

# **Consistency of students' mathematical difficulties may allow reliable performance predictability**

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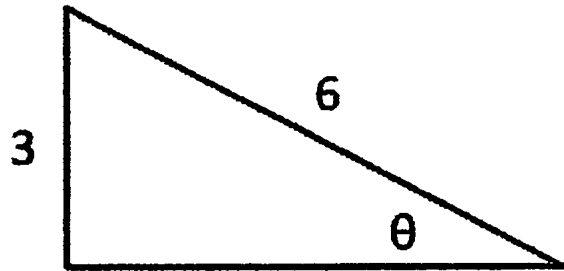
# Overview

We have given pre-college-math diagnostic tests to over 6000 physics students in introductory courses at four universities. We find:

- Error rates are consistently in the 30-60% range, suggesting that difficulties with basic operations might have significant impact on course performance.
- Class-average performance on *individual* test items is highly predictive of performance on the overall 13-item test
- Individual student performance on test items from any one topic (algebra, trigonometry, graphing, and geometry) is highly correlated with performance on items from the other topics
- Performance on conceptual physics items is correlated with math performance

# Examples of Test Items

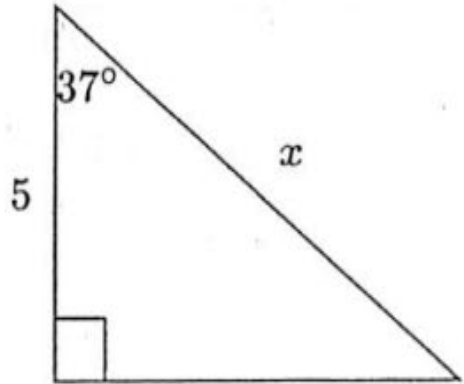
# Find Unknown Angle



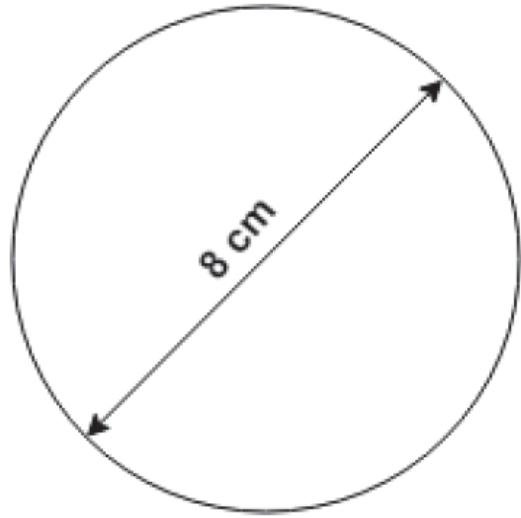
What is the value of  $\theta$ ?

# Find Unknown Side

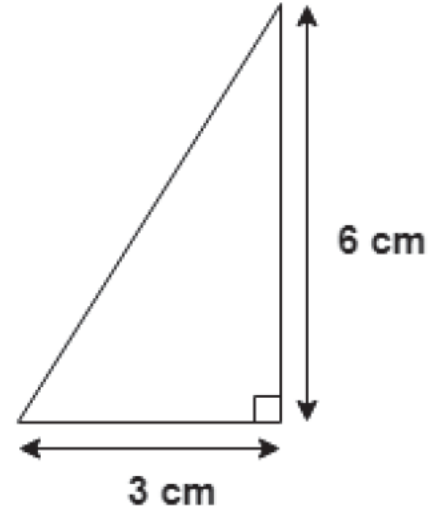
1. What is the length of side  $x$ ?



# Find Area



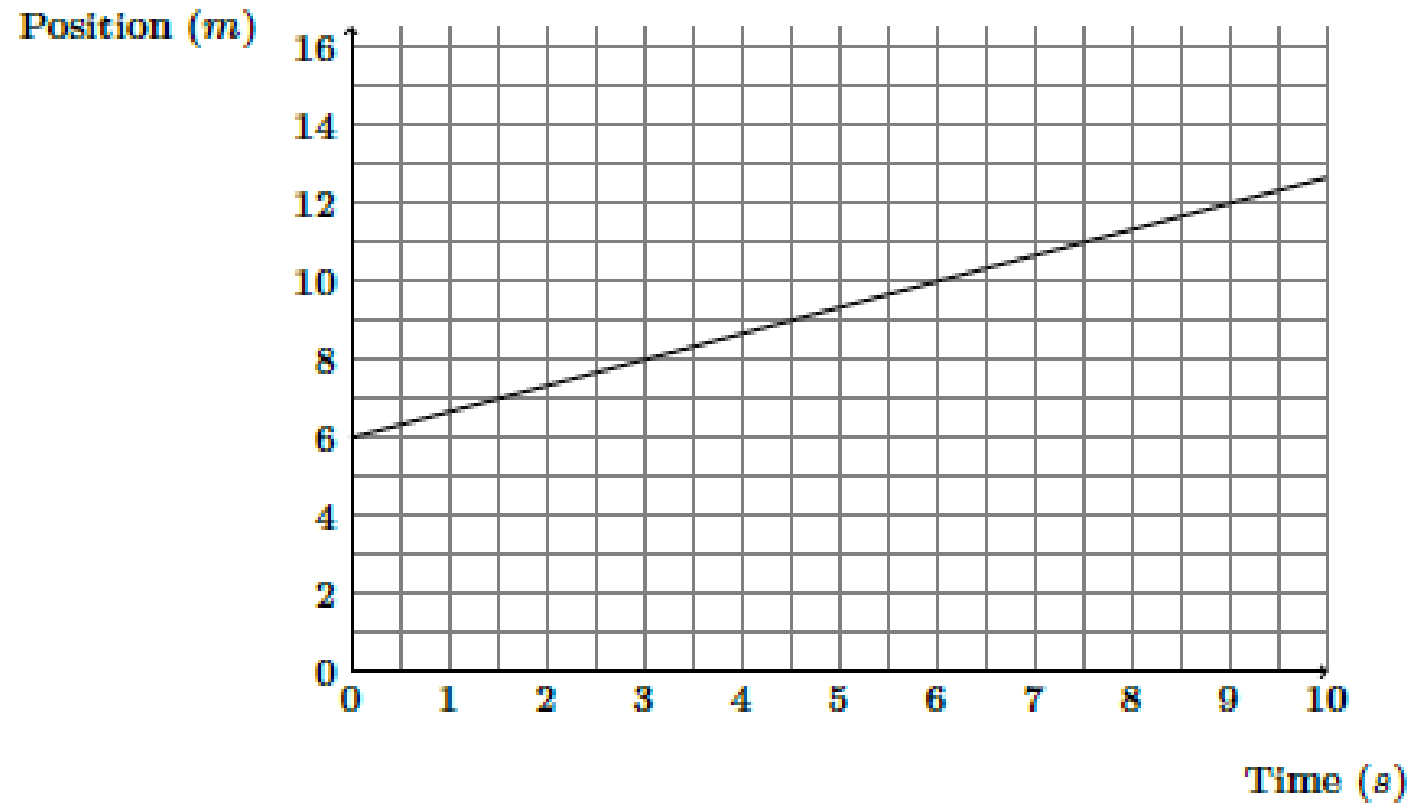
(a) Area of the circle =



(b) Area of the triangle =

# Find Slope of Graph

What is the slope of the graph below?





# Simultaneous Equations, Numeric Coefficients

What is the numerical value of  $x$ ?

$$0.5y = 2x$$

$$78.4 - y = 8x$$

# Simultaneous Equations, Symbolic Coefficients

$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

4. Find the value of each of the following.

$$\cos(0^\circ) = ?$$

$$\sin(90^\circ) = ?$$

$$\tan(0^\circ) = ?$$

6. Solve for  $\theta$ .

$$\gamma\theta + \eta = \lambda\theta + \omega$$

7. Solve for  $x$ .

$$ax + b = cx + d$$

$$10. 2\left(\frac{3}{4}\right) = ?$$

(Some) Other Items

$$11. \frac{a/b}{c^2/d} = ?$$

$$\text{A. } \frac{ac^2}{bd} \quad \text{B. } \frac{ad}{bc^2} \quad \text{C. } \frac{bd}{ac^2} \quad \text{D. } \frac{bc^2}{ad}$$

$$17. ax - dx = c$$

$$x = ?$$

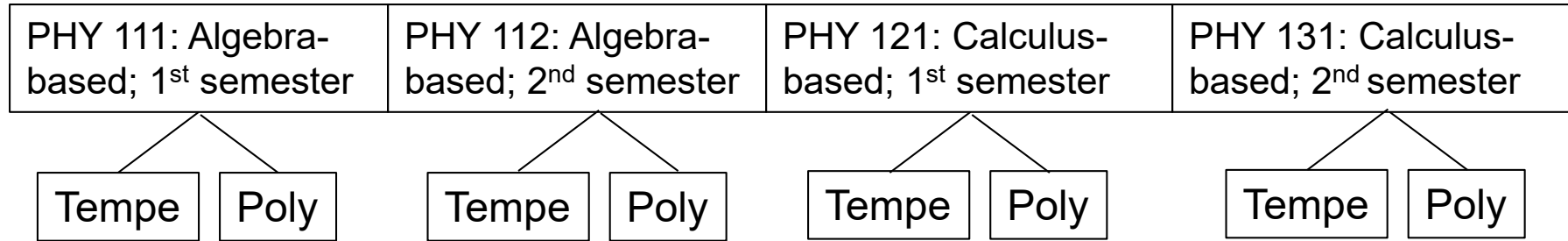
# Our Primary Sample Populations

(Arizona State University)

PHY 111: Algebra-based; 1 <sup>st</sup> semester	PHY 112: Algebra-based; 2 <sup>nd</sup> semester	PHY 121: Calculus-based; 1 <sup>st</sup> semester	PHY 131: Calculus-based; 2 <sup>nd</sup> semester
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# Our Primary Sample Populations

