

# Relationship between course grades in introductory physics and pre-instruction assessment scores

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# Acknowledgments

- Diagnostic data have been provided by (among others):
  - Vince Coletta (Loyola Marymount University)
  - Steven Pollock (University of Colorado, Boulder)
  - Christopher Varney (University of West Florida)

# Assessment Pretests

- Diagnostic pretest covering pre-college mathematics (“Math”)
  - calculators allowed
- Pre-instruction tests of scientific reasoning skill and physics concept knowledge:
  - Lawson Test of Scientific Reasoning (“Lawson”)
  - Force Concept Inventory (FCI)
- Why do this? Perhaps ultimately we can offer special assistance to those students who need it most.

# Sample Description

- 30 introductory physics classes from 4 universities, over 2000 total students
- Instruction in most classes was “non-traditional,” generally highly interactive using research-based instructional materials and methods

# Course and Institution Code

Alg-1: Algebra-based course, first semester

Alg-2: Algebra-based course, second semester

Calc-1: Calculus-based course, first semester

Calc-2: Calculus-based course, second semester

ASU-P: Arizona State University, Polytechnic campus

ASU-T: Arizona State University, Tempe campus

LMU: Loyola Marymount University

UWF: University of West Florida

CU: University of Colorado, Boulder

# Comparing probabilities of high\* and low\* grades

- What is the probability of a student with a *high* score on a pre-instruction assessment getting a high grade in the class?
- How does that compare to a *low-scoring* student's probability of getting a high grade?

(and, same questions for probabilities of getting a *low* grade)

\*In this context, “high” and “low” mean “top quartile” and “bottom quartile”

*Consistent result:*

***High*** scorers on the diagnostic pretests were much more likely to get ***high*** grades than were low scorers

## High Course Grade vs. Mathematics Diagnostic Pretest Score

Course	Campus	N	Top-quartile Math: % with top-quartile grades	Bottom-quartile Math: % with top-quartile grades	High-grade odds ratio
Alg-1 2021a	ASU-P	39	51%	10%	5.0
Alg-1 2021b	ASU-P	42	44%	10%	4.6
Alg-1 2022a	ASU-P	40	27%	6%	4.4
Alg-1 2022b	ASU-P	52	49%	10%	5.1
Alg-1 2023a	ASU-P	42	39%	10%	4.1
Alg-1 2023b	ASU-P	46	64%	9%	7.3
Alg-2 2022	ASU-P	75	46%	21%	2.2
Alg-2 2023	ASU-P	92	41%	13%	3.2
Alg-2 2024	ASU-P	99	51%	8%	6.1
<b>Alg-2 2021</b>	<b>ASU-T</b>	<b>129</b>	<b>30%</b>	<b>39%</b>	<b>0.8</b>
Calc-1 2021a	UWF	53	43%	0%	"∞"
Calc-1 2021b	UWF	42	43%	0%	"∞"
Calc-2 2021	UWF	58	43%	14%	3.1
<b>AVERAGE</b>	<b>(unweighted)</b>	<b>(809)</b>	<b>44%</b>	<b>12%</b>	<b>3.8</b>



## High Course Grade vs. Mathematics Diagnostic Pretest Score

Course	Campus	N	Top-quartile Math: % with top-quartile grades	Bottom-quartile Math: % with top-quartile grades	High-grade odds ratio
Alg-1 2021a	ASU-P	39	51%	10%	5.0
Alg-1 2021b	ASU-P	42	44%	10%	4.6
Alg-1 2022a	ASU-P	40	27%	6%	4.4
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<b>AVERAGE</b>	<b>(unweighted)</b>	<b>(809)</b>	<b>44%</b>	<b>12%</b>	<b>3.8</b>

## High Course Grade vs. Mathematics Diagnostic Pretest Score

Course	Campus	N	Top-quartile Math: % with top-quartile grades	Bottom-quartile Math: % with top-quartile grades	High-grade odds ratio
Alg-1 2021a	ASU-P	39	51%	10%	5.0
Alg-1 2021b	ASU-P	42	44%	10%	4.6
Alg-1 2022a	ASU-P	40	27%	6%	4.4
Alg-1 2022b	ASU-P	52	49%	10%	5.1
Alg-1 2023a	ASU-P	42	39%	10%	4.1
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Alg-2 2022	ASU-P	75	46%	21%	2.2
Alg-2 2023	ASU-P	92	41%	13%	3.2
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## High Course Grade vs. Mathematics Diagnostic Pretest Score

Course	Campus	N	Top-quartile Math: % with top-quartile grades	Bottom-quartile Math: % with top-quartile grades	High-grade odds ratio
Alg-1 2021a	ASU-P	39	51%	10%	5.0
Alg-1 2021b	ASU-P	42	44%	10%	4.6
Alg-1 2022a	ASU-P	40	27%	6%	4.4
Alg-1 2022b	ASU-P	52	49%	10%	5.1
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Alg-2 2023	ASU-P	92	41%	13%	3.2
Alg-2 2024	ASU-P	99	51%	8%	6.1
<b>Alg-2 2021</b>	<b>ASU-T</b>	<b>129</b>	<b>30%</b>	<b>39%</b>	<b>0.8</b>
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<b>AVERAGE</b>	<b>(unweighted)</b>	<b>(809)</b>	<b>44%</b>	<b>12%</b>	<b>3.8</b>

## High Course Grade vs. Mathematics Diagnostic Pretest Score

Course	Campus	N	Top-quartile Math: % with top-quartile grades	Bottom-quartile Math: % with top-quartile grades	High-grade odds ratio
Alg-1 2021a	ASU-P	39	51%	10%	5.0
Alg-1 2021b	ASU-P	42	44%	10%	4.6
Alg-1 2022a	ASU-P	40	27%	6%	4.4
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<b>AVERAGE</b>	<b>(unweighted)</b>	<b>(809)</b>	<b>44%</b>	<b>12%</b>	<b>3.8</b>



## High Course Grade vs. Mathematics Diagnostic Pretest Score

Campus	N	Top-quartile Math: % with top-quartile grades	Bottom-quartile Math: % with top-quartile grades	High-grade odds ratio
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**AVERAGE**

(unweighted)

(809)

44%

12%

3.8

**High scorers on math pretest were 3.8 times more likely to get a high grade than were low scorers**

## High Course Grade vs. Lawson Test of Scientific Reasoning Pretest Score

Course	Campus	N	Top-quartile Lawson: % with top-quartile grades	Bottom-quartile Lawson: % with top-quartile grades	High-grade odds ratio
Alg-1 2021a	ASU-P	35	46%	23%	2.0
Alg-1 2021b	ASU-P	38	32%	8%	4.0
Alg-1 2022a	ASU-P	41	49%	10%	5.0
Alg-1 2022b	ASU-P	54	57%	10%	5.6
Alg-1 2023a	ASU-P	36	39%	33%	1.2
Alg-1 2023b	ASU-P	44	55%	9%	6.0
Alg-2 2022	ASU-P	73	41%	6%	7.6
Alg-2 2023	ASU-P	92	52%	10%	5.0
Alg-2 2024	ASU-P	90	42%	5%	9.2
Alg-1	CU	469	45%	8%	5.5
Calc-2	CU	276	57%	8%	6.9
Alg-1 2007	LMU	24	50%	0%	"∞"
Alg-1 2009	LMU	51	34%	11%	3.2
Alg-1 2011	LMU	57	53%	18%	2.9
Alg-1 2012	LMU	44	64%	6%	10.5
Alg-1 2013	LMU	30	53%	12%	4.6
Alg-1 2014	LMU	33	61%	0%	"∞"
Alg-1 2015	LMU	24	63%	0%	"∞"
Alg-1 2016	LMU	35	41%	0%	"∞"
Alg-1 2018	LMU	47	54%	9%	6.3
Alg-1 2021	LMU	27	44%	0%	"∞"
<b>AVERAGE</b>	<b>(unweighted)</b>	<b>(1620)</b>	<b>49%</b>	<b>9%</b>	<b>5.5</b>



