Physics Students' Mathematical Difficulties with Operations and Algebra

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Students' difficulties with symbols
Torigoe and Gladding (2011)

Numeric version:

A car can go from 0 to 60 m/s in 8 s. At what distance *d* from the start at rest is the car traveling 30 m/s?

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{\Delta v}{\Delta t}$

$\Delta v = 60$

$\Delta t = 8$

$v = 30$

$d = ?$

Our version (pure math)

Symbolic version:

A car can go from 0 to v_1 in t_1 seconds. At what distance *d* from the start at rest is the car traveling (v_1/2)?

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{v_1}{t_1}$

$v = \frac{v_1}{2}$

$d = ?$

Our version (pure math)

Difference in Procedure

Examples of student work

Numeric:

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{\Delta v}{\Delta t}$

$\Delta v = 60$

$\Delta t = 8$

$v = 30$

$d = ?$

$30^2 = 0^2 + 2\left(\frac{\Delta v}{\Delta t}\right)d$

$900 = 0 + 2\left(\frac{60}{8}\right)d$

$900 = 15d$

$d = 60$

Skills required:

Arithmetic

Symbolic:

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{v_1}{t_1}$

$v = \frac{v_1}{2}$

$d = ?$

$v^2 = 2ad$

$\left(\frac{v_1}{2}\right)^2 = 2\left(\frac{v_1}{t_1}\right)d$

$\frac{v_1^2}{4} = \frac{2v_1}{t_1}d$

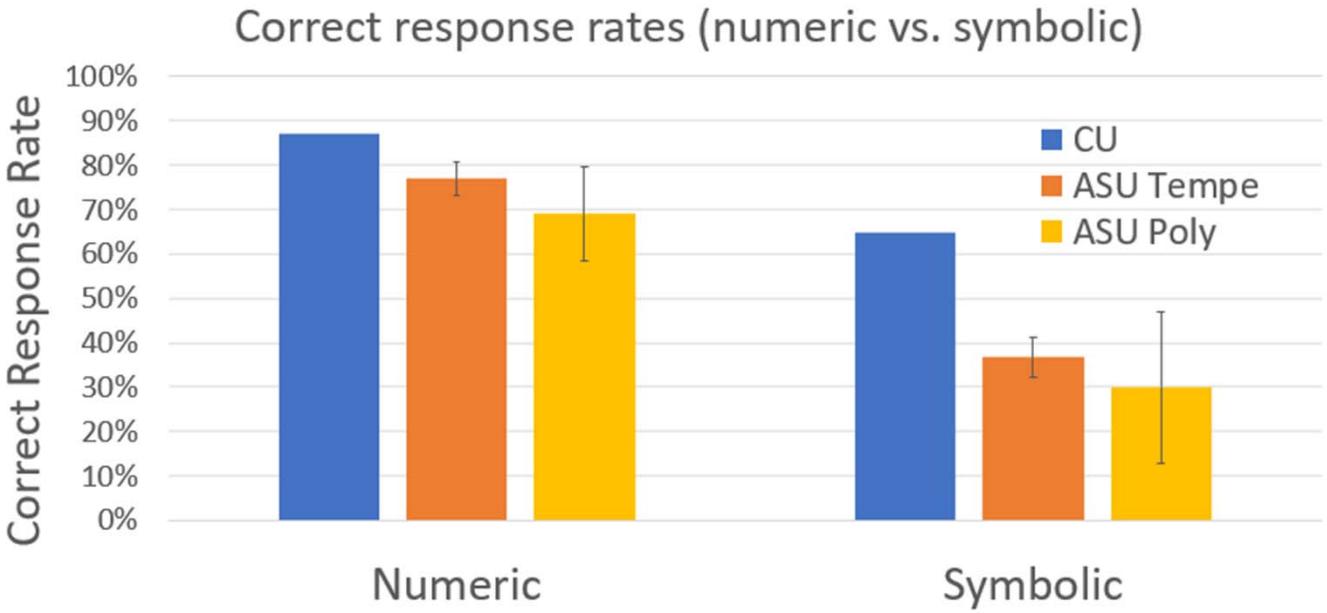
$d = \frac{v_1^2}{4} \cdot \frac{t_1}{2v_1}$