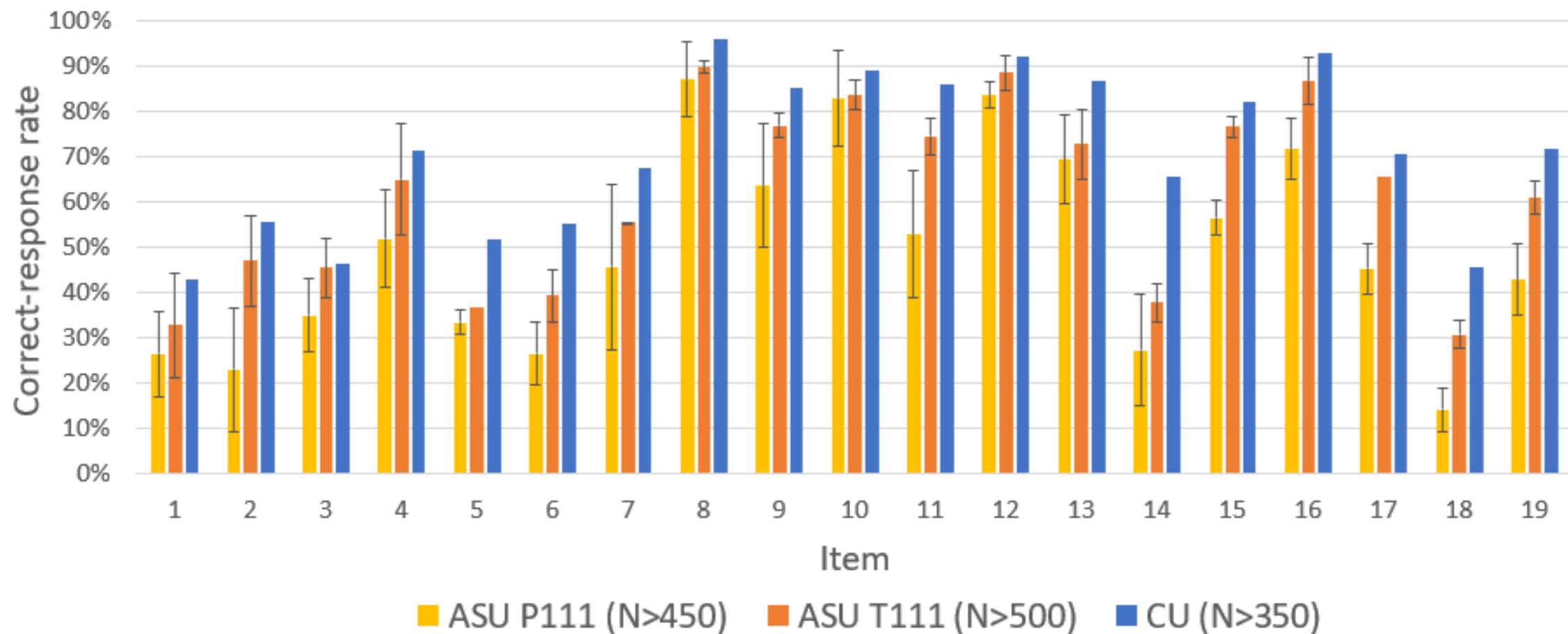
(Arizona State University)



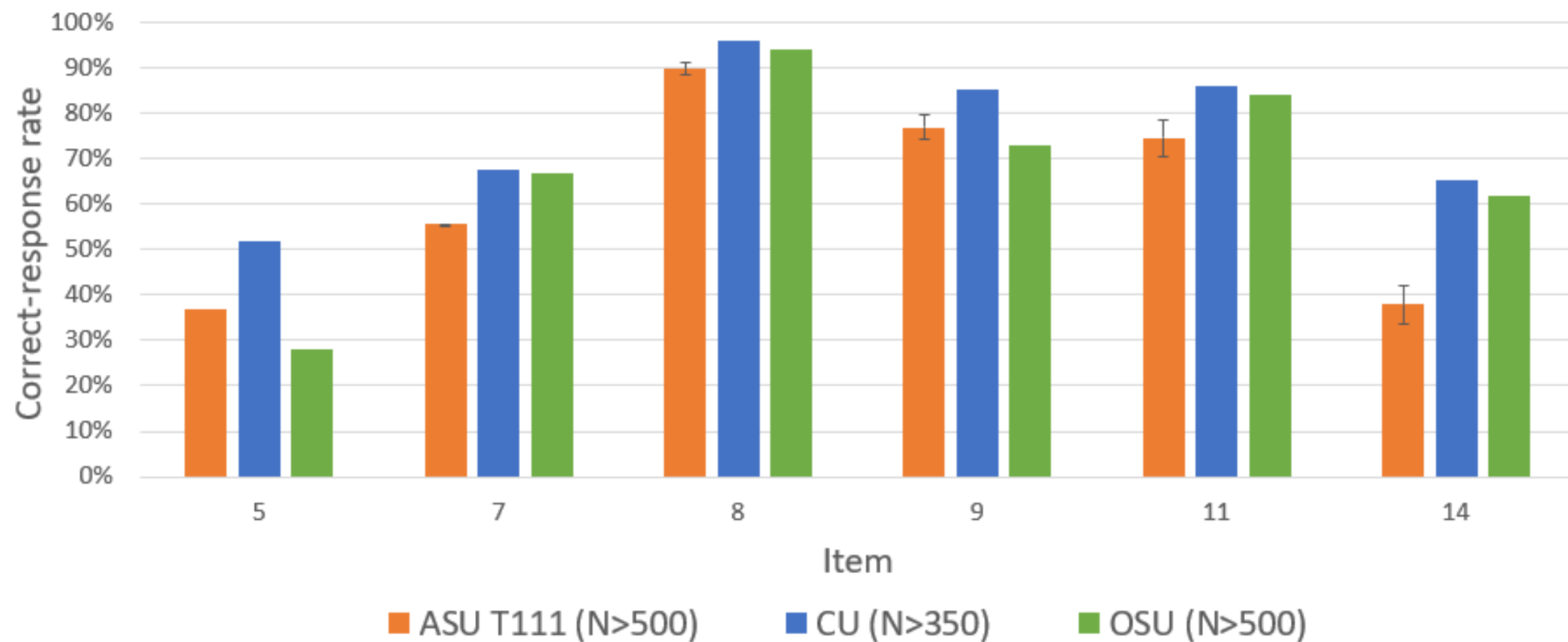
# Primary Findings

- Difficulties with pre-college mathematical operations are widespread among students in both algebra- and calculus-based physics courses.
  - Results were highly consistent among five different campuses at four different state universities: Arizona State University (ASU), Tempe and Poly campuses; University of Colorado (CU); Ohio State University (OSU); University of West Florida (UWF)
- Despite the great diversity of diagnostic item types, students' item responses were *highly* correlated with each other, and with *total* score.

## Correct-response rates: algebra-based course

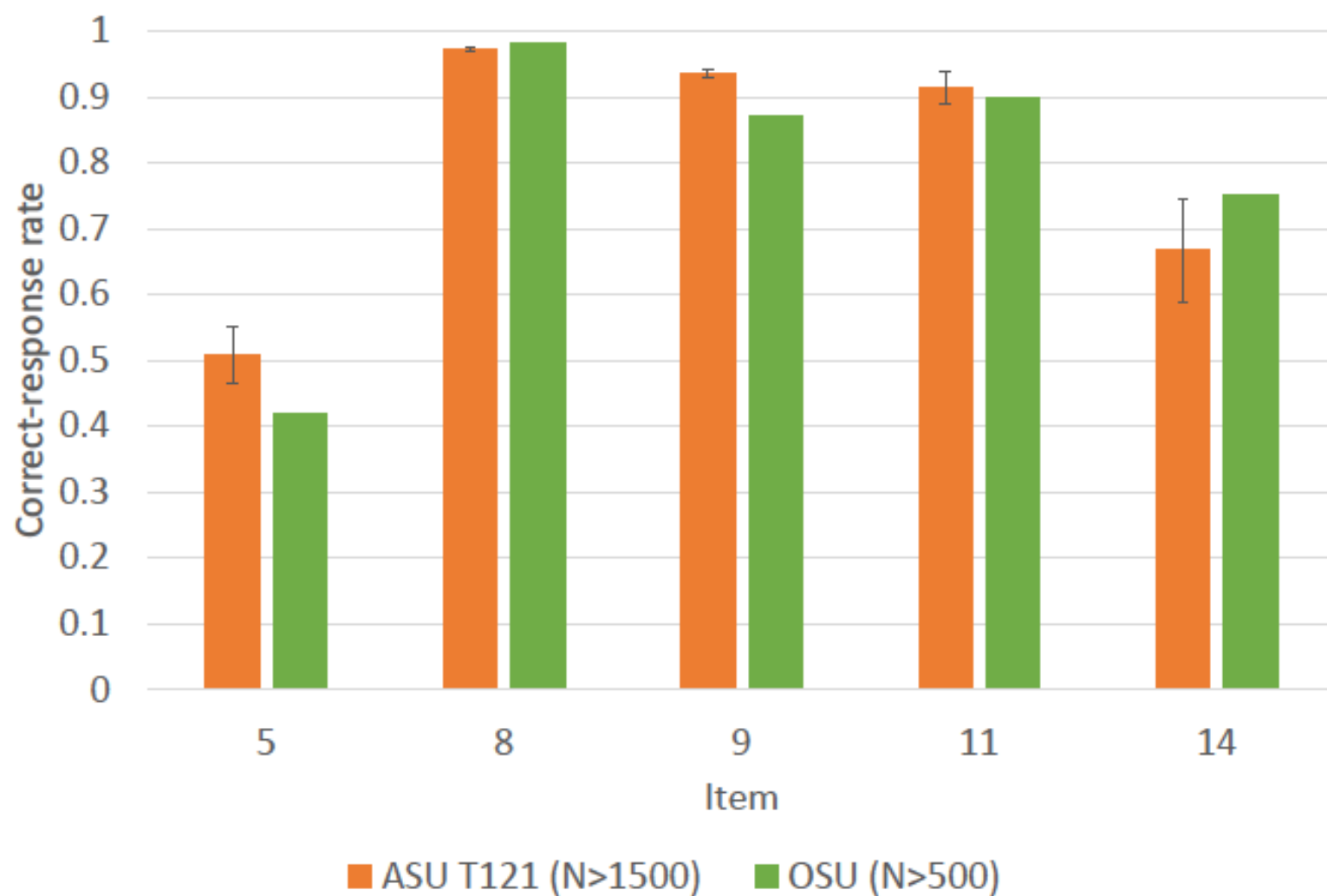


## Correct-response rates: algebra-based course





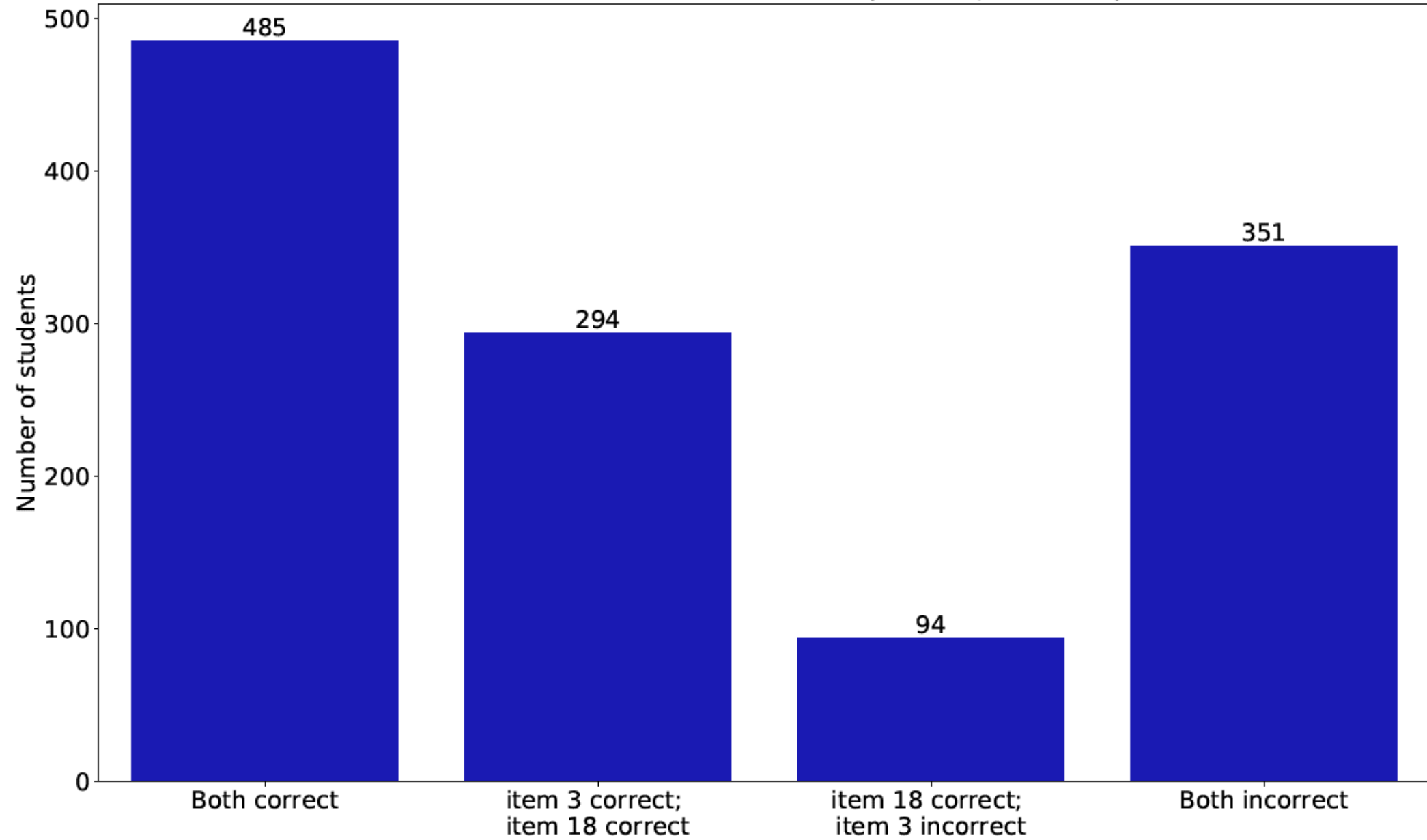
Correct-response rates: calculus-based course



# Primary Findings

- Difficulties with pre-college mathematical operations are widespread among students in both algebra- and calculus-based physics courses.
  - Results were highly consistent among five different campuses at four different state universities.
- **Despite the great diversity of diagnostic item types, students' item responses were *highly* correlated with each other, and with *total* score.**
  - Example: Items #3 and #18

Performance on items 3 and 18 ( $r=0.40$ ;  $N=1224$ )



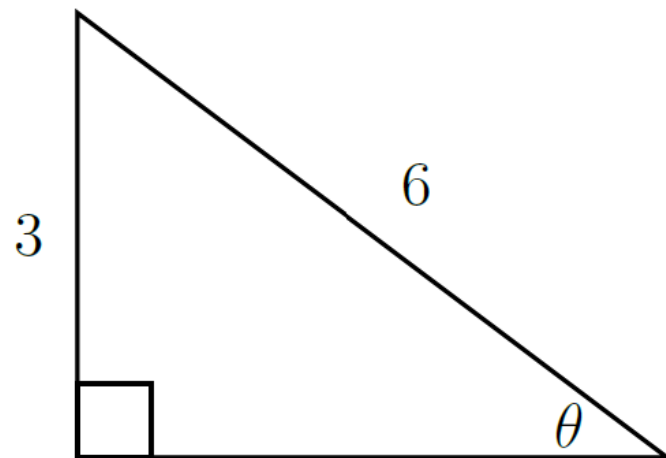
# Predictability at Individual-Student Level

- Performance on a single test item can predict probable correct-response rate on remaining 13 items

Examples:

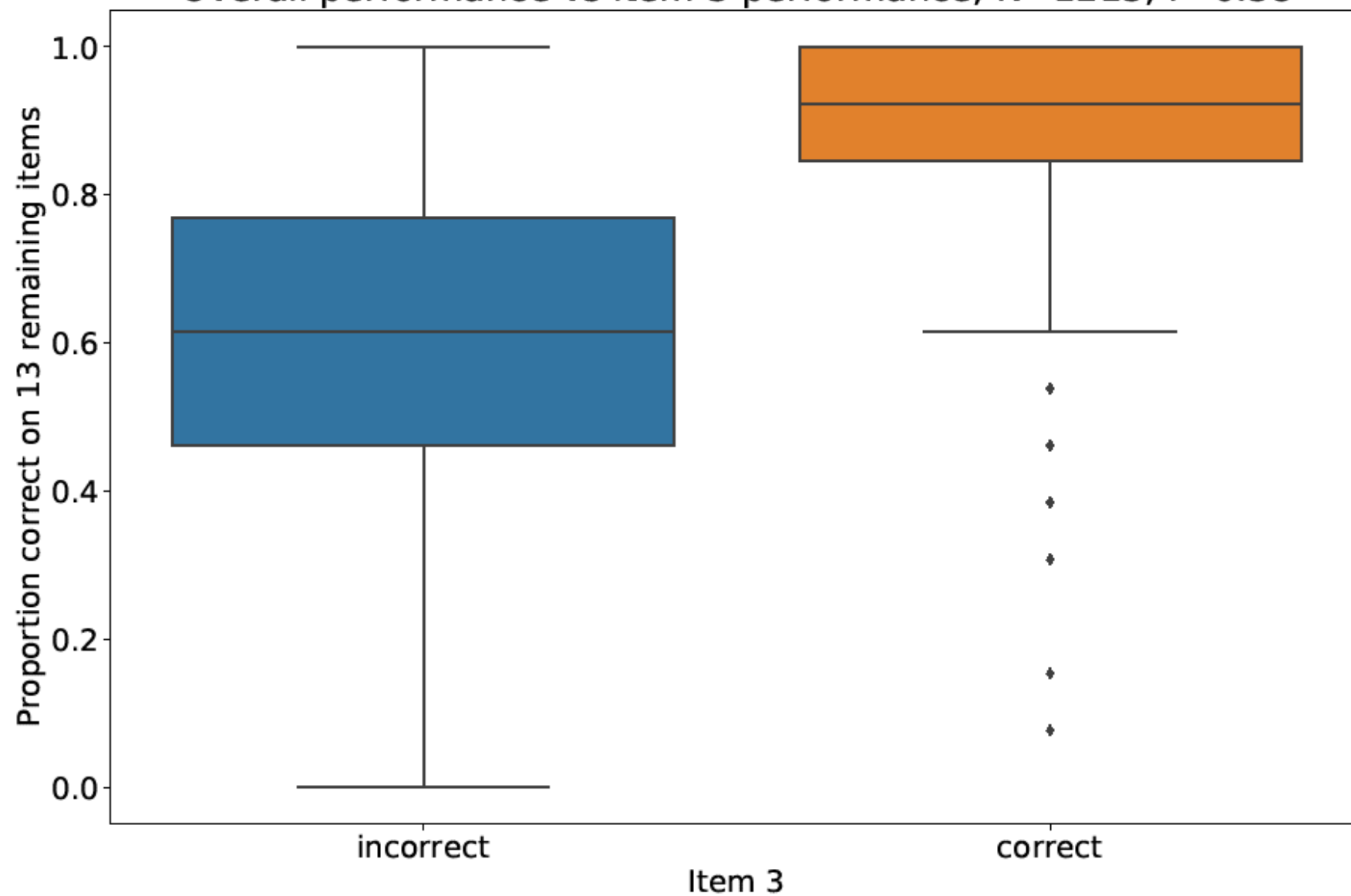
- Item #3
- Item #7

3. What is the value of  $\theta$ ?



Predictor: item 3

Overall performance vs item-3 performance; N=1215;  $r=0.58$



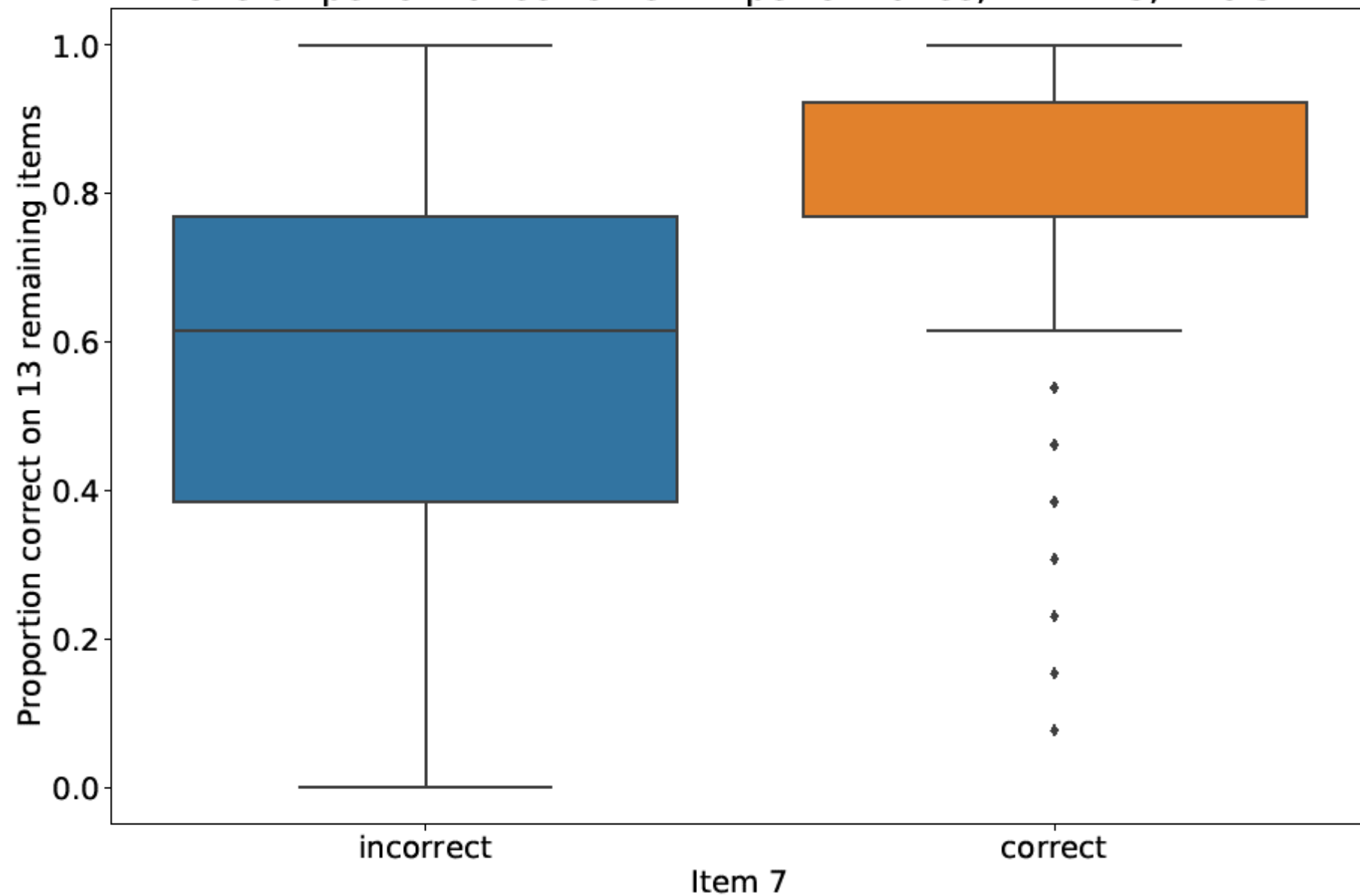
7. Solve for  $x$ .

$$ax + b = cx + d$$

(Your answer for  $x$  should have a, b, c, and d in it. Please isolate  $x$  to one side of your answer.)

Predictor: item 7

Overall performance vs item-7 performance; N=1215;  $r=0.51$





# Predictability at Individual-Student Level

- Performance on 3-item subset can accurately predict correct-response rate on remaining 11 items

Examples:

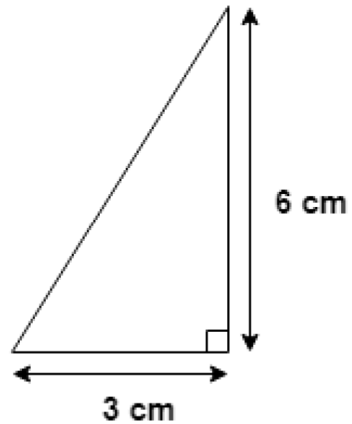
[#9, #14, #16]

[#3, #9, #15]

9.  $2\left(\frac{a}{b}\right) = ?$

A.  $\frac{2a}{b}$     B.  $\frac{2a}{2b}$     C.  $\frac{a}{2b}$

16. Find the area of the circle and triangle below.



Area of the triangle =

14.  $v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{v_1}{t_1}$

$v = \frac{v_1}{2}$

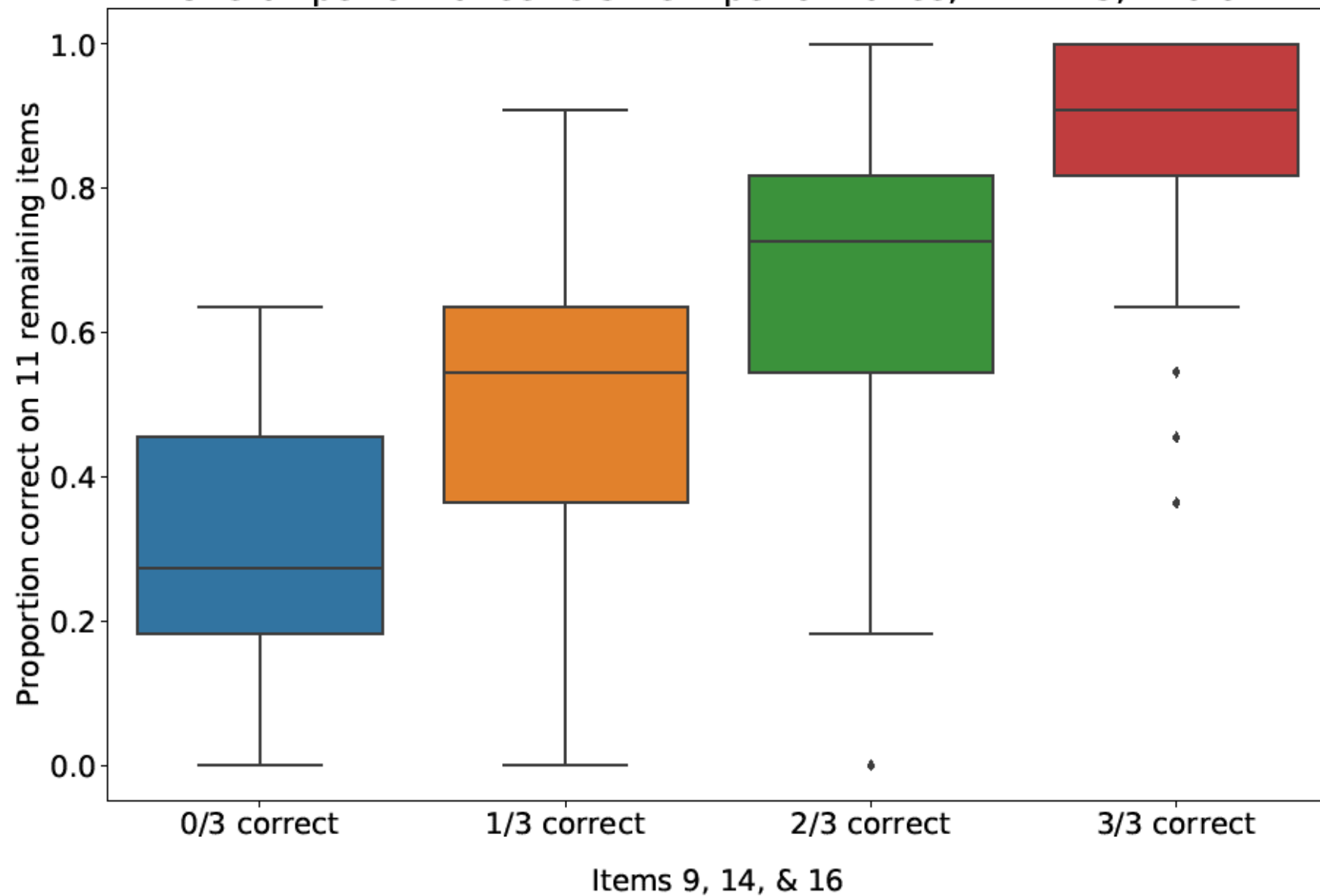
$d = ?$

(Please clearly *circle* your answer and show all work.)

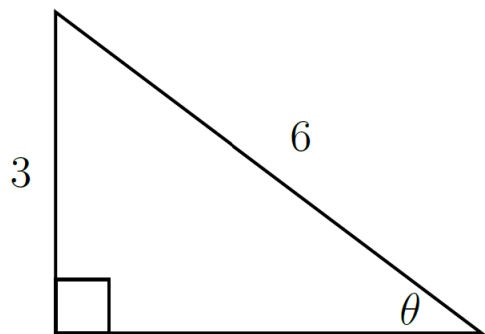
A.  $d = v_1 t_1$     B.  $d = \frac{v_1 t_1}{2}$     C.  $d = \frac{v_1 t_1}{4}$     D.  $d = \frac{v_1 t_1}{8}$     E.  $d = \frac{v_1 t_1}{16}$

Predictors: items 9, 14, & 16

Overall performance vs 3-item performance; N=1215;  $r=0.67$



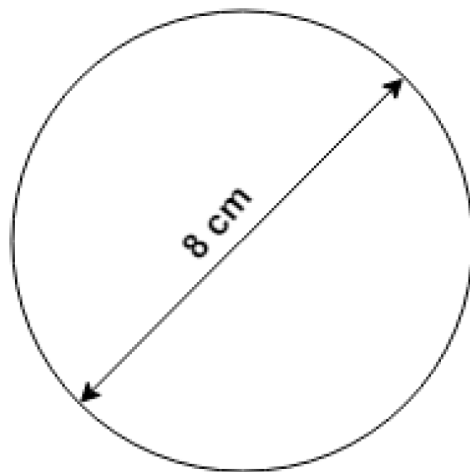
3. What is the value of  $\theta$ ?



9.  $2\left(\frac{a}{b}\right) = ?$

A.  $\frac{2a}{b}$     B.  $\frac{2a}{2b}$     C.  $\frac{a}{2b}$

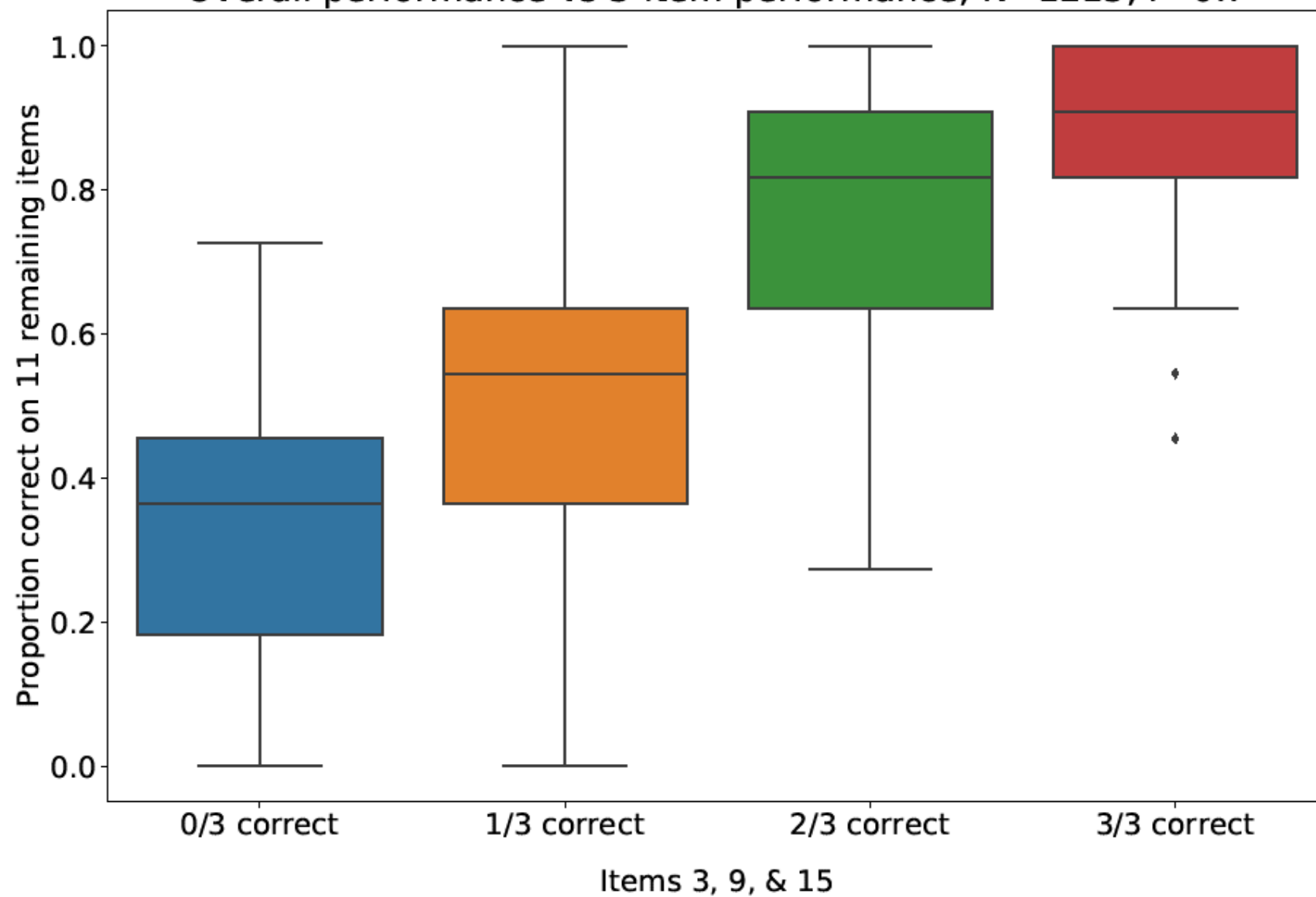
15. Find the area of the circle and triangle below.



Area of the circle =

Predictors: items 3, 9, & 15

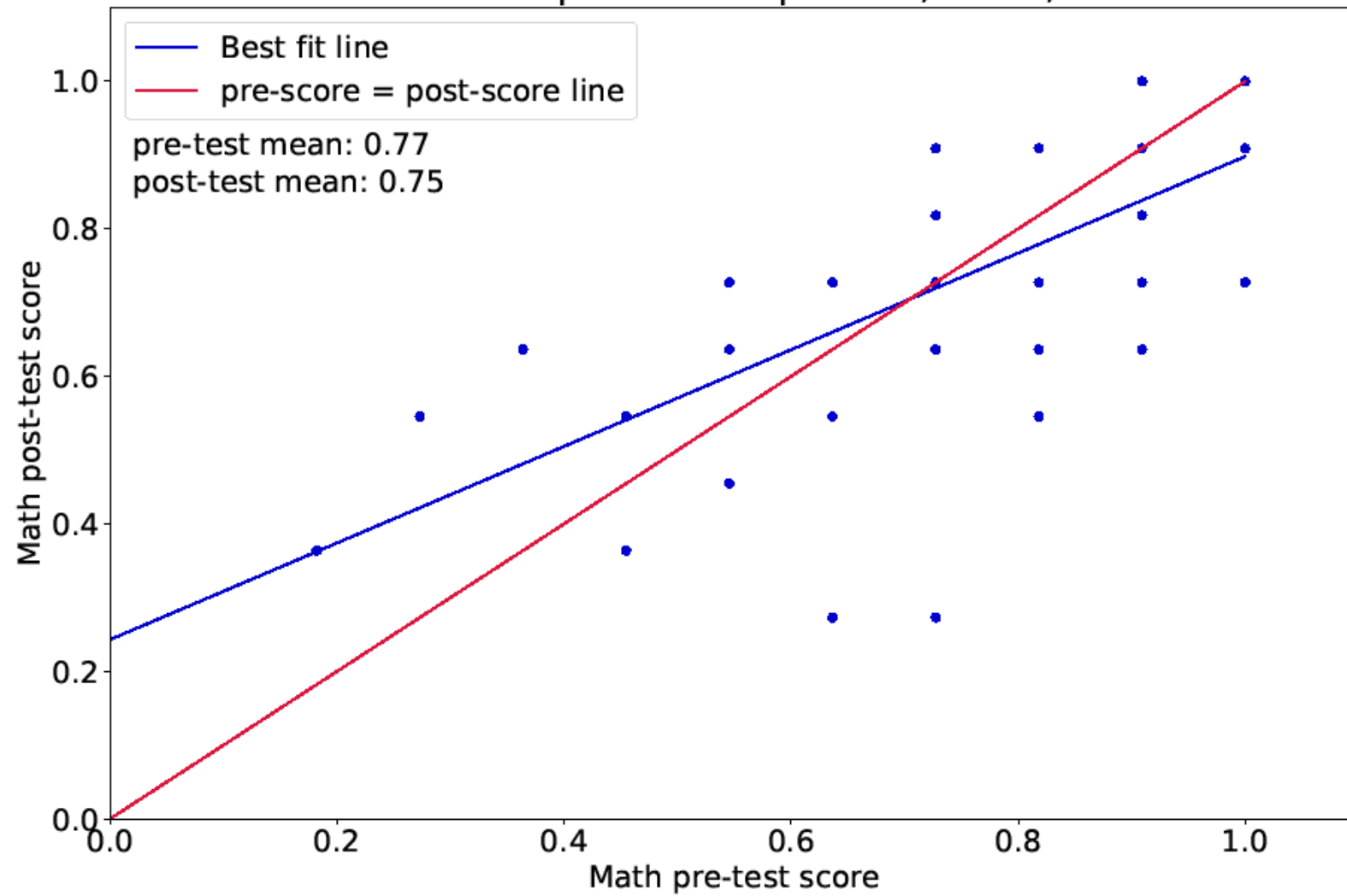
Overall performance vs 3-item performance; N=1215;  $r=0.7$



# Predictability at Individual-Student Level

- Individual student pretest score on online version is moderately predictive of post-test score

Online version: post-test vs pre-test; N=51;  $r=0.66$

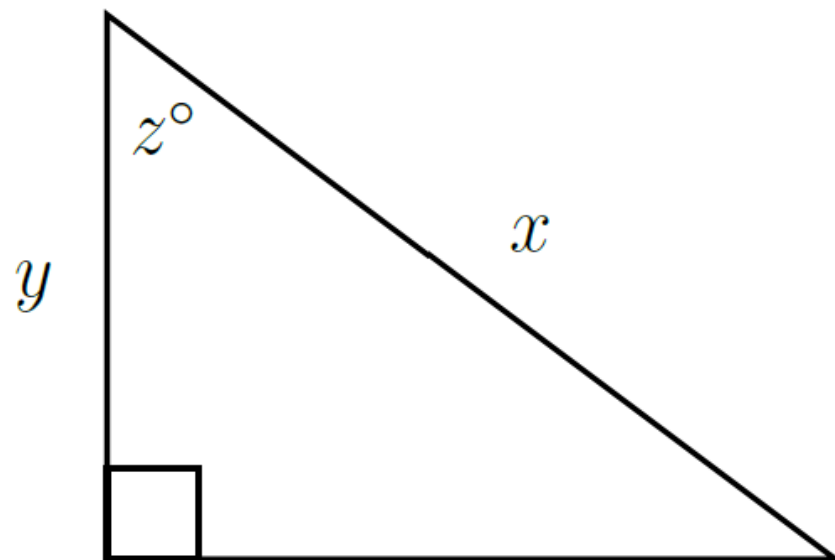


# Predictability at Whole-Class Level

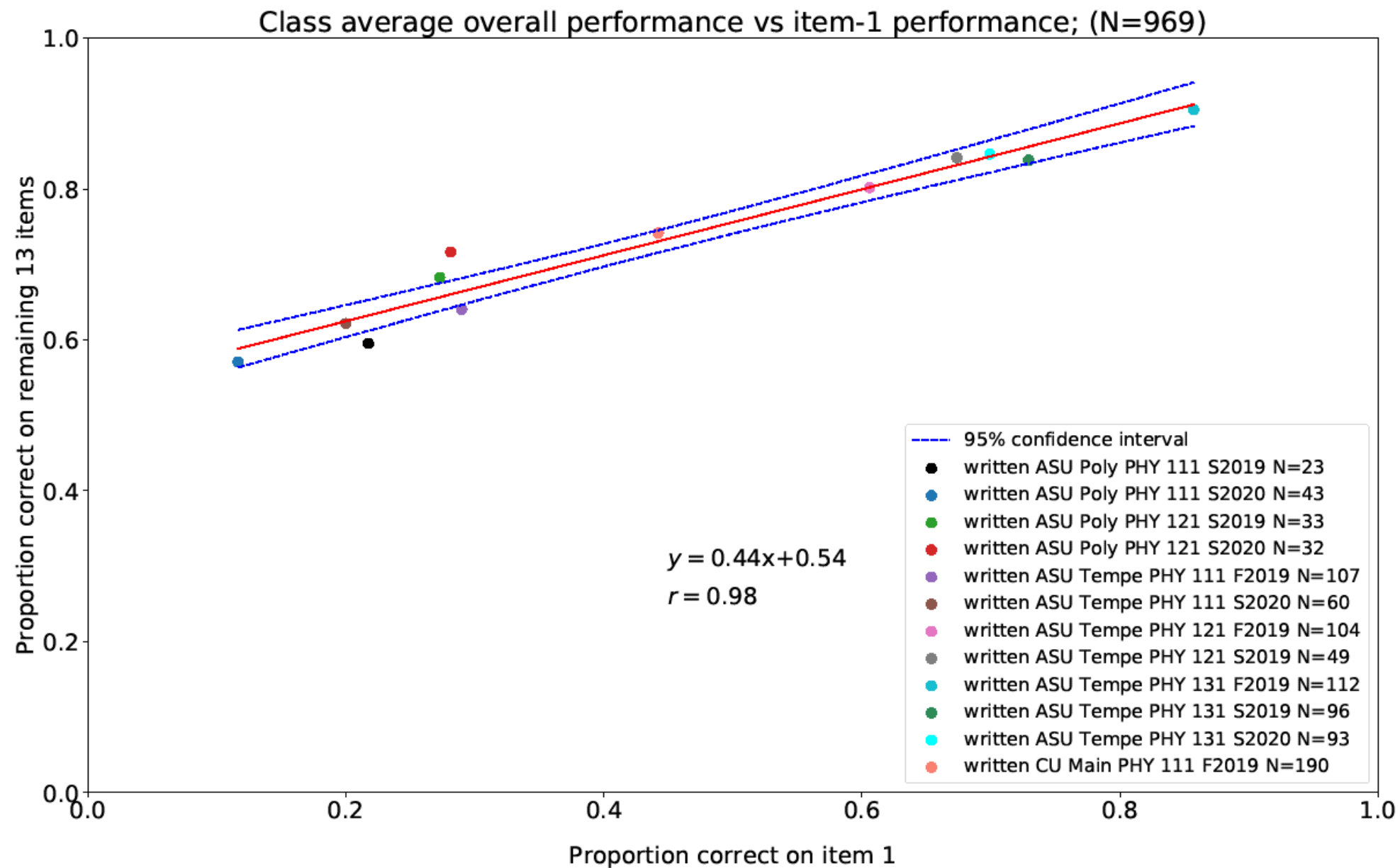
- Class-average performance on *single test item* can accurately predict class-average correct-response rate on remaining 13 items



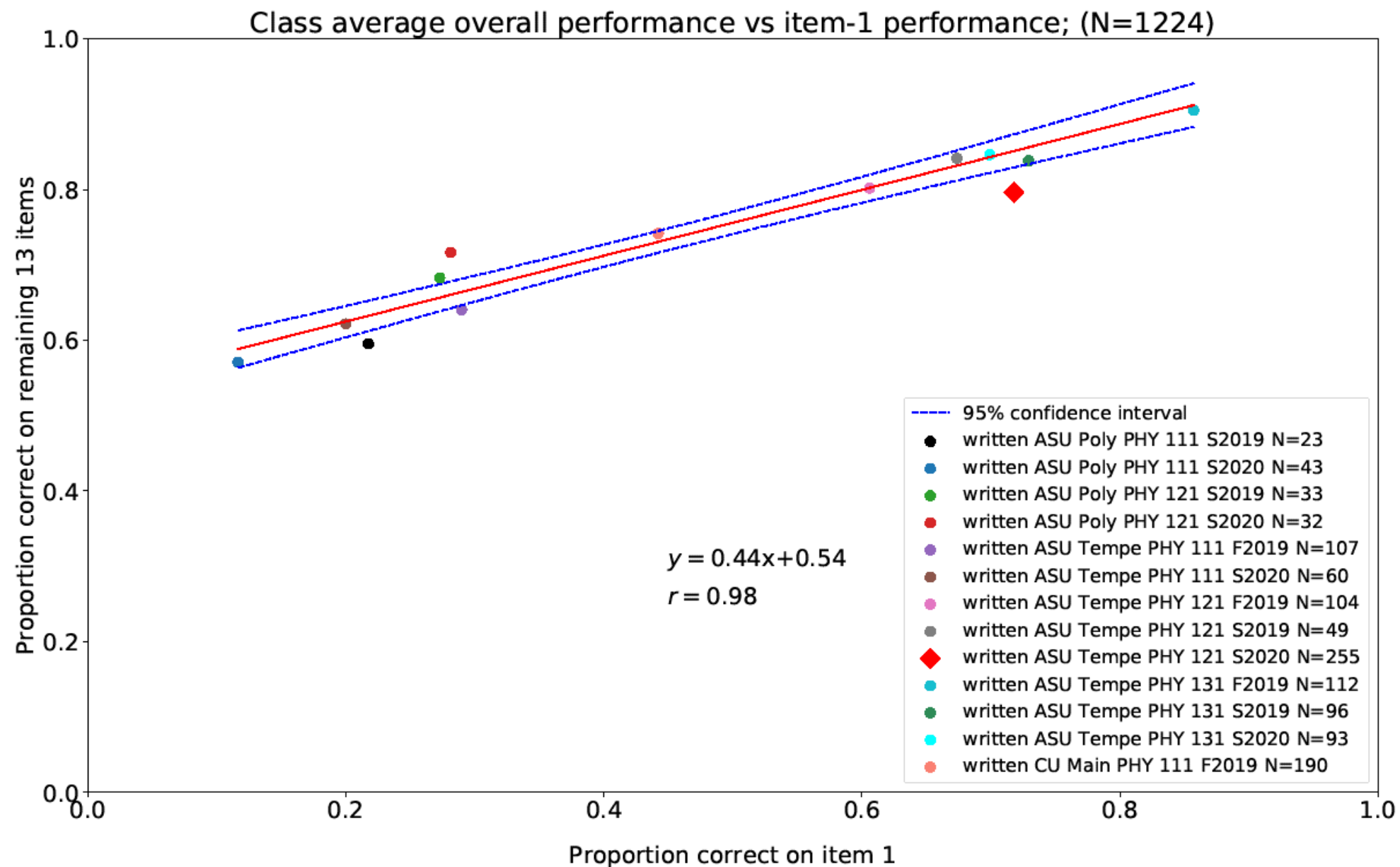
1. What is the length of side  $x$ ?



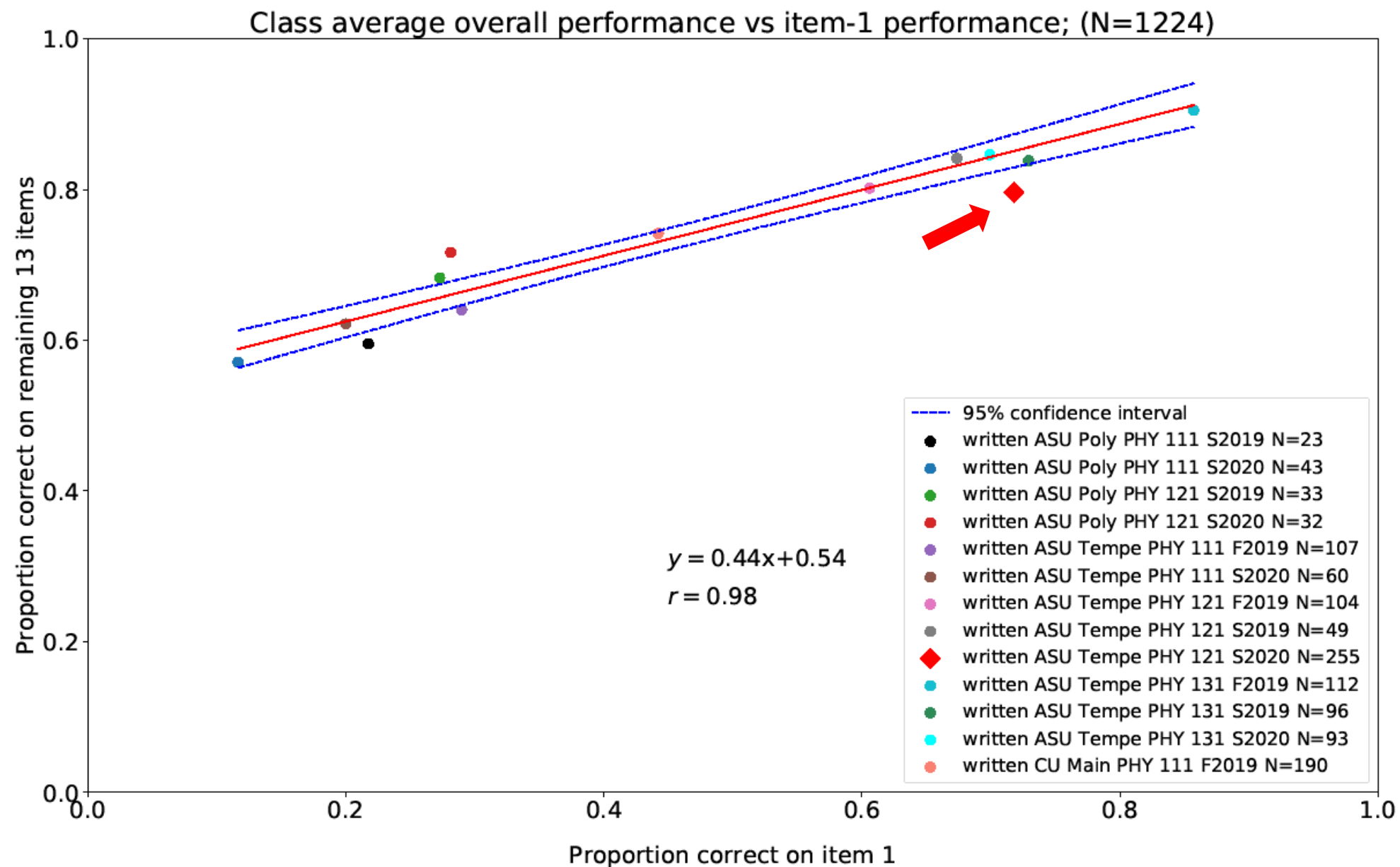
Predictor: item 1



Predictor: item 1



Predictor: item 1



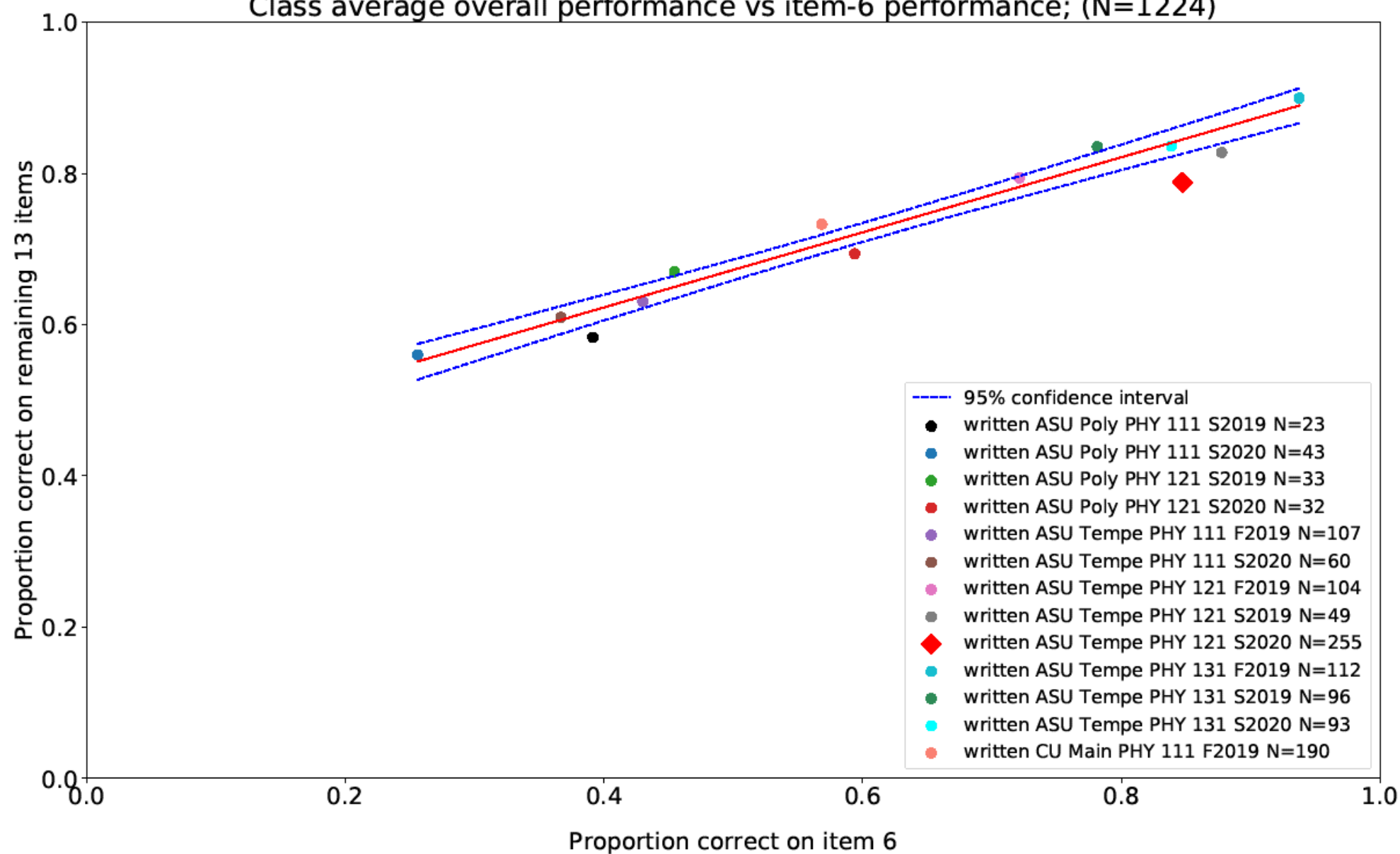
6. Solve for  $\theta$ .

$$\gamma\theta + \eta = \lambda\theta + \omega$$

(Your answer for  $\theta$  should have  $\gamma$ ,  $\eta$ ,  $\lambda$ , and  $\omega$  in it. Please isolate  $\theta$  to one side of your answer.)

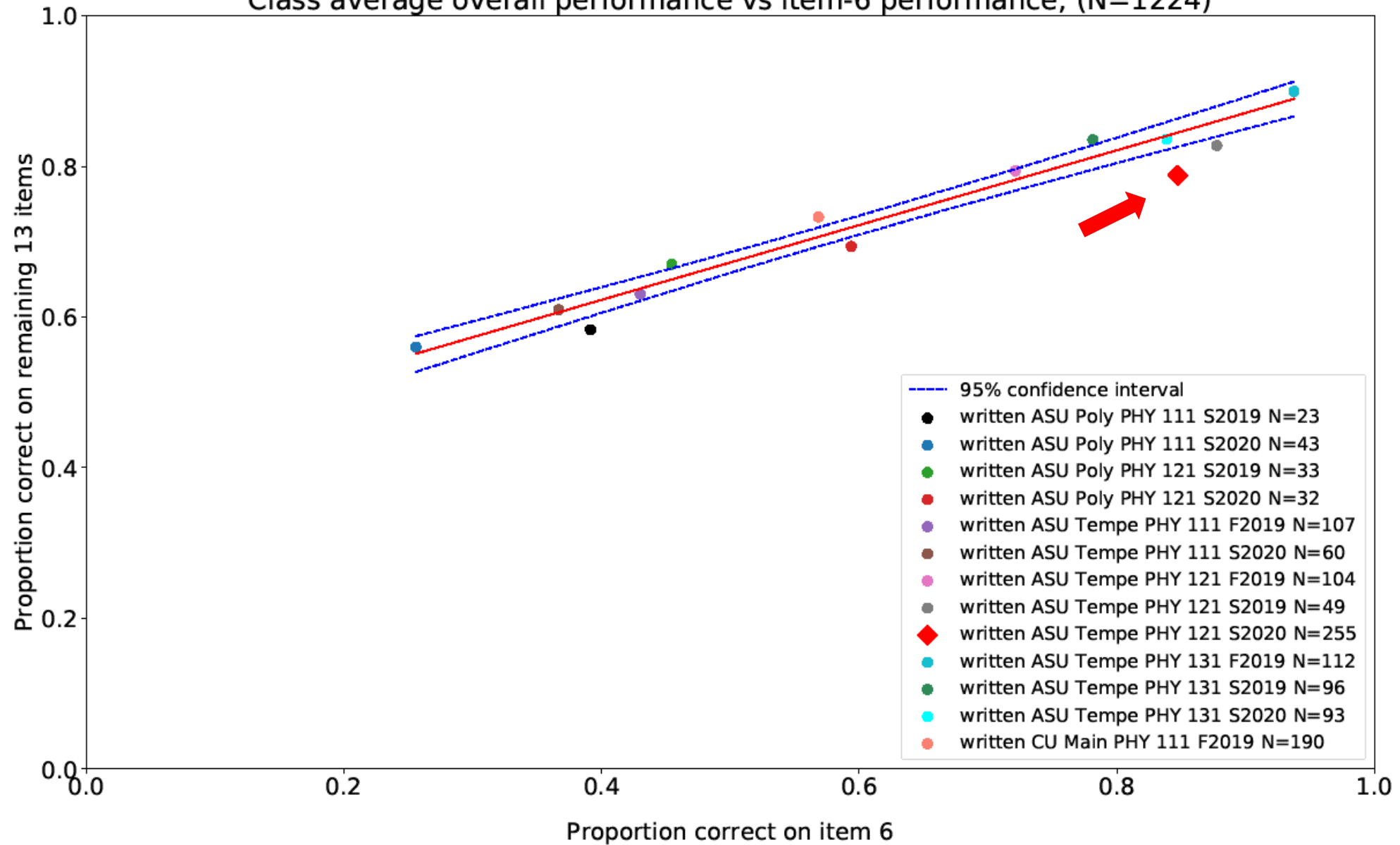
# Predictor: item 6

Class average overall performance vs item-6 performance; (N=1224)



# Predictor: item 6

Class average overall performance vs item-6 performance; (N=1224)



18.  $cy = dx$

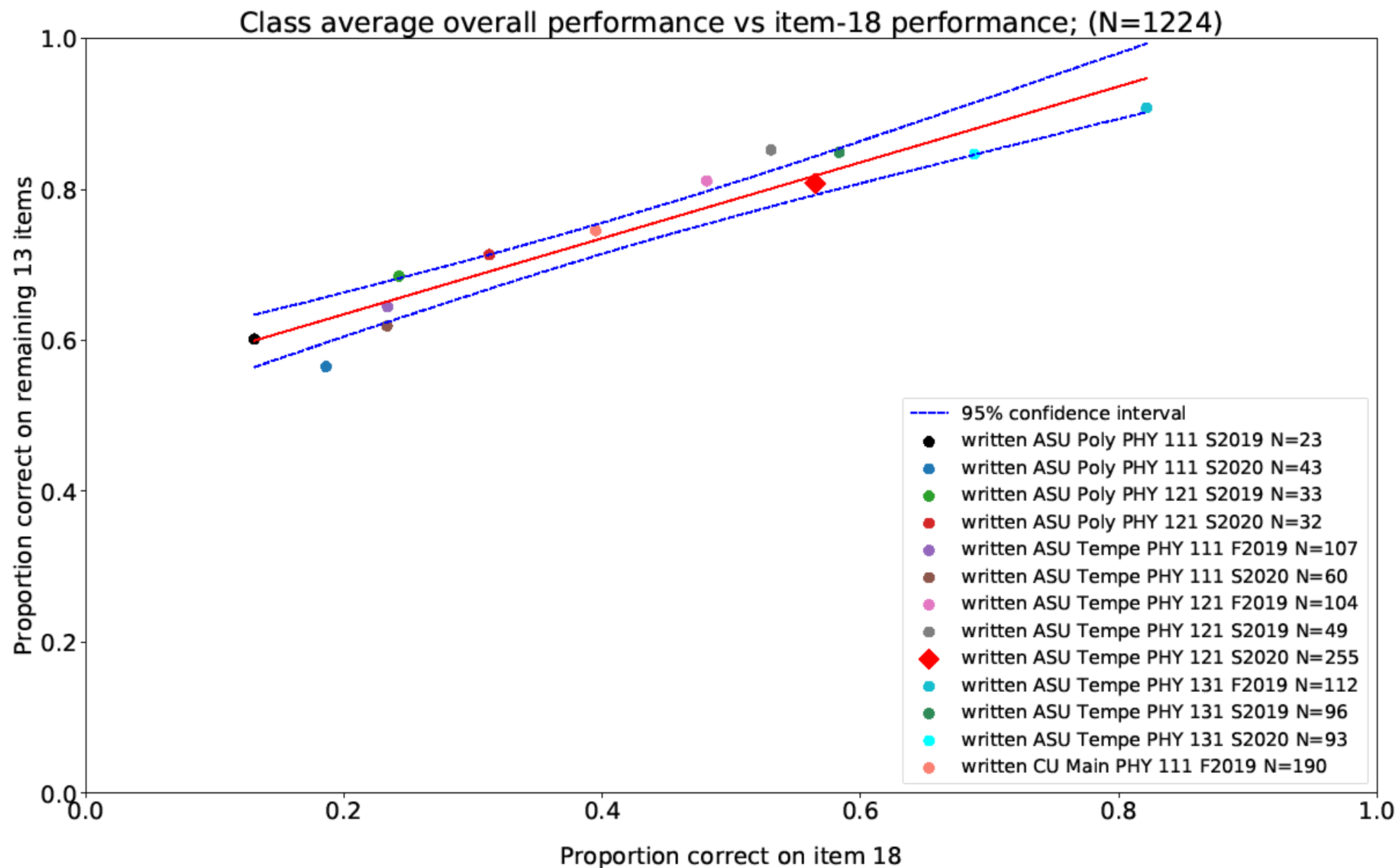
$$a - y = bx$$

$$x = ?$$

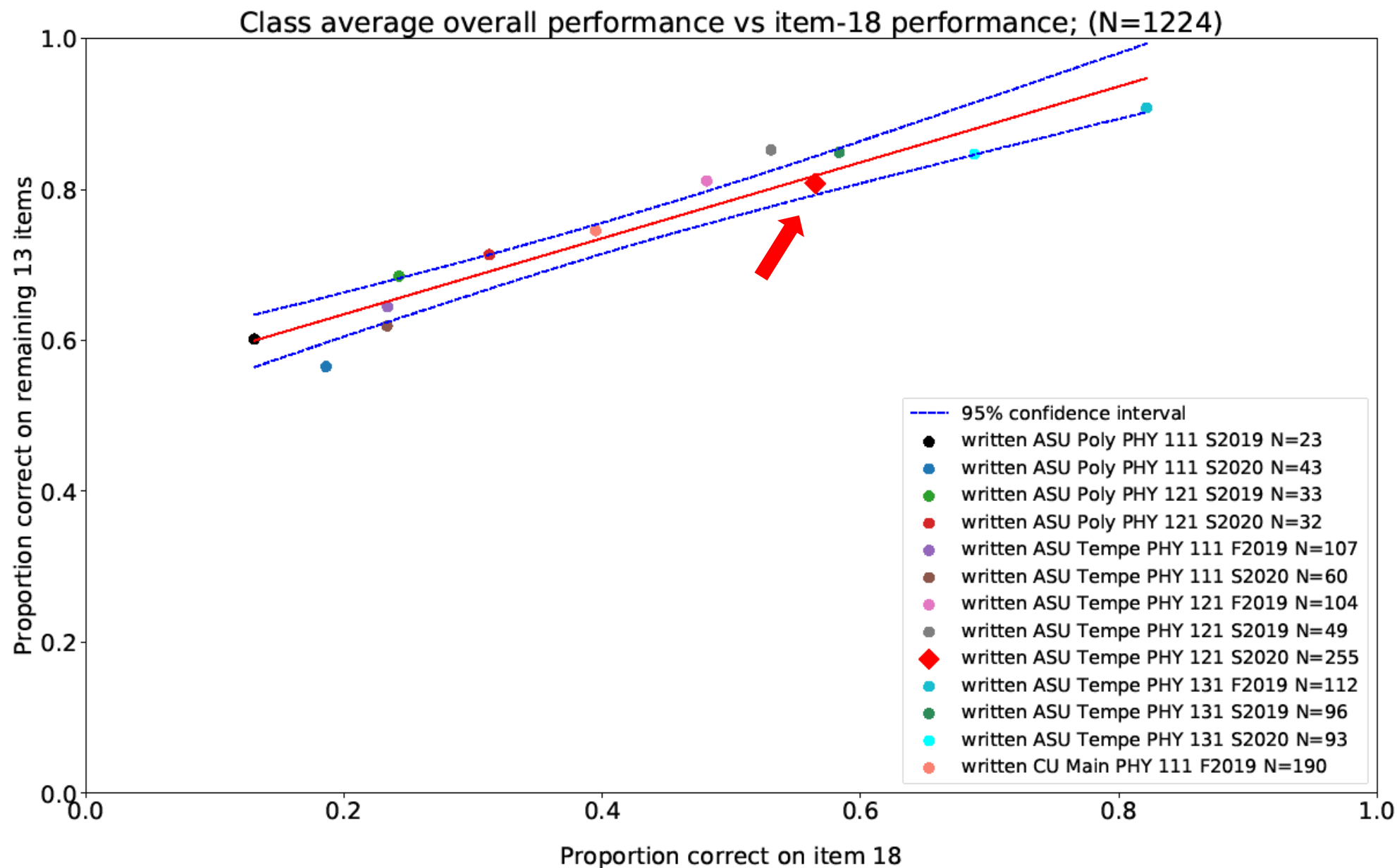
(Your answer for  $x$  should have  $a$ ,  $b$ ,  $c$ , and  $d$  in it, but *not*  $y$ . ***Your answer should have  $x$  only on one side.***)



Predictor: item 18



Predictor: item 18



# Relationship between physics and math performance

- For online version, student performance on conceptual physics questions is positively correlated with performance on math diagnostic
- [4-item physics cluster vs 14-item math diagnostic]

**PHY 1.**

A rock, attached to a rope, is being pulled straight up at constant, unchanging speed. While it is being pulled, the forces acting on the rock are:

- A. An upward force from the rope that is larger than the downward force of gravity
- B. An upward force from the rope that is equal in magnitude to the downward force of gravity
- C. An upward force from the rope that is smaller than the downward force of gravity
- D. Only the downward force of gravity
- E. Only an upward force from the rope

**PHY 2.**

A rock, attached to a rope, is being pulled straight up; it is speeding up at a steady rate (with constant acceleration) while it is being pulled. The downward force of gravity on the rock is nearly constant and unchanging. The upward force from the rope is:

- A. Larger than the force of gravity, and increasing in strength
- B. Larger than the force of gravity, and nearly constant in strength
- C. Equal to the force of gravity
- D. Smaller than the force of gravity, and decreasing in strength
- E. Smaller than the force of gravity, and nearly constant in strength

**PHY 3.**

A small woman is pushing on a stalled car and the car is very gradually speeding up. While she is pushing on the car,

- A. The force of woman on the car is larger than the force of the car on the woman
- B. 

The force of the woman on the car is equal in magnitude to the force of the car on the woman
--
- C. The force of the woman on the car is smaller than the force of the car on the woman
- D. The woman exerts a force on the car but the car does not exert any force on the woman
- E. The car exerts a force on the woman but the woman does not exert any force on the car

**PHY 4.**

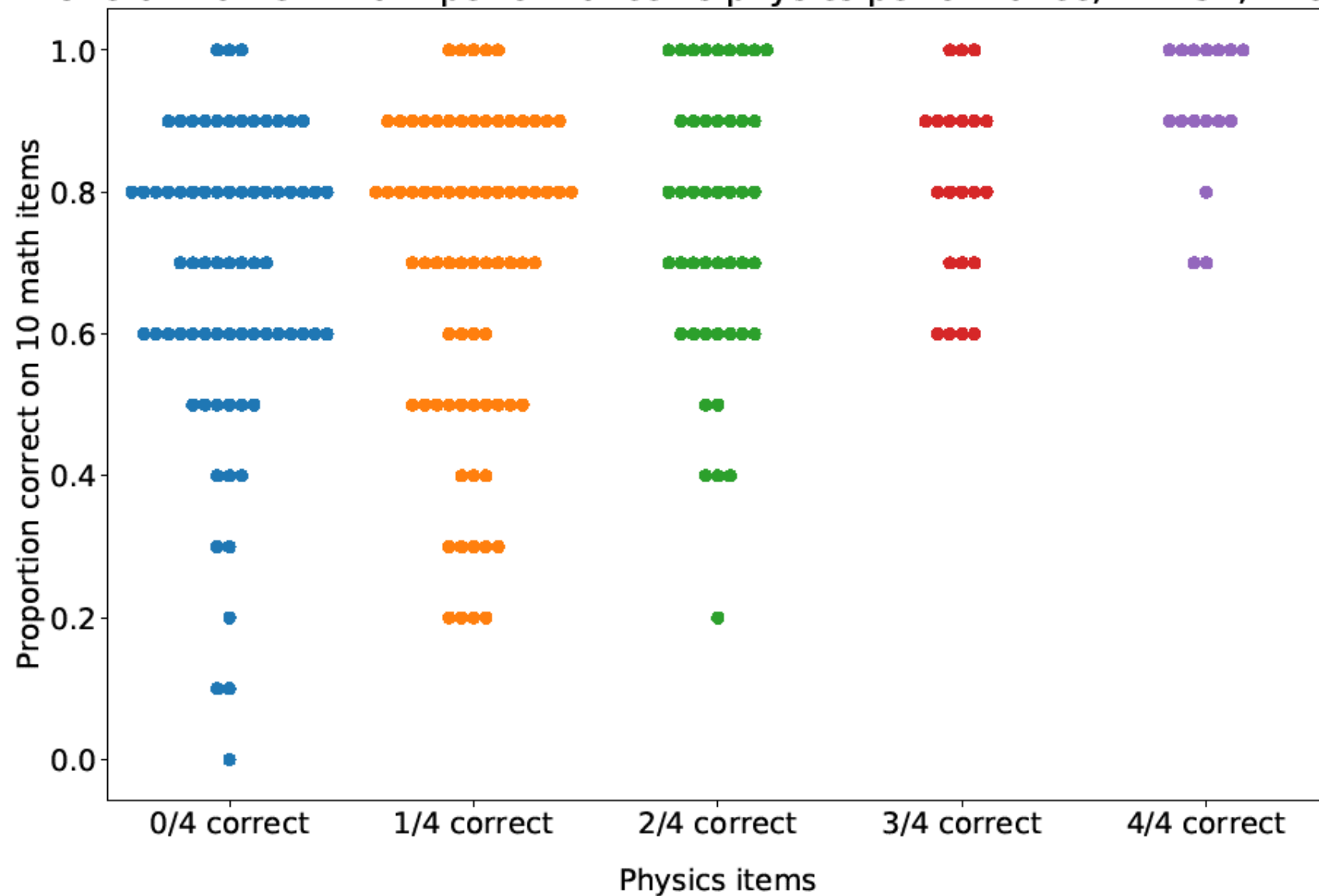
A child is sitting on a sled that is stranded in the middle of a frozen pond. The ice is so wet and slippery that *friction can be assumed to be zero*. Workers in a truck on land attach a fishing line to the sled and pull on the line; a scale attached to the line ensures that it pulls with unchanging force. However, after a few moments the line breaks. Then:

- A. The sled moves at constant speed while the line pulls and at a constant but slower speed after the line breaks
- B. The sled speeds up while the line pulls and slows down after the line breaks
- C. 

The sled speeds up while the line pulls and moves at constant speed after the line breaks
---
- D. The sled speeds up while the line pulls and keeps gaining speed after the line breaks, but not as rapidly as before
- E. The sled moves at constant speed the whole time

# Predictors: 4 physics items

Overall 10-item math performance vs physics performance; N=232;  $r=0.29$



# Summary

- Physics students' difficulties with basic mathematical operations are not confined to one particular topic (e.g., trigonometry, algebra, graphing); rather, difficulties on one topic are highly correlated with difficulties on other topics
- Performance on single math test items, or on small item subsets, is predictive of overall individual-student math performance, and *highly* predictive at the class-average level