (Fast-moving Walker) should be included.]

Line #1 (the slow-moving Walker)

t_1 = _______; P_1 = _______

t_2 = _______; P_2 = _______ **Slope "*m*"** $= \frac{P2 - P1}{t2 - t1} =$ _________

Line #2 (the rule *P* = 3*t*)

t_1 = _______; P_1 = _______

t_2 = _______; P_2 = _______ **Slope "*m*"** $= \frac{P2 - P1}{t2 - t1} =$ _________

Now we will consider the case when the Walker does <u>not</u> start at the 0-meter point.

12) Reproduce the walking/timing activity, but now have the Walker start from a position *different* than 0 meters; call this new position P_{initial}, and call the starting time t_{initial}, where $t_{\text{initial}} = 0$ seconds. If you don't have time to carry out the entire activity again, you can use the "Sample Data" below to complete this exercise. Here, $P_{\text{initial}} = 2$ meters. **Note that this Walker is *never* at position = 0 meters.**

<u>Sample Data: Starting position</u> not at 0 meters

TABLE FOR EACH GROUP							
Position (meters)	**0**	**2**	**4**	**6**	**8**	**10**	**12**
Time (average of three timers, in seconds)	—	0.0	3.0	5.8	9.1	12.3	14.8

13) Plot the data points (using either the Sample Data or your own observations), and draw a best-fit straight line using a ruler. [This line should ***not*** go through the origin (0 s, 0 m).] Calculate the slope as before. What is the value of the slope? **ANSWER:**

14) Since P_{initial} is paired with an initial time t_{initial} in this current example, we can use the point-slope formula to rewrite the equation of a line using $(t_{\text{initial}}, P_{\text{initial}})$;

Instead of $P - P_1 = m(t - t_1)$ we will write: $P - P_{\text{initial}} = m(t - t_{\text{initial}})$

However, since we have chosen $t_{\text{initial}} = 0$ seconds, we can now also write:

$$P - P_{\text{initial}} = m(t - 0) = mt$$

Finally, we can re-arrange this last equation into the following form:

$$P = mt + P_{\text{initial}}$$

This is called the **slope-intercept form** of the equation for a straight line. In our current example, P_{initial} corresponds to the value of position at time $t = 0$ seconds.

Draw a small circle around the point on your new graph that corresponds to P_{initial}. Explain how you can determine the value of P_{initial} from a graph of the function $P = mt + P_{\text{initial}}$.

ANSWER:

15) We can see that P_{initial} represents the point on the vertical axis (here: "position axis," more generally, "*y* axis") where the line intersects a point with a value of zero on the horizontal axis (here: "time axis," more generally, "*x* axis"). In our case the intersection point is called P_{initial} but, more generally, it is represented by the letter "*b*," known as the **y-intercept**. This form of the equation is usually written $\boldsymbol{y = mx + b}$ where m represents slope, and x and y are variables.

What values of intercept b correspond to the measurements on your three graphs?

ANSWERS:

Review:
We have found that the equation $P = mt + P_{\text{initial}}$ is a good representation of motion at constant speed, where m represents the slope of the position-time graph. We found that this slope was related to the speed at which the Walker was moving. The technical term for the slope of the position-time graph is *velocity*, symbolized by the letter v. Velocity can be either positive or negative—it can also be zero—corresponding to the sign of the slope. The word "speed" is defined to be the absolute value of velocity, so speed is always positive. We will usually write the equation of constant-velocity motion in the form $P = P_{\text{initial}} + vt$.

ACTIVITY 3: Lesson Two, Part 1—Plotting Slopes

MTE 301

For this week's activities, we will revisit some of the procedures we did last week with some slight changes. We will explore how these slight changes in motion impact the slope.

Groups: In the first part of this activity, we will be gathering data as a class. Students will be placed in groups of at least 15, preferably 20. There will be one "Starter," one "Walker," and everybody else will be "Timers." Ideally, there should be 3 timers for each of the meter marks labelled 2 meters, 4 meters, 6 meters, 8 meters, 10 meters and 12 meters (described in more detail below).

Equipment Needed (per group):

- Stopwatches—Every person but two people in the group will need one (sometimes available on smart phones, or as a free downloadable app)
- One-meter stick or long tape measure with metric units
- 1 stick of sidewalk chalk or large stick of chalk;

1) Find an open space; it should be about 12 meters long and 2 or 3 meters wide.

2) With a tape measure or meter sticks, lay out a 10-to-15-meter straight-line path, with chalk markings at every meter, starting from a position marked "0 meters" at one end. A tick mark should be drawn every two meters and labeled with that distance. It should look like this:

12 Meters --

10 Meters --

8 Meters --

6 Meters --

4 Meters --

2 Meters --

0 Meters --

3) One person—the "Walker"—will walk the distance while all of the others time him/her. Decide who will be the Walker. You will also need someone to start the data collection. Decide who will be the Starter. All other group members will stand near one of the labeled meter distances. For example, you might have 2 to 3 students with stopwatches at each of the labeled distances (except for 0 meters).

4) Working in your group of 20 or so you will first carry out a few practice runs so that you are confident in how to gather the data.

5) The Starter will begin the data collection by loudly counting down so everyone can hear. He/she will say “3, 2, 1, Go!” At the exact moment the Starter says the word “Go!,” every Timer will start their stopwatch. The Walker will start out at initially at a slow steady pace but then switching to continuously speeding up as he/she walks. (More practice runs than before will probably be needed.) It is essential that the speed-up phase be gradual, so: first slightly faster, then still faster, then faster still, all the way until the end of the track is reached.—starting from the position marked “0 meters” and continuing until he/she passes the point marked “12 meters.” As he/she passes the students with the stopwatch who are standing at the meter marks, the students with the stopwatches will stop their watches. For example, if you are standing at “6 meters” and the Walker walks by and crosses the chalk line, stop your stopwatch at the exact moment the Walker crosses your line. After the trial, compare your findings with other people at the same meter mark. Were your times similar? The times recorded by Timers at your meter mark should be really close to each other. Run at least 3 practice trials to make sure you are consistent in gathering your data; there should be 3 students with stopwatches at each of the labeled distances (except for 0 meters).

6) Once you have finished at least 3 practice runs and you have consistency in gathering your data, you are ready to collect formal data. The Starter will begin the data collection by loudly counting down so everyone can hear. He/she will say “3, 2, 1, Go!” The Walker will start out at initially at a slow steady pace but then switching to continuously speeding up as he/she walks. It is essential that the speed-up phase be gradual, so: first slightly faster, then still faster, then faster still, all the way until the end of the track is reached.—starting from the position marked “0 meters” and continuing until he/she passes the point marked “12 meters.” Record the times in the table to the nearest tenth of a second and then calculate the arithmetic mean (the average) for each of the three timers.

Data Collection

TABLE FOR EACH GROUP							
Position (meters)	**0**	**2**	**4**	**6**	**8**	**10**	**12**
Time (seconds), timer #1	0.0						
Time (seconds), timer #2	0.0						
Time (seconds), timer #3	0.0						
Time (average of three timers, in seconds)	0.0						

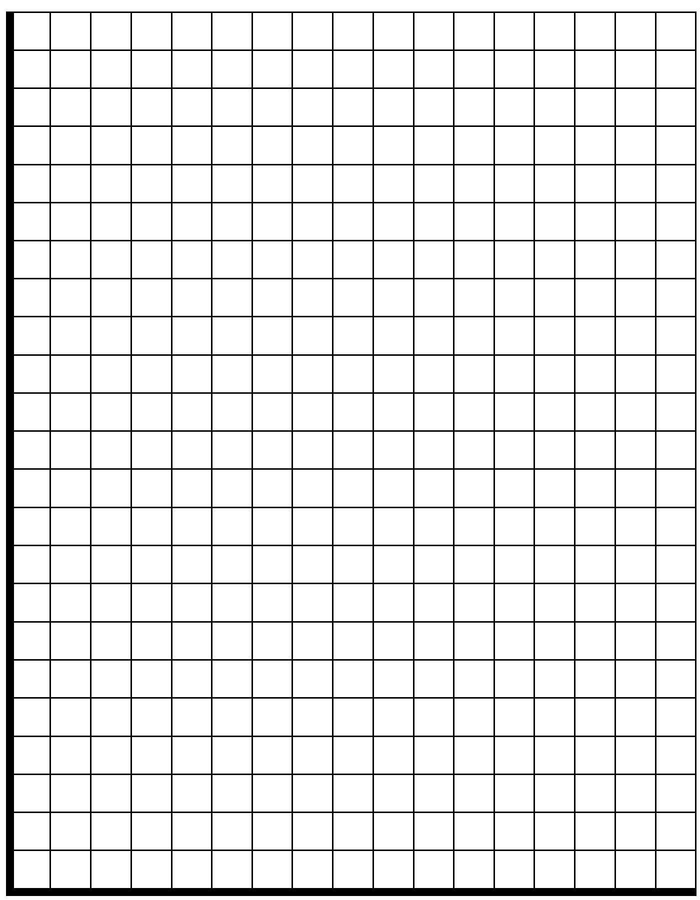
7) Make sure each student has a copy of the data from this lesson. Each student should have his/her own copy of the completed tables. The data will be used in the next part of the lesson. The data used from the next section is from the table entitled “Data Collection.”

To continue this lesson, you will need to regroup. Working with someone who helped gather the data get into groups of 2 or 3. (Sample data are shown in the table below.)

Position (meters)	**0**	**2**	**4**	**6**	**8**	**10**	**12**
Time (average of 3 timers, seconds)	0.0	1.95	3.00	3.84	4.52	4.93	5.43

8) Use graph paper (below); label it with "Position (meters)" along the vertical line and "Time (seconds)" along the horizontal line, with numbers along each line beginning with 0 (meters) and 0 (seconds) at the place where the lines meet. The vertical and horizontal lines are called **axes**, and the place where they meet is called the **origin**. The numbers are called the **scale**. Numbers should run from 0 to 20 along the vertical "position" axis, and from 0 to 15 along the horizontal "time" axis.

   Place a large dot at each grid point that matches up with a position-time pair from the table of values above; this process is called **plotting** the points representing our data. Since we only had a limited number of timing teams, we were not able to tabulate times for every possible position of the student walker, such as 3 meters, 7 meters, or 9 meters. However, we know that the Walker did pass by all of those positions.



9) In the last activity, we drew a best-fit straight line. Try this again. Take a clear plastic ruler and lay it down on the graph so that its edge goes through the origin, and lies as close as possible to the points that were plotted; some points should lie slightly to one side of the edge, and other dots on the other side of the edge. With a pencil, draw a straight line along the ruler's edge. We say that this is a line-graph representation of position as a function of time, and we call the line that we have drawn a best-fit *straight* line for the data points. Is this line appropriately and accurately aligning with the data? Why or why not?

ANSWER:

10) Using your best-fit straight line, estimate the time at which the walker passed by the positions 3 meters, 7 meters, and 9 meters. Write your estimates below:

- time at 3 meters: _________
- time at 7 meters: _________
- time at 9 meters: _________

11) It may not be possible to use a straight line as a "best fit" to a set of data points. In that case, we can try to use a best-fine line that is *curved* instead of straight. It is often easiest to draw such a curved best-fit line freehand, without using a ruler or other mechanical aid. Try drawing a curved line that is a good fit to the data points you plotted on the graph. You should draw a smooth curve, that is, a line that curves slowly and gently and appears "smooth," rather than one that contains several sudden up-and-down variations or that appears "choppy."

12) Using your curved best-fit line, estimate the time at which the walker passed by the positions 3 meters, 7 meters, and 9 meters. Write your estimates below:

- time at 3 meters: _________
- time at 7 meters: _________
- time at 9 meters: _________

Compare the values predicted by the *curved* best-fit line to the values predicted by the best-fit *straight* line; what do you notice?

ANSWER:

Would the equation of a straight line $P = mt + P_{\text{initial}}$ be a good model for the data? Why or why not? [*Note:* When describing motion, we usually use "v" (velocity) instead of m.]

ANSWER:

13) Try drawing a new best-fit straight line using a ruler or straight edge; however, this time, try to fit *only* the three data points for positions = 0 meters, 2 meters, and 4 meters. What do you notice about the points for positions 8 meters, 10 meters, and 12 meters? Are they all on one side of the straight line, or do they fall on both sides of it? Which point is closest to the line, and which is farthest away?

ANSWER:

14) If the points all fall on one side of the best-fit straight line, then we may be able to find a new equation that would be a good match to the data by adding a term to the equation for the straight line; in other words, we would have $P = P_{\text{initial}} + mt$ + [additional term that depends on t, but is not simply a constant multiplied by t].

15) You should see that the additional term will be relatively small when time is small, but will grow rapidly as time gets larger. (This would account for the fact that a straight line is a good fit for the low-time part of the curve, but not for the higher-time part.) Can you make any guesses as the nature or form of the additional term?

ANSWER:

ACTIVITY 4: Lesson Four—Rolling Cart

MTE 301

Groups: Students will work at their tables in groups of 4.

Equipment Needed (per group): If possible, each group should be supplied with a track, a cart, a motion sensor, and a GLX device. A couple of thick books or a medium-sized box will also be needed to lift up one end of the track.

Overview:

For this lesson, we will start by observing the motion of a metal cart rolling down an inclined 2-meter track. Our goal is to find a mathematical equation that will accurately model the motion of the cart as it rolls down the incline, and then we will represent the motion with various graphs.

Procedures:

1. In preparation for this lesson, students should watch two brief videos describing the use of the carts, tracks, and GLX devices. The videos just provide an outline, and additional hands-on guidance from the instructor will be needed as the students practice using the equipment.

   Link to video, "4. Using the GLX device to collect motion data":
   https://youtu.be/PNyJ49kNMd4

   Link to video, "5. Description of track arrangement for speeding up":
   https://youtu.be/EkEqws7suWc

2. To begin this lesson, watch the YouTube video showing the motion of a cart rolling down the inclined track; The link is below.

   Link to video, "6. Collecting data for the cart rolling down the track":
   https://youtu.be/CXd-HSgL6f0

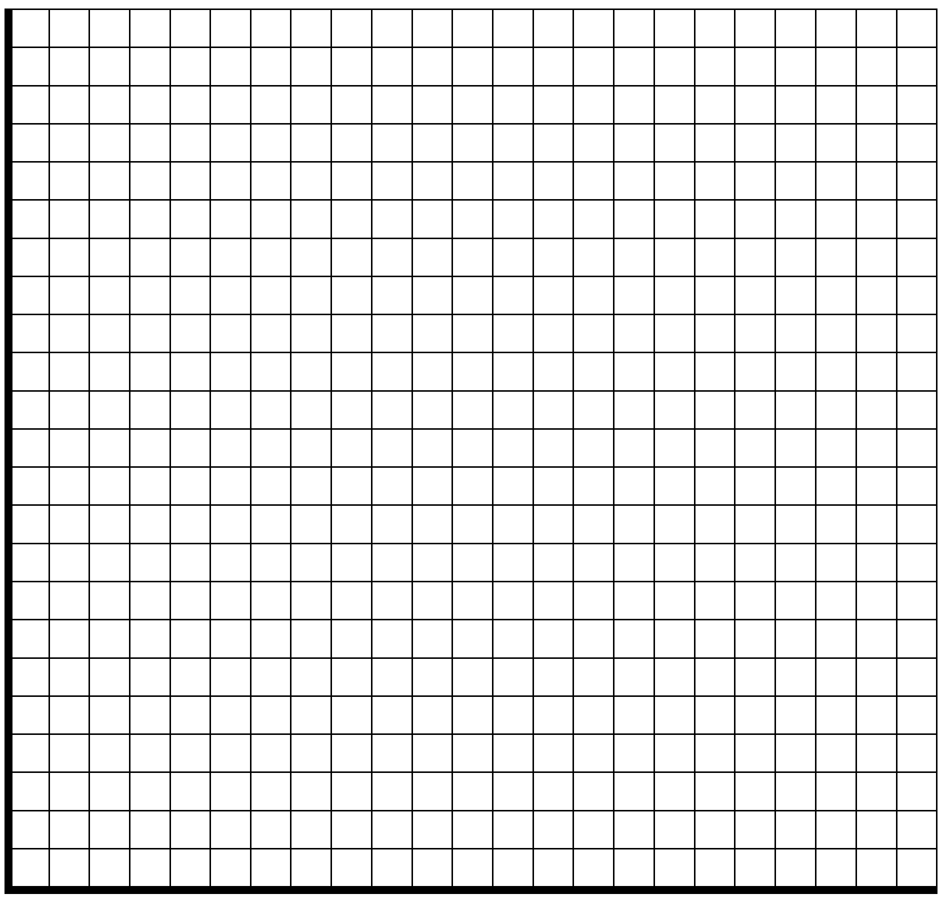
3. In the video, you will see the metal cart held motionless near the top of the 2-meter track. A motion sensor at the top of the track (it looks like a small blue box) will detect the position of the cart as it rolls down the track. The GLX portable graphing unit will record the exact position of the cart from moment to moment as it rolls down the track.

4. However, for this activity, we can *not* make use of the data that comes directly from the GLX data tables without modification. (That *will* be possible, for some future activities.) If we did, we would find that the mathematical equation we need is more complicated than the one we want to use to study this motion. So, instead, we will use data that has been adjusted slightly from the actual values produced by the GLX device. In the table below, the time elapsed after the cart was released to roll down the track is paired with a measurement of the cart's position on the track, relative to the

origin defined by the motion sensor. We will be using this data table for the activities that follow.

Time (seconds)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Position (meters)	0.19	0.20	0.25	0.30	0.40	0.51	0.68	0.82	1.01	1.26	1.47

5. Using the graph below, draw in the appropriate tick marks and numbers to indicate position (meters) on the vertical axis and time (seconds) on the horizontal axis. The vertical axis should run from 0.0 meters to about 2.0 meters, while the horizontal axis goes from 0.0 seconds to 1.1 seconds.
6. Draw a best-fit curved line "freehand" through the plotted points.



7. As you can see, a curved line is a better fit to the data than is a straight line. Since the line is curved, its slope does *not* have a constant, unchanging value; instead, the slope varies from one point to the next on that curved line.

When we wrote down the equation of a straight line, we made use of the slope m that had a specific constant value for all points on the line. While we don't expect to find a constant value for slope m for this line, we will still try to find some other quantity that is constant along the line. In the same way that the constant slope m helped us write a mathematical equation to model the straight line, some other constant might help us come up with the best mathematical equation to fit this curved line.

8. To begin our search for some quantity that is constant, fill in the appropriate values in the table below, making use of the data in the previous table. *Note:* In this table, the letter P stands for "Position," and "P_{initial}" represents the value of position at time $t = 0$ seconds. Many of the table values have already been filled in, so you just need to fill in those that remain.

Position (meters)	**t (seconds)**	**t^2 (seconds^2)**	**P-P_{initial} (meters)**	**$(P-P_{\text{inital}})/t$**	**$(P-P_{\text{inital}})/t^2$**
0.19	0.0	0.0	0.0	[NO VALUE]	[NO VALUE]
0.25	0.2	0.04	0.06		
0.40	0.4	0.16	0.21		
0.68	0.6	0.36	0.49		
1.01	0.8	0.64			
1.47	1.00	1.00			
Mean (average) value					

9. Find the mean (average) values of the numbers in the last two columns. For which of the two columns is the mean value a good match to *most* of the values in that column? For that case, let us assign a symbol to the mean value; we'll call it "c" for now; that is, c = mean value of $[(P-P_{\text{initial}})/t]$ or $[(P-P_{\text{initial}})/t^2]$, whichever is the better fit.

ANSWER:

10. Now, how can we use the constant c to build an equation to represent the curved line? Depending on which was found to be a better fit, we will know that one of the following two relationships (i) or (ii) is a better model of our curved line:

(i) $(P - P_{\text{initial}})/t = c$

or:

(ii) $(P - P_{\text{initial}})/t^2 = c$

The only difference is that in one case we have a t, and in the other case it's the squared value, t^2.

Let's try to rearrange the equations so that we isolate P (the position variable) on the left side of the equation, and put everything else on the right side of the equation. First we can multiply both sides of equation (i) by t, and both sides of equation (ii) by t^2, and so we get this new pair of equations:

(iii) $P - P_{\text{initial}} = ct$

(iv) $P - P_{initial} = ct^2$

Go ahead and complete the rearrangement of equations (i) and (ii) so that the variable P is isolated on the left side of the equation.

ANSWER:

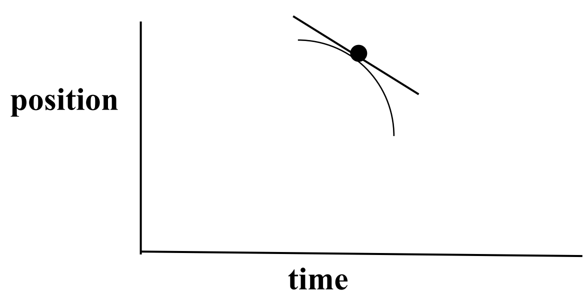
11. Eventually we should arrive at the equation $P = P_{\text{initial}} + ct^2$ as a good model for our curved line. This is an example of a "nonlinear" equation, because it represents a line graph that is *not* a straight line; in that sense, it is different from what we have seen up until now. This particular equation, and the line graph associated with it, are examples of "quadratic variation"; quadratic variation is the term applied to relationships that can be modeled with a term such as ct^2 that contain the square of a quantity; the quantity that is squared here is t, representing the time.

12. Straight lines, such as those we dealt with in Lessons One and Two, can be characterized by a single value of the slope. In those cases, the slope is a value that

characterizes every part of the line, whether the beginning, middle, or end of the line. Curved lines are not so easy to characterize in terms of their slope. Can you explain why that is?

ANSWER:

13. The slope of a curved line varies from point to point along that line; the slope at any particular point on the line can be estimated by drawing a "tangent" line at that point; the tangent is a straight line that touches the curve at only a single point. See diagram.



14. The slope of the tangent line is a good approximation to the slope of the curved line at the point where the tangent line touches the curved line.

Can you explain why values of the slope would differ from one point to another along the curved line, based on the tangent-line method?

ANSWER:

In the diagram above, is the slope of the tangent line negative or positive?

ANSWER:

Practice videos for self-timing experiments:

1. Description of Walking Track:
   https://youtu.be/ioZxLJtUTRU

2. Walking at constant speed:
   https://youtu.be/JrZbkwDNRMM

3. Walking, speeding up:
   https://youtu.be/cS7zZ5vGHa4

Using the lab equipment:

4. Using the GLX to collect data:
   https://youtu.be/PNyJ49kNMd4

5. Description of track arrangement for speeding up:
   https://youtu.be/EkEqws7suWc

6. Collecting data for the cart rolling down the track:
   https://youtu.be/CXd-HSgL6f0

Videos of specific motions:

7. Constant positive position:
   https://youtu.be/vUW-yhBP740

8. Constant positive velocity:
   https://youtu.be/KUeY_naer3g

9. Constant negative velocity:
   https://youtu.be/Ry4ZU2QnZxo

10. Positive velocity, speeding up:
    https://youtu.be/SqKL66BjIz8

11. Negative velocity, speeding up:
    https://youtu.be/Md9olQPWyJ0

12. Positive velocity, slowing down:
    https://youtu.be/PCdOzXLvRr0

13. Negative velocity, slowing down:
    https://youtu.be/hnHXCH2Uupg

14. Changing direction, positive to negative:
    https://youtu.be/reXgOJM8Ia8

15. Changing direction, negative to positive:
    https://youtu.be/bN4n4nmFSZo

16. Changing direction (negative to positive) with positive acceleration:
    https://youtu.be/1X9XD4Kp4FI

17. High velocity compared to low velocity:
    https://youtu.be/Z3n4i7A7RiU

18. Speeding up slowly compared to speeding up quickly:
    https://youtu.be/l-IPZRcdZqA