## High Course Grade vs. FCI

Course	Campus	<i>N</i>	Top-quartile FCI: % with top-quartile grades	Bottom-quartile FCI: % with top-quartile grades	High-grade odds ratio
Alg-1 2018	ASU-P	48	40%	8%	4.8
Alg-1 2019	ASU-P	63	38%	13%	3.0
Alg-1 2021a	ASU-P	35	57%	0%	"∞"
Alg-1 2021b	ASU-P	37	32%	17%	1.9
Alg-1 2022a	ASU-P	41	21%	15%	1.4
Alg-1 2022b	ASU-P	52	26%	7%	3.9
Alg-1 2023a	ASU-P	40	30%	20%	1.3
Alg-1 2023b	ASU-P	47	55%	18%	3.1
Alg-1	CU	470	41%	12%	3.5
Alg-1 2007	LMU	23	87%	0%	"∞"
Alg-1 2009	LMU	51	63%	0%	"∞"
Alg-1 2012	LMU	44	50%	0%	"∞"
Alg-1 2013	LMU	30	51%	0%	"∞"
Alg-1 2014	LMU	33	43%	12%	3.6
Alg-1 2015	LMU	24	67%	0%	"∞"
Alg-1 2016	LMU	34	71%	0%	"∞"
Alg-1 2018	LMU	47	34%	14%	2.4
Alg-1 2021	LMU	27	44%	0%	"∞"
Calc-1 2012	ASU-P	40	43%	0%	"∞"
Calc-1 2013a	ASU-P	18	44%	0%	"∞"
Calc-1 2013b	ASU-P	48	54%	17%	3.3
Calc-1 2021a	UWF	62	29%	26%	1.1
Calc-1 2021b	UWF	53	40%	15%	2.6
<b>AVERAGE</b>	<b>(unweighted)</b>	<b>(1367)</b>	<b>46%</b>	<b>8%</b>	<b>5.4</b>



- High scorers on the pretests were about *5 times* more likely to get a high grade than low scorers.
- Low scorers on the pretests were about *4 times* more likely to get a low grade than high scorers.

High and low grades for high and low scorers were compared for more than 2000 students in 30 distinct classes at 4 universities, yielding a total of 114 high/low comparisons.

➤ ***The quartile ratios were greater than 1.0 in 111 of the 114 cases (97%).***



# Relevant Questions

- Which, if any, of the diagnostic pretests is most predictive of students' performance?
- Does using multiple predictor variables offer greater predictive power than using just one of them?
- Can an “accurate” predictive model be created that incorporates multiple predictor variables?
- Does better performance on one pretest indicate that another pretest is more (or less) predictive? (This would be an “interaction” effect.)

# Relevant Questions

- Which, if any, of the diagnostic pretests is most predictive of students' performance? *Varies with the course*
- Courses and instructors differ on the relative emphasis placed on conceptual problems, mathematical problem solving, and problems requiring significant reading and reasoning skills.
- There are also many possible ways to compare relative “predictability”
  - ...for example, compare high/low grade ratios for different pretests

# High Grade Odds Ratios

Course	Campus	<i>N</i> (# classes)	High-grade odds ratio, Math (average)	High-grade odds ratio, Lawson (average)	High-grade odds ratio, FCI (average)
Alg-1	ASU-P	6	5.0	3.0	3.6
Alg-1	CU	1	--	5.5	3.5
Alg-1	LMU	9	--	12.2	19.6
Alg-2	ASU-P	3	3.3	6.4	--
Calc-1	UWF	2	"∞"	--	1.7

## High Grade Odds Ratios

Course	Campus	<i>N</i> (# classes)	High-grade odds ratio, Math (average)	High-grade odds ratio, Lawson (average)	High-grade odds ratio, FCI (average)
Alg-1	ASU-P	6	5.0	3.0	3.6
Alg-1	CU	1	--	5.5	3.5
Alg-1	LMU	9	--	12.2	19.6
Alg-2	ASU-P	3	3.3	6.4	--
Calc-1	UWF	2	"∞"	--	1.7

## Low Grade Odds Ratios

Course	Campus	<i>N</i> (# classes)	Low-grade odds ratio, Math (average)	Low-grade odds ratio, Lawson (average)	Low-grade odds ratio, FCI (average)
Alg-1	ASU-P	6	2.8	3.9	3.2
Alg-1	CU	1	--	4.4	1.1
Alg-1	LMU	9	--	6.8	5.4
Alg-2	ASU-P	3	3.7	2.6	--
Calc-1	UWF	2	4.2	--	3.1

# Another Approach: Multiple Regression

- Fit an equation including all predictor variables to the data using ordinary least squares, e.g.:

$$\text{Grades} = \beta_0 + \beta_1 * \text{Lawson Pretest} + \beta_2 * \text{Math Pretest} + \beta_3 * \text{FCI Pretest}$$

- Problem: most sample sizes too small to yield significant results, and too much between-class variation to combine samples
- Partial solution:
  - choose one very large sample (Alg-1 CU;  $N = 466$ ) to compare Lawson and FCI
  - combine three very similar classes taught by same instructor (Alg-2 ASU-P, 2022-23-24;  $N = 216$ ) to compare Lawson and Math

## ***Reminder: Results for High Grade Odds Ratios for CU sample***

<b>Course</b>	<b>Campus</b>	<b><i>N</i></b>	<b>High-grade odds ratio, Math</b>	<b>High-grade odds ratio, Lawson</b>	<b>High-grade odds ratio, FCI</b>
Alg-1	CU	466	--	5.5	3.5

## **Low Grade Odds Ratios**

<b>Course</b>	<b>Campus</b>	<b><i>N</i></b>	<b>Low-grade odds ratio, Math</b>	<b>Low-grade odds ratio, Lawson</b>	<b>Low-grade odds ratio, FCI</b>
Alg-1	CU	466	--	4.4	1.1



Lawson pretest score seems to be more predictive than FCI pretest score

Alg-1 CU,  $N = 466$

*Results of Multiple Regression*

## Alg-1 CU, $N = 466$

**Model:**  $\text{Grades} = 54.6109 + 0.2635 \cdot \text{Lawson pre} + 0.0814 \cdot \text{FCI pre}$



# Alg-1 CU, N = 466

Lawson more "influential"\* than FCI

Model: Grades = 54.6109 + 0.2635 · Lawson pre + 0.0814 · FCI pre

Predictor	Coefficient	Estimate	Standard Error	t-statistic	p-value
Constant	$\beta_0$	54.6109	2.3996	22.7587	0
Lawson pre	$\beta_1$	0.2635	0.0344	7.6579	0
FCI pre	$\beta_2$	0.0814	0.0353	2.3046	0.0216

Both variables statistically significant

## Summary of Overall Fit

R-Squared:	$r^2 = 0.1632$
Adjusted R-Squared:	$r^2_{adj} = 0.1596$
Residual Standard Error:	10.7314 on 463 degrees of freedom.
Overall F-statistic:	45.1624 on 2 and 463 degrees of freedom.
Overall p-value:	0

But correlation is quite low

\*weighted more heavily when predicting grades

(Interaction effect not significant)

## High Grade Odds Ratios for Alg-2 ASU-P

Course	Campus	<i>N</i>	High-grade odds ratio, Math	High-grade odds ratio, Lawson	High-grade odds ratio, FCI
Alg-2	ASU-P, 2022-23-24	216	2.8	4.1	--

## High Grade Odds Ratios for Alg-2 ASU-P

Course	Campus	<i>N</i>	High-grade odds ratio, Math	High-grade odds ratio, Lawson	High-grade odds ratio, FCI
<b>Alg-2</b>	<b>ASU-P, 2022-23-24</b>	216	2.8	4.1	--

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Alg-2	ASU-P, 2022-23-24	216	2.8	4.1	--

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Alg-2	ASU-P, 2022-23-24	216	4.8	2.4	--



No consistent “best” predictor

# Alg-2 ASU-P, 2022-23-24, $N = 216$

Math more “influential” than Lawson?

