Historical Sketch of Research on Mathematization in Physics Education

David E. Meltzer
Arizona State University

http://www.physicseducation.net/

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Development of Students' Mathematical Thinking

- Most college physics students receive their initial mathematical preparation in middle school and high school
- The "mathematical landscape" of physics students' thinking must be traced back to these formative years…

The High School-College Connection

- Until the 1870s, high school physics instruction was largely qualitative, light on math
- As high schools spread and enrollment increased, they became increasingly important sources for college admissions
- High school physics instruction began to resemble college more closely: emphasis on quantitative measurement and mathematical problem solving

The Teacher Preparation Dilemma

- Very weak high school physics teacher education in the early 1900s contributed to ineffective preparation of students for college physics
- College entrance exam scores were low, and high school instructors began to rebel at the heavy influence of the colleges
- A “new movement” of physics teachers attempted to re-focus high school instruction on inductive, lab-centered learning

“De-Mathematization” and Backlash

Some suggested “de-mathematization” of high school physics:

I would therefore propose—for discussion—the feasibility of a plan for the teaching of physics which avoids as far as possible the use of mathematics even of the most elementary kind —and which gives to the science of measurement only a secondary importance.

Prof. A. A. Michelson (1909)

A backlash developed as some investigators tried to show that “de-mathematization” was leading to poor learning.

Studies of Physics Students' Math Skills

- Beginning in 1918 and continuing today, investigators have probed physics students' mathematics preparation and asked whether it's adequate for college physics.
- Many mathematics diagnostic tests have been administered to high school and college physics students.
- Almost always, students' mathematics preparation has been found wanting.

Representative Results from Diagnostic Tests

- **Randall, Chapman, and Sutton (1918)** claimed poor performance showed that “de-mathematization” was damaging physics students’ ability to have “thorough understanding” and “exact thinking.”
- **Hughes (1924)** argued to the contrary, that poor math performance by university students showed that it was *not* possible to “mathematize” high school physics to any great extent and still get satisfactory achievement.
- **Lohr (1925)** concluded that it was necessary for university physics teachers to “re-teach until [they are] sure of assimilation of the mathematics involved before attempting to give the physics using these principles.”
- **Kilzer (1929)** concluded that there was a need for “maintenance drills” covering the math needed in high school physics courses.
- **Breitenberger (1992)** found that new physics *graduate* students were deficient in math skills and mathematical thinking!

Probes of Math’s Impact on Physics Performance…

- **Bless (1932)** found a very high correlation between university students’ physics grades and their scores on an arithmetic/algebra diagnostic test.

- **Carter (1932)** found a similarly high correlation among high school students.
  – *However*, he noted that the correlation was sharply reduced when student’s “intelligence” (determined by an IQ test) was held constant

- **Kruglak and Keller (1950)** found a high correlation between math course grades and physics course grades of university students.

Additional Analogous Results Based on Grades or Diagnostic Test Scores, by:

- Blumenthal (1961)
- Bolte (1966)
- Larkin and Brackett (1974)
- Hudson and McIntire (1977)
- Cohen, Hillman, and Agne (1978)
- Champagne, Klopfer, and Anderson (1980)
- Hudson and Rottmann (1981)
- Champagne and Klopfer (1982)
- Hudson and Liberman (1982)
- Wollman and Lawrenz (1984)
- Griffith (1985)
- Halloun and Hestenes (1985)
- Hudson (1986)
- McCammon, Golden, and Wuensch (1988)
- Linder and Hudson (1989)
- Hart and Cottle (1993)
- Alters (1995)
- Sadler and Tai (2001)

But the Problem is More Complicated…

- Weak calculational skills are only part of the problem.
- Many early studies were flawed by conflating difficulties with *physics* concepts together with weak mathematical skills, and presuming the combination was "problems with math."
- Undeveloped technology limited the tools available for visualization of functional relationships, as well as real-time interaction with the outcomes of calculations.
  - (Those of us who went to college in the late 1960s-early 1970s did most of our calculations for course exams on a slide rule!)
- Up until the 1970s, there was virtually no research on which to base efforts to improve the situation.

Glimpses of the Future…

- **Lapp (1940)** showed that university students who were taught to give a *qualitative* analysis of a physics problem—that is, describing in words *exactly* how they could go about solving it—had substantially higher scores on a standard physics diagnostic test compared to students who were taught merely to find quantitative solutions to the same problems.

- **Black (1931)** carried out a rare investigation of physics students' thinking in a *physics* context, using written diagnostics and one-on-one student interviews…

From Black (1931):

Table 34.

PART 2. QUESTION 2a.

An object is compressed to half its size. What effect has this on its mass?

Grade	—8	—9	—10	—11	—12	+9	+10	+11	+12
Number	114	120	115	125	100	172	86	178	192
No effect	1.7	4.2	5.2	3.3	7.0	4.7	27.9	44.9	37.5
Mass is half	17.6	23.3	20.8	25.6	37.0	35.5	62.8	32.6	53.1
Mass is doubled	0.0	1.7	0.0	0.0	3.0	3.5	2.3	3.4	3.1
No answer	80.7	70.8	74.0	71.2	53.0	56.3	7.0	19.1	6.3

PART 2. QUESTION 2c.

An object is compressed to half its size. What effect has this on its Mass Why?

Grade	—8	—9	—10	—11	—12	+9	+10	+11	+12
Vol. (size) half, therefore, mass is half	12.3	19.2	17.4	20.8	25.0	24.4	41.9	28.0	42.2
Mass or quantity of matter always same (or) mass independent of volume	0	0	0	0	0	2.9	2.3	23.6	21.9
*Miscellaneous	3.5	3.3	4.3	4.8	5.0	7.0	9.3	14.6	12.5
No reason	84.2	77.5	78.3	74.4	70.0	65.7	46.5	33.8	23.4

Interlude

- Lots of research by physicists during the 1960s and 1970s on students' mathematical and scientific reasoning processes (Robert Karplus et al.)
  - However, not directly in the context of physics
- Research on general problem solving, carried out in a physics context (Fred Reif et al.)
  - Not *directly* linked to studies of mathematization
  - Direct precursor to future investigations by many PER workers focused on methods of *physics* problem solving
- "Workshop on Physics Teaching and the Development of Reasoning" by Karplus et al. (1975), and "The Various Language" by Arons (1977) both contributed greatly to future advances in research-based curriculum development

McDermott and the PEG

- Lillian McDermott and the University of Washington Physics Education Group (PEG) demonstrated that physics students' mathematical skills, physics ideas, and reasoning abilities are not easily disentangled, and must often be studied *together*, in the context of authentic physical systems.
- The PEG investigated students' abilities to work with multiple representations of physics ideas, including graphs and diagrams.

Some Examples

- Probing students' thinking regarding ratios of differences, e.g. $\Delta v/\Delta t$
  - Trowbridge and McDermott, 1981
- Investigating students' ideas about graphical representations of motion
  - McDermott, Rosenquist, and Van Zee, 1987
- Examining the utility of computer simulations to probe student thinking
  - McDermott, 1990; Grayson and McDermott, 1996

Further Work

- In the mid-1980s, David Trowbridge developed some of the very first physics simulation software ("Graphs and Tracks") based on his Ph.D. research directed by McDermott.
- Beginning in 1986, Thornton, Sokoloff, and Laws utilized the PEG's research to help develop curriculum based on real-time data acquisition and visualization using "probeware" and computers (computer graphing– the "microcomputer-based labs," or "MBL")
  - Thornton (1987); Laws (1989); Thornton and Sokoloff (1990)
- Beichner (1994) made use of PEG work in developing his diagnostic test on kinematics graphs.

Redish and the Maryland PER Group

From early on, Joe Redish emphasized that efficient accessing of ideas could be as important as forming them in the first place:

> *Corollary 1.2: It is not sufficient for students to "know" the relevant correct statements of physics. They also have to be able to gain access to them at the appropriate times; and they have to have methods of cross checking and evaluating to be certain that the result they have called up is truly relevant.*

-- Redish (1994)

Mental Models for Mathematization

The further work of the Maryland group explored the mental models that influence students' learning and use of mathematics for physics:

> ***Corollary 1.5: Many of our students do not have appropriate mental models for what it means to learn physics.***
> This is a "meta" issue. People build mental models not only for content, but also for how to learn and what actions are appropriate under what circumstances.

-- Redish (1994)

Later Work by the Maryland Group

- Students often fail to make use of specific mathematical tools that they *do* know how to use, because they don’t recognize their applicability to a physics problem
  - Bing and Redish, 2009; Gupta and Elby, 2011

Summary

- Despite nearly a century of efforts, research on mathematization in physics education tended to circle around and repeat similar studies with few new insights through the 1950s
- Work by physicists Arons, Karplus, and Reif during the 1960s and 1970s had an enormous impact and helped set the stage for modern PER
- McDermott, Redish, and their students and collaborators, helped catalyze critical breakthroughs that continue to guide much of the work done today.