$= \frac{v_1 t_1}{8}$

Skills required:

Squaring, multiplying, and dividing symbolic fractions; handling of “complex” symbolic expressions

Correct response rate difference: symbolic vs. numeric



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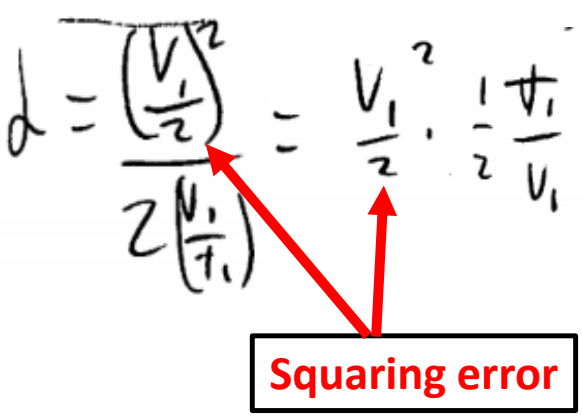
$d = ?$

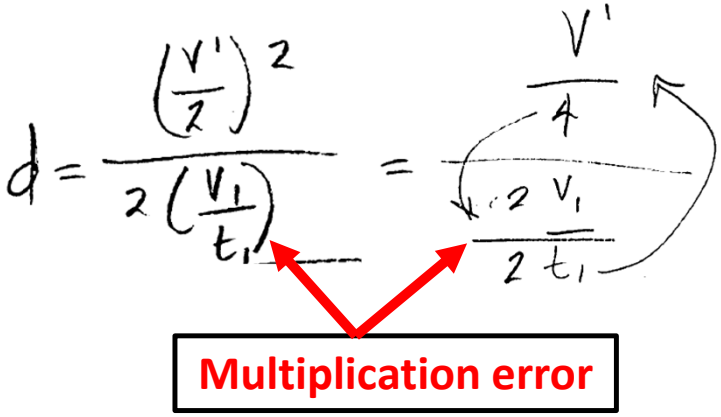
- CU: *N*=196, 191
- ASU Tempe: *N=471, 461* (average from 3 semesters)
- ASU Poly: *N =147, 136* (average from 3 semesters)
- Error bars are ±σ

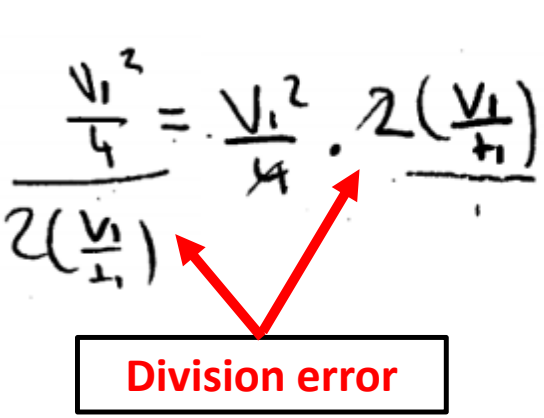
- **Significant** differences between numeric and symbolic version; ≈20-40% difference in correct response rates

Errors observed on the symbolic version

- Incorrectly squaring and multiplying/dividing fractions







➢High frequency of these errors motivated us to investigate students' difficulties at the level of operations

Types of difficulty with symbols

Operations → **Multi-step** → **Physics**

$$\left(\frac{a}{3}\right)^3 = ?$$

$$2\left(\frac{a}{b}\right) = ?$$

$$\frac{a/b}{c^2/d} = ?$$

"Operations"

$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{v_1}{t_1}$$

$$v = \frac{v_1}{2}$$

$$d = ?$$

"Multi-step"

A car can go from 0 to v_1 in t_1 seconds. At what distance *d* from the start at rest is the car traveling (v_1/2)?

Types of difficulty with symbols

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$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{v_1}{t_1}$$

$$v = \frac{v_1}{2}$$

$$d = ?$$

A car can go from 0 to v_1 in t_1 seconds. At what distance d from the start at rest is the car traveling ($v_1/2$)?

Mastery of "operations" in "multi-step" problems is required to correctly answer the physics problem

“Correct” students

$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{v_1}{t_1}$$

$$v = \frac{v_1}{2}$$

$$d = ?$$

Students who respond correctly to this problem will be referred to as “Correct” students

“Incorrect” students

$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

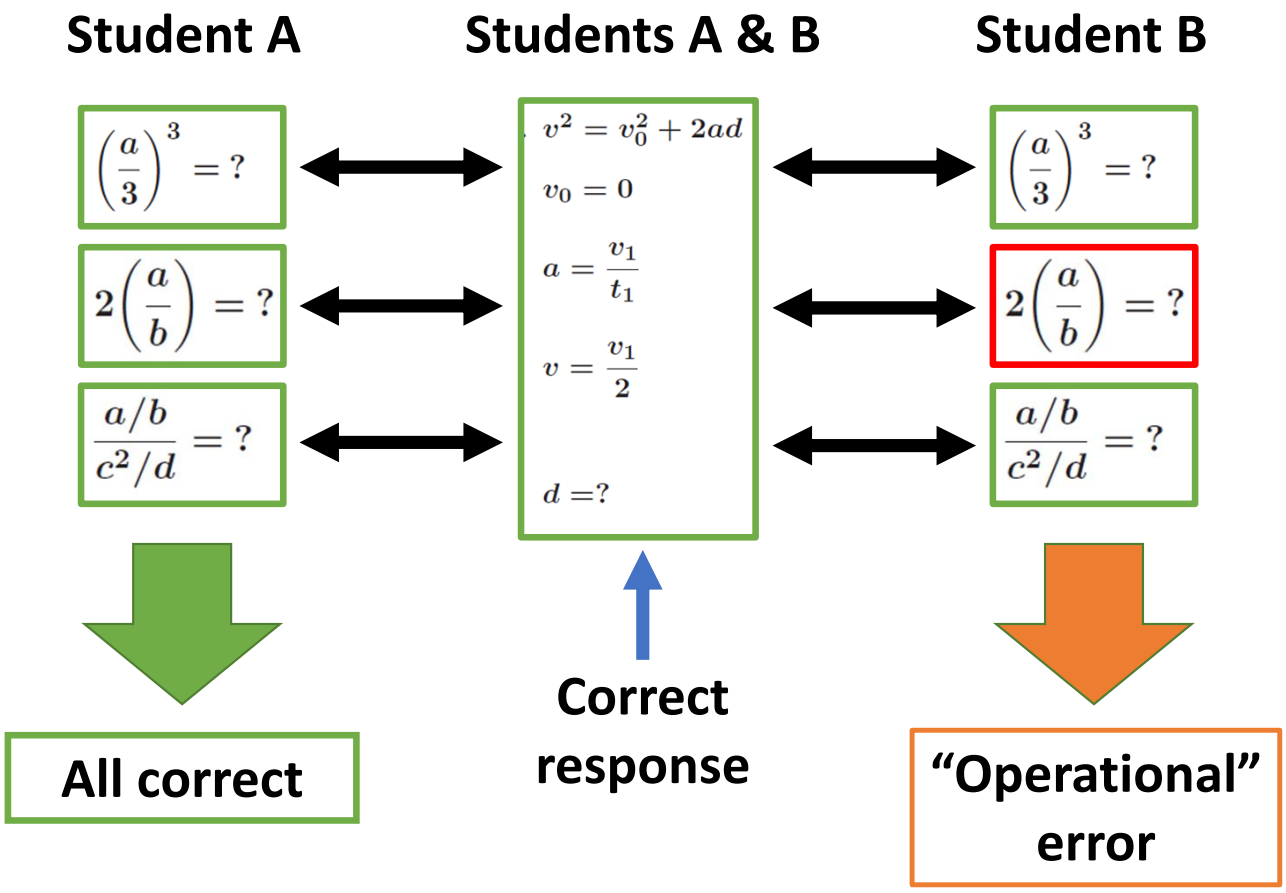
$$a = \frac{v_1}{t_1}$$

$$v = \frac{v_1}{2}$$

$$d = ?$$

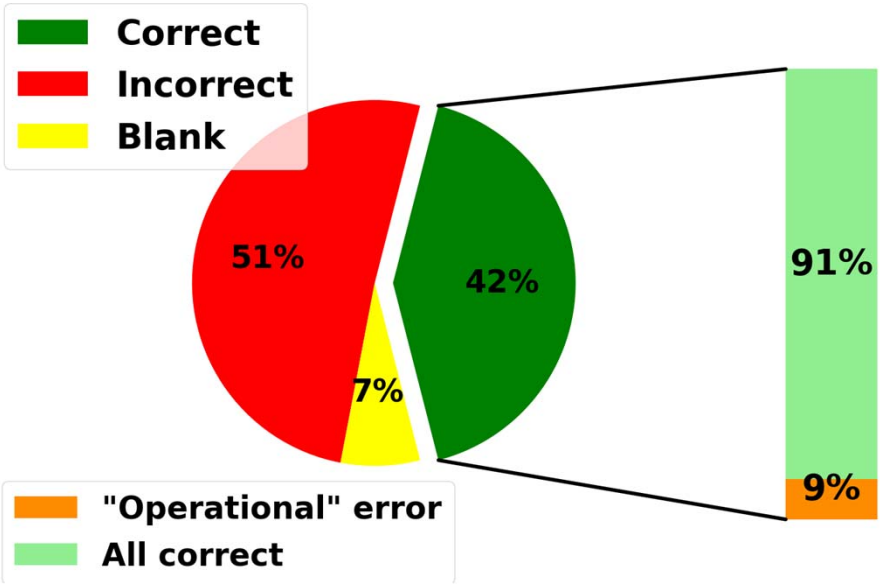
Students who respond incorrectly to this problem will be referred to as “Incorrect” students

Grouping “Correct” students on kinematics problem

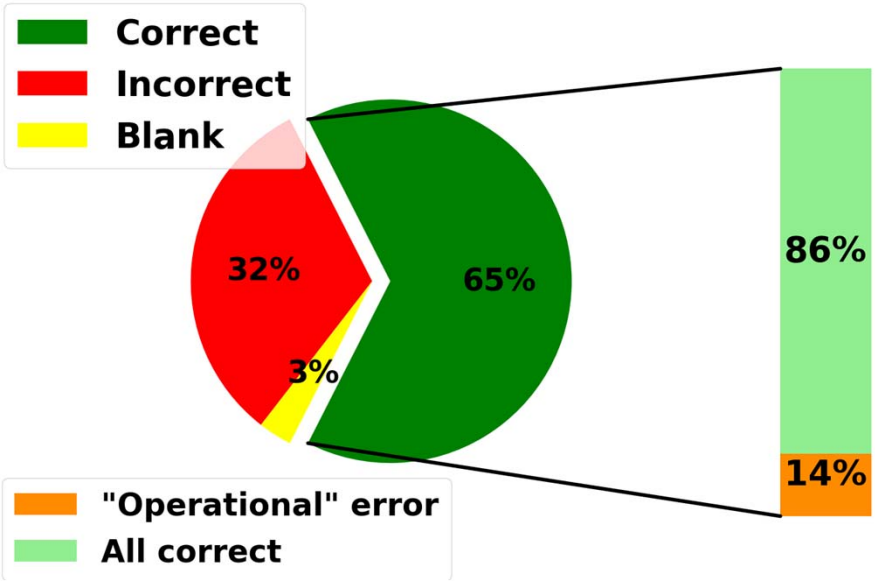


Quantifying the “Correct” groups (All correct and “Operational” error)

PHY 111 Tempe Fall 2019 (*N*=106)



CU Fall 2019 (*N*=191)

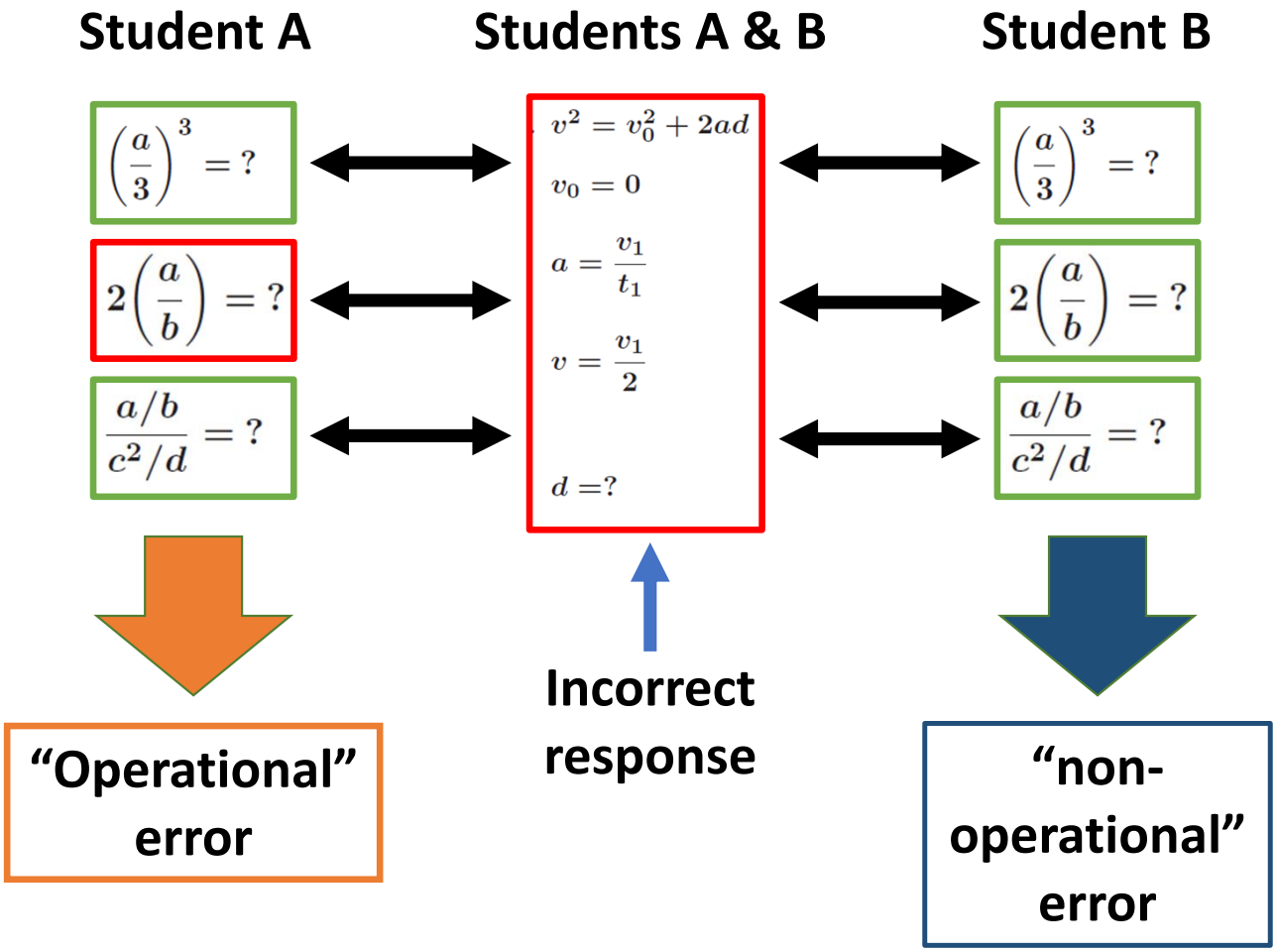


Most students who solve the symbolic kinematics problem correctly, solve all “operation” problems correctly.

“Operational” and “non-operational” errors

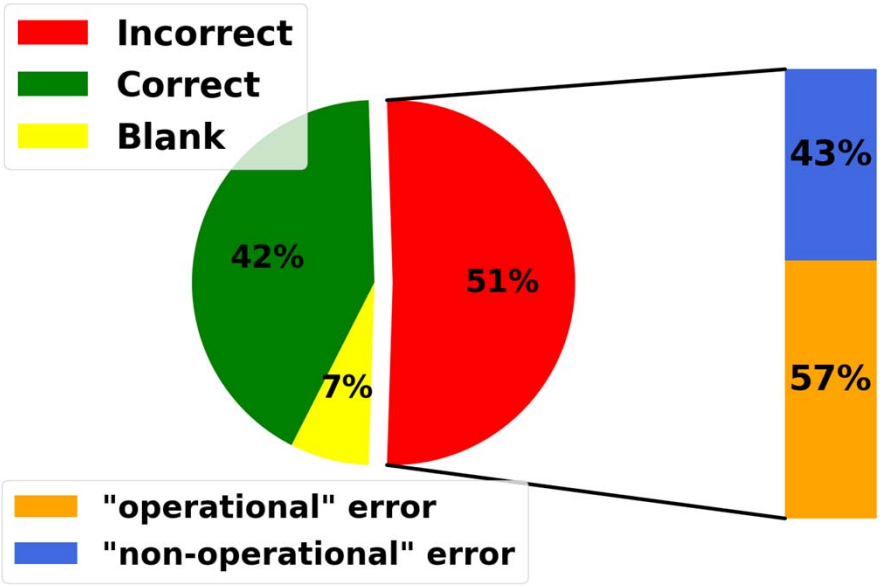
- We now examine the “incorrect” students and group them into two groups: “operational” errors and “non-operational” errors
- Define errors on any of the three fraction problems as “operational” errors
- Define “non-operational errors” as errors that occur when the student answers all fraction problems correctly while also answering the kinematic problem *incorrectly*

Grouping “Incorrect” students on kinematics problem



Quantifying the “Incorrect” groups (“operational” and “non-operational”)

PHY 111 Tempe Fall 2019 (*N*=106)



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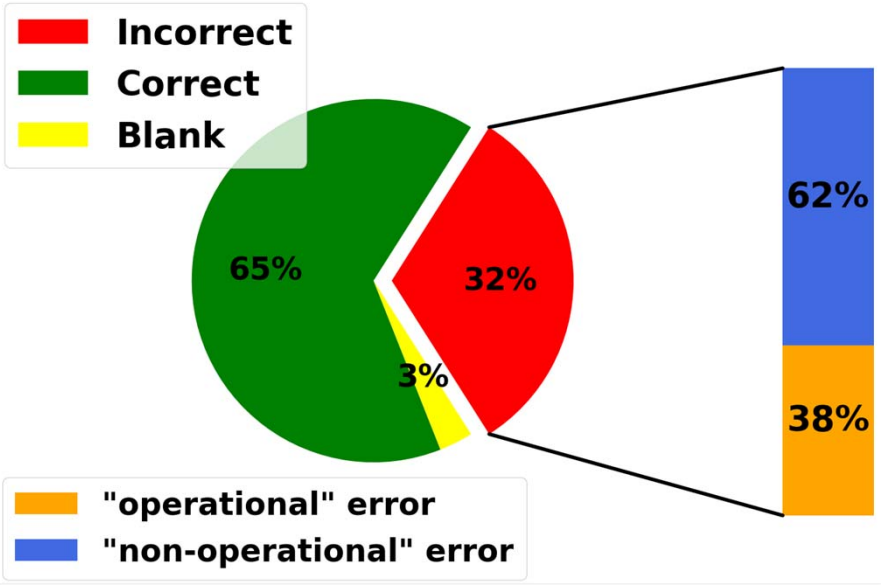
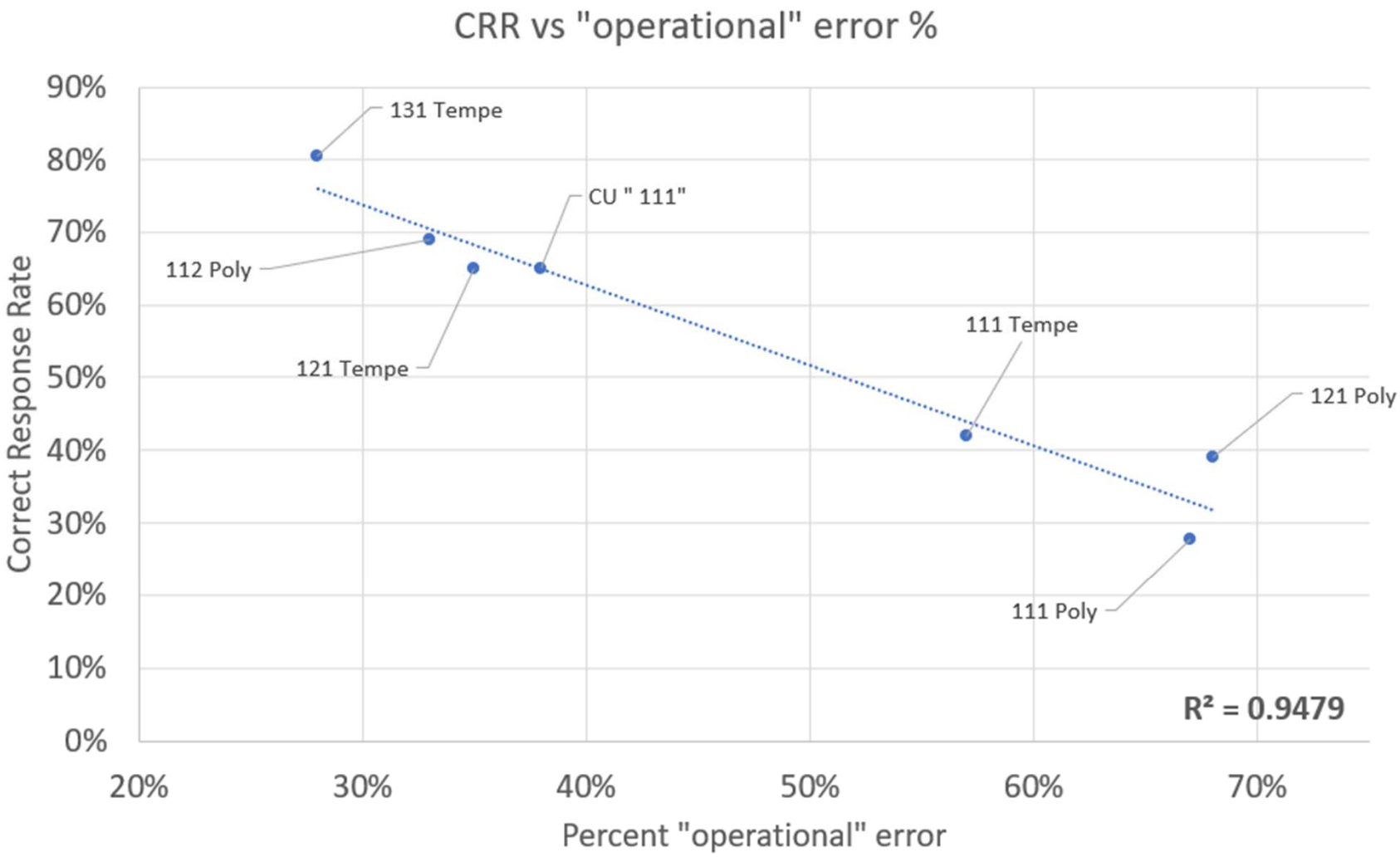
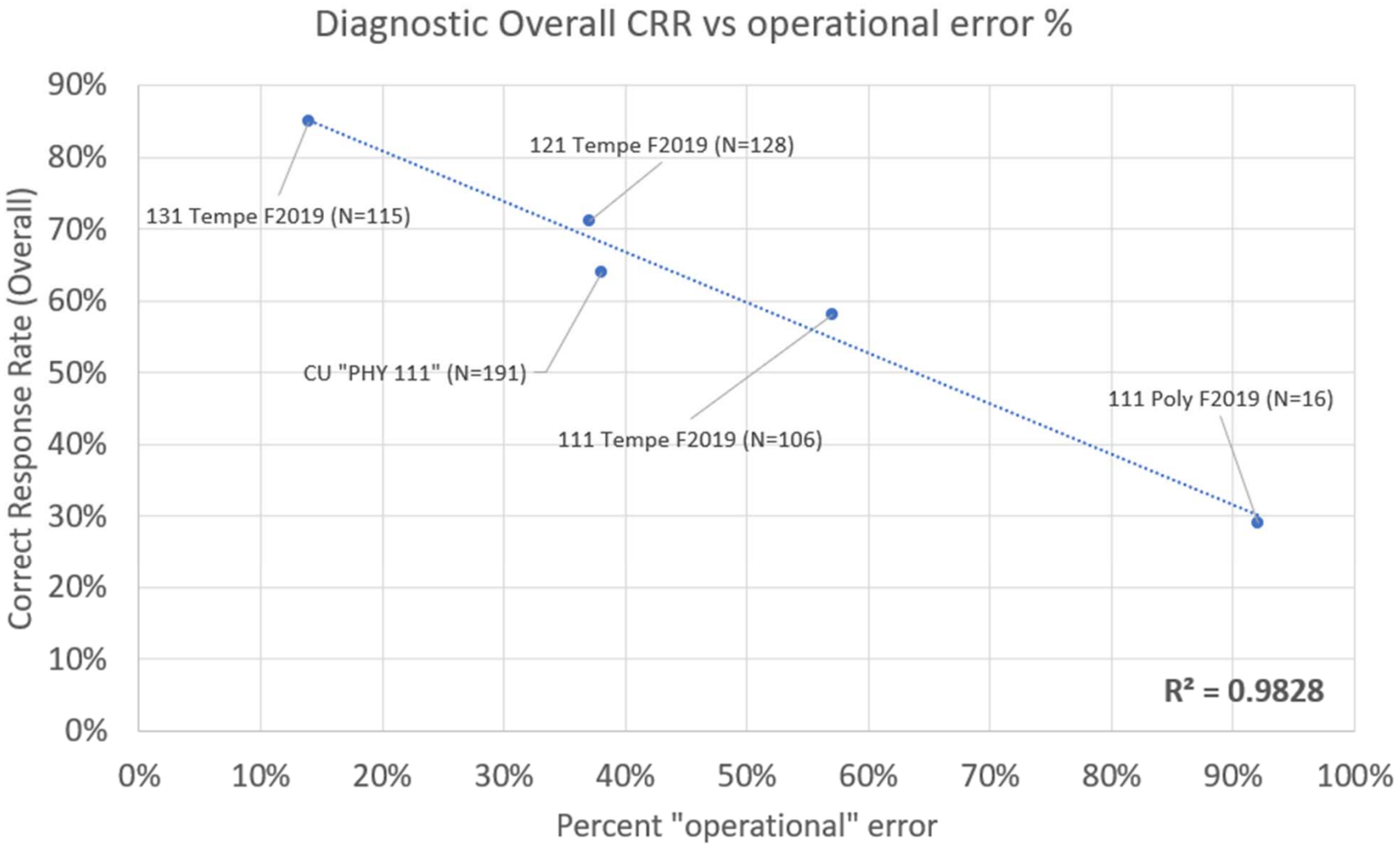


Courses with lower operational error rates achieve higher correct response rates


CRR vs "operational" error %
Correct Response Rate
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
131 Tempe
CU " 111"
112 Poly
121 Tempe
111 Tempe
121 Poly
111 Poly
R² = 0.9479
20%
30%
40%
50%
60%
70%
Percent "operational" error


Diagnostic Overall CRR vs operational error %
Correct Response Rate (Overall)
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
131 Tempe F2019 (N=115)
121 Tempe F2019 (N=128)
CU "PHY 111" (N=191)
111 Tempe F2019 (N=106)
111 Poly F2019 (N=16)
R² = 0.9828
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
Percent "operational" error

Interpretation of results

- Students often apparently possess the operational tools necessary to solve a problem, but make non-operational mistakes
- These non-operational mistakes may be due to carelessness, such as the lack of checking over work
- Some students' difficulties are at the level of operations
- Possibility of significant improvement by way of:
  - Operational skill practice
  - Work-checking strategies

Future research

- Develop and implement practice assignments (called *Math Practice*) in collaboration with researchers at Ohio State University
- Goal: improve students' success in introductory physics through improvement of math fluency
  - Test and practice relevant math skills, with and without physics context
  - Analyze impact from student performance data (grade and retention)
  - Evaluate impact of "operational" and "non-operational" errors on performance