Model:  $\text{Grades} = 26.363 + 0.185 \cdot \text{Lawson Pretest} + 0.295 \cdot \text{Math Pretest}$

Predictor	Coefficient	Estimate	Standard Error	$t$ -statistic	$p$ -value
Constant	$\beta_0$	26.363	4.35	6.061	0
Lawson Pretest	$\beta_1$	0.185	0.067	2.775	0.006
Math Pretest	$\beta_2$	0.295	0.067	4.391	0

Both variables statistically significant

## Summary of Overall Fit

R-Squared:  $r^2 = 0.154$   
Adjusted R-Squared:  $r^2_{\text{adj}} = 0.146$   
Residual Standard Error: 26.73 on 213 degrees of freedom.  
Overall  $F$ -statistic: 19.427 on 2 and 213 degrees of freedom.  
Overall  $p$ -value: 0

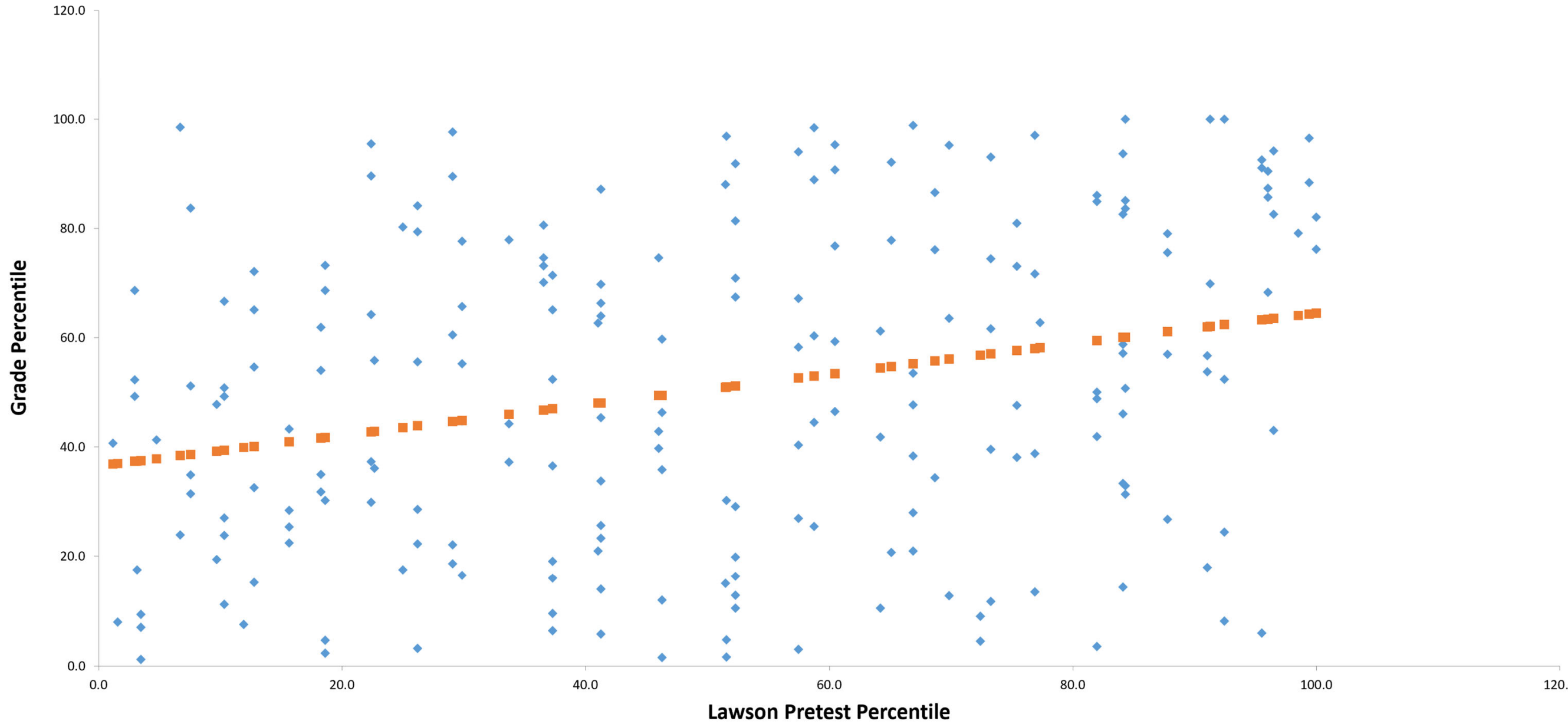
But correlation is still quite low

(Interaction effect not significant)

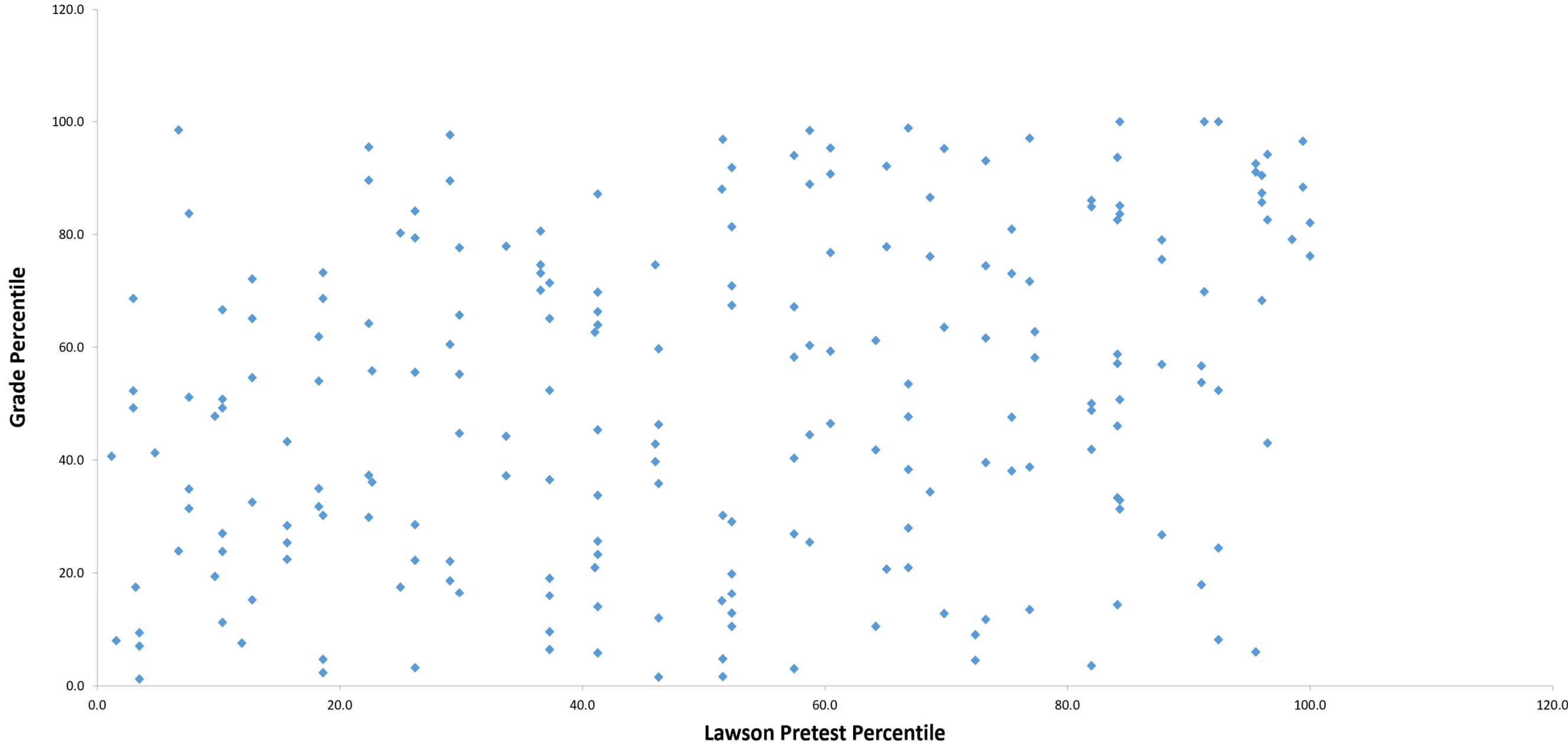
# Regression analysis can be misleading

- High scatter in the data leads to relatively low correlation
- However, quartile comparison can reveal highly significant differences between low and high scorers

# ASU-P Alg-2 (2022, 2023, 2024); Grade Points vs. Lawson Pretest (N = 216, r = 0.28)

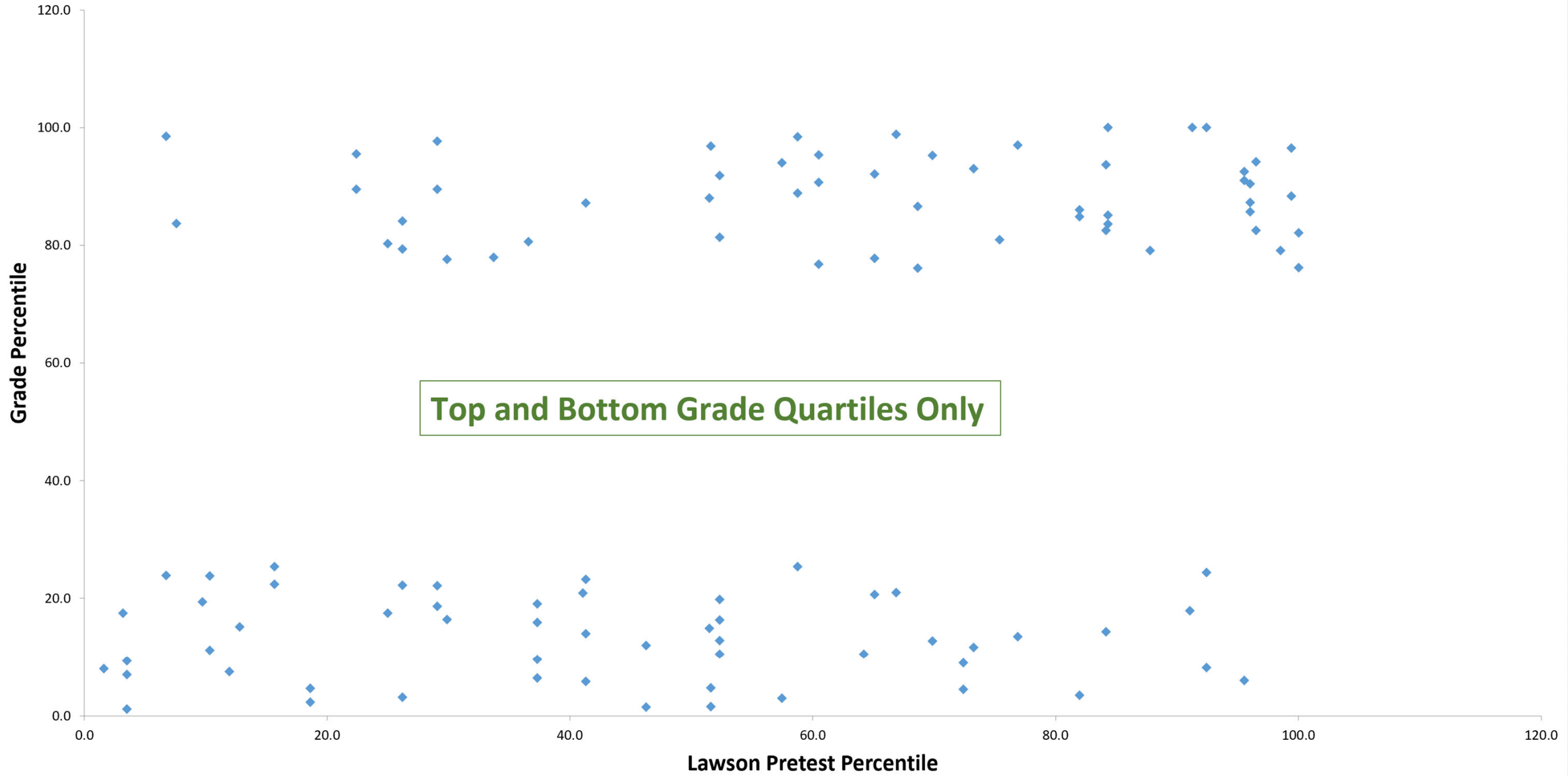


# ASU-P Alg-2 (2022, 2023, 2024); Grade Points vs. Lawson Pretest (N = 216, r = 0.28)

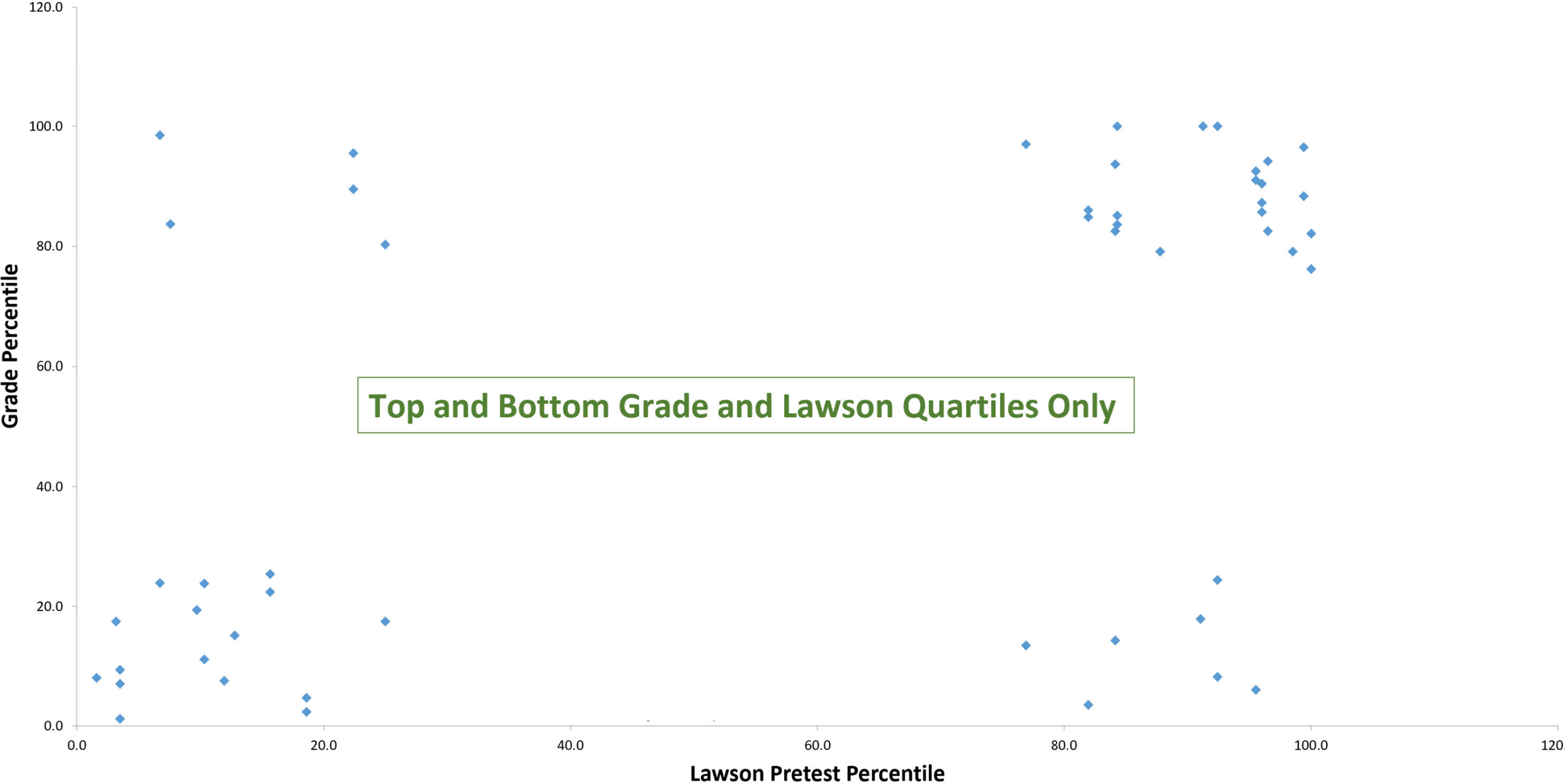




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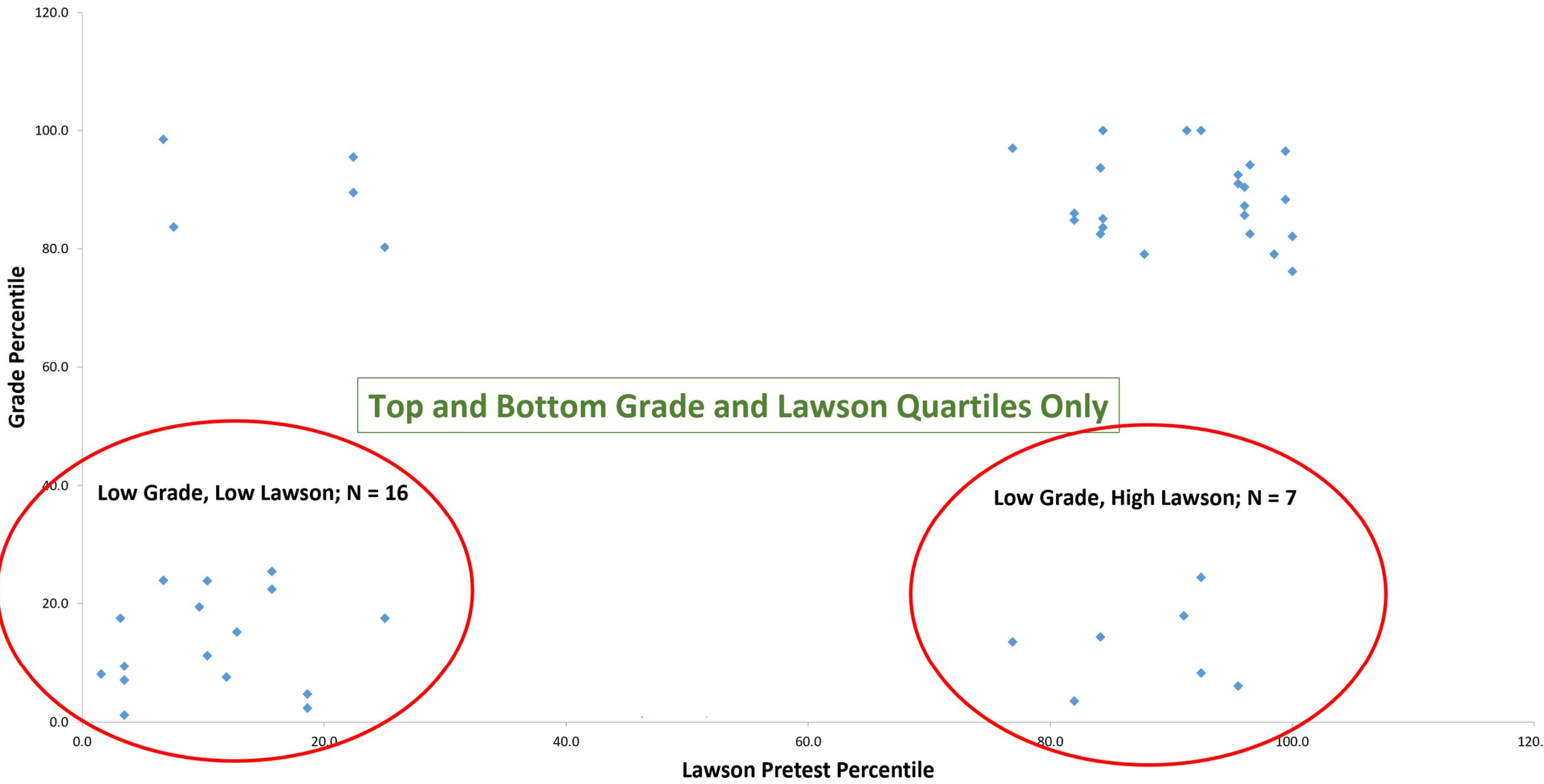
High Grade, Low Lawson; N = 5

High Grade, High Lawson; N = 23

Top and Bottom Grade and Lawson Quartiles Only



# ASU-P Alg-2 (2022, 2023, 2024); Grade Points vs. Lawson Pretest (N = 216, r = 0.28)



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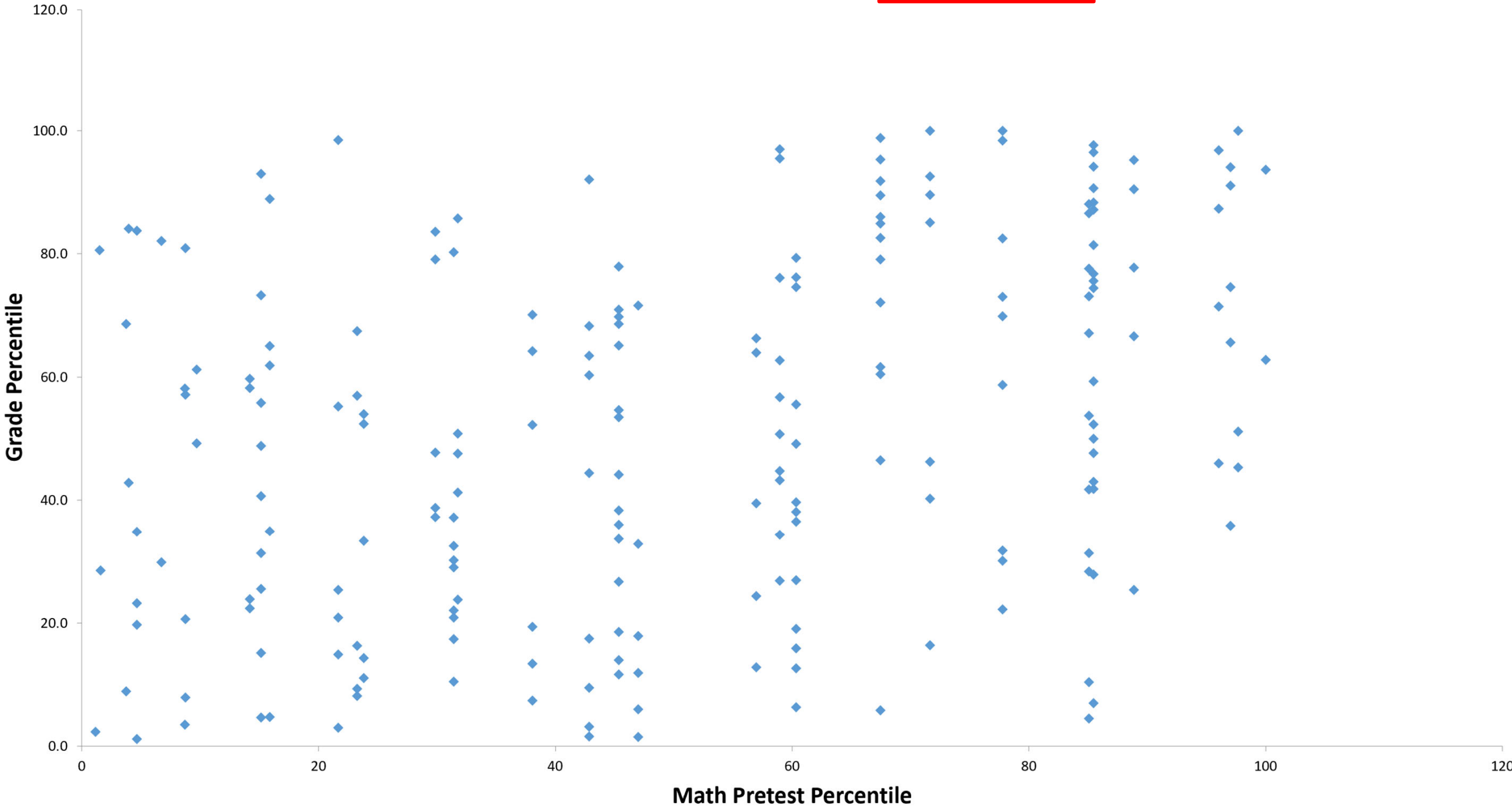


Top and Bottom Grade and Lawson Quartiles Only

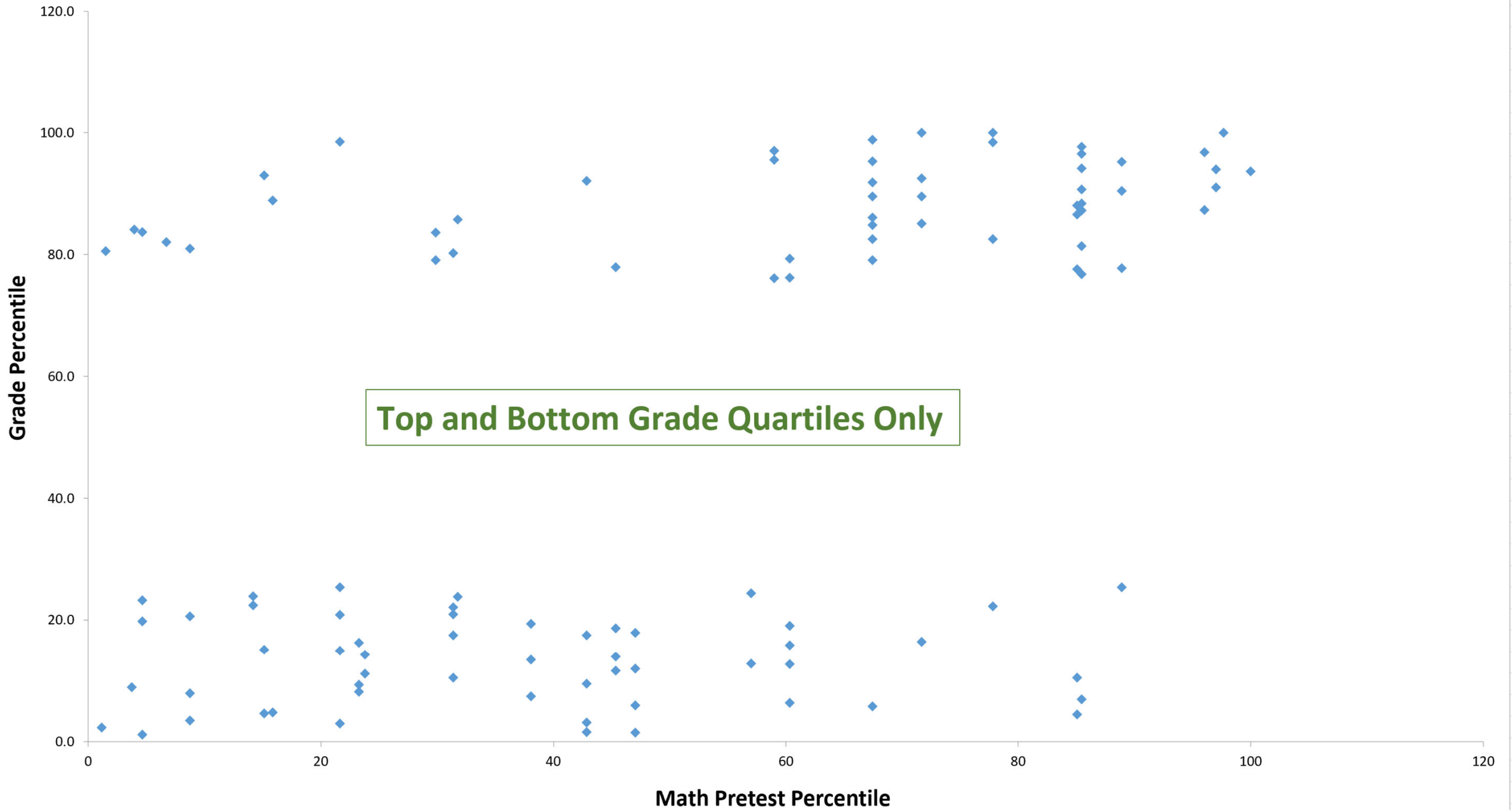
Low Grade, Low Lawson; N = 16

Low Grade, High Lawson; N = 7

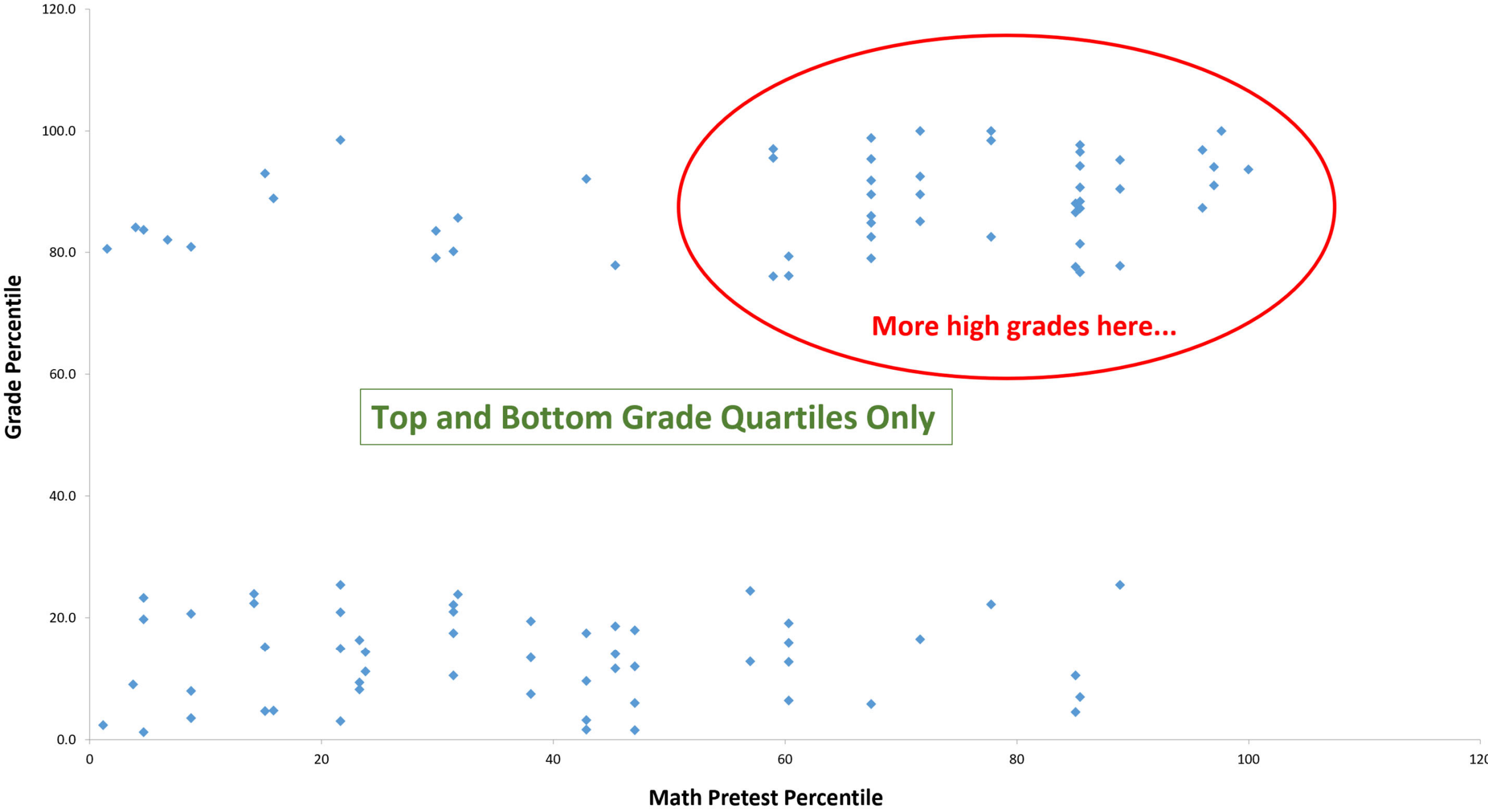
**ASU-P Alg-2 (2022, 2023, 2024); Grade Points vs. Math Pretest (N = 216, r = 0.35)**



# ASU-P Alg-2 (2022, 2023, 2024); Grade Points vs. Math Pretest (N = 216, r = 0.35)



# ASU-P Alg-2 (2022, 2023, 2024); Grade Points vs. Math Pretest (N = 216, r = 0.35)

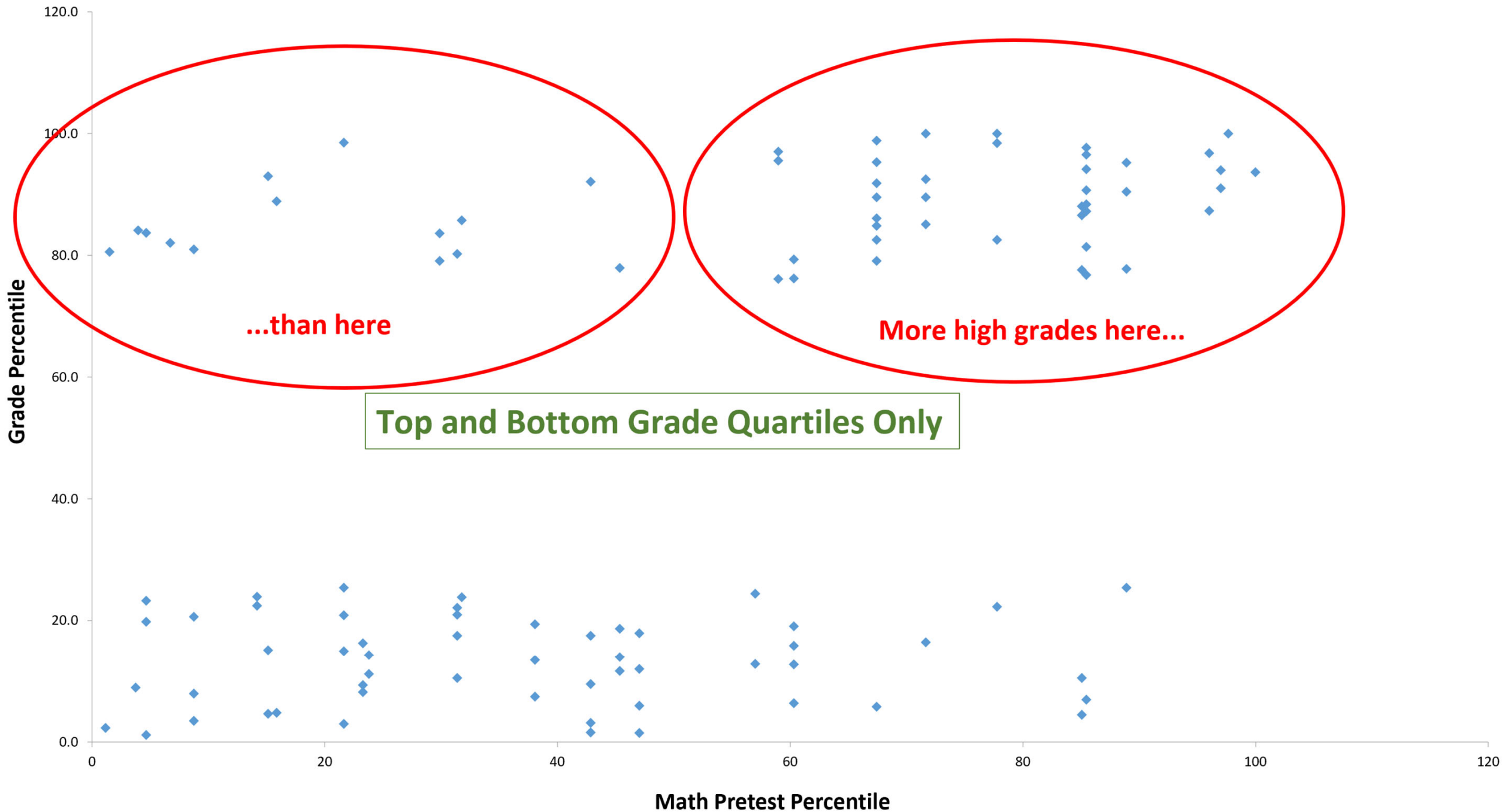


More high grades here...

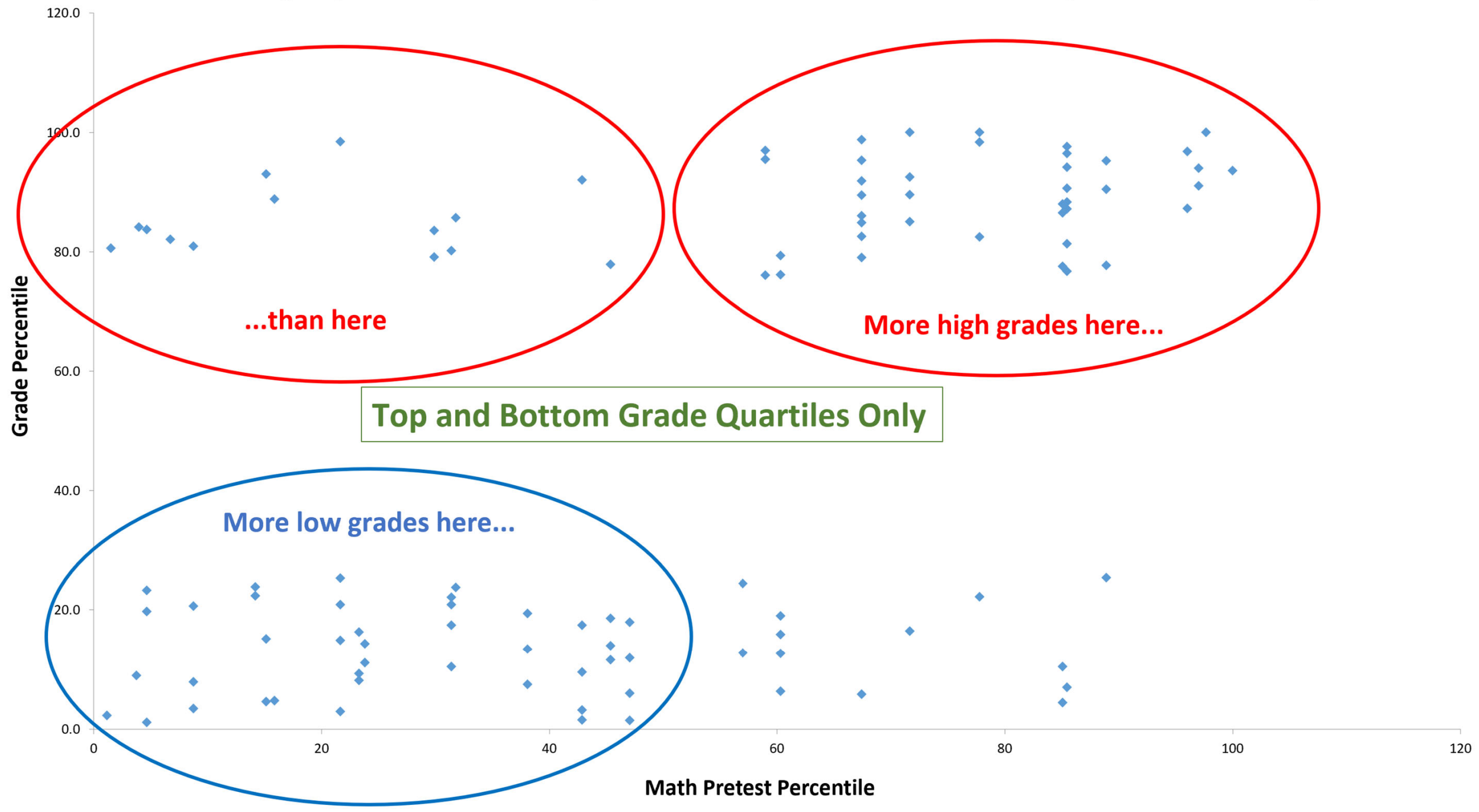
Top and Bottom Grade Quartiles Only



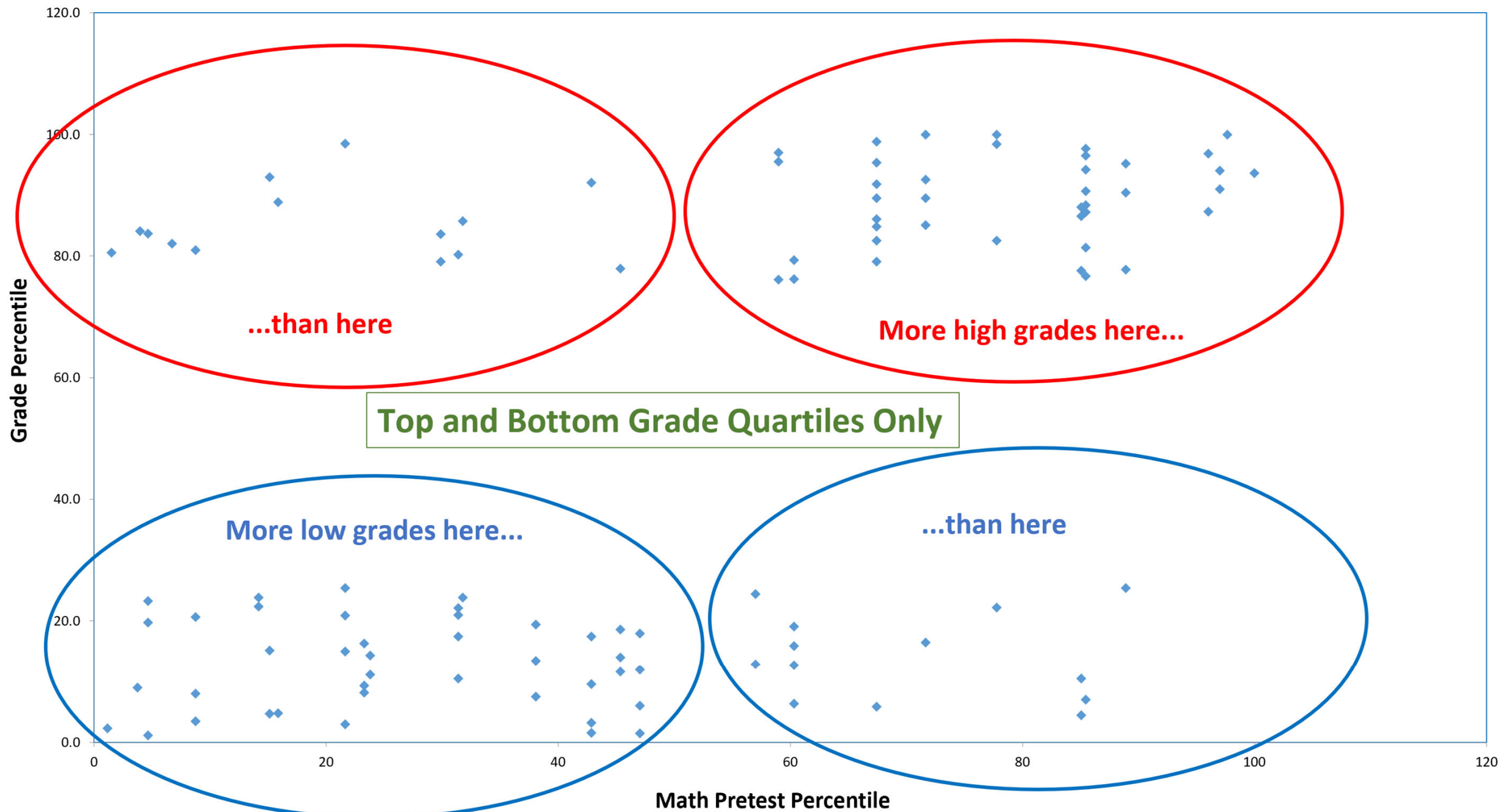
# ASU-P Alg-2 (2022, 2023, 2024); Grade Points vs. Math Pretest (N = 216, r = 0.35)



# ASU-P Alg-2 (2022, 2023, 2024); Grade Points vs. Math Pretest (N = 216, r = 0.35)



# ASU-P Alg-2 (2022, 2023, 2024); Grade Points vs. Math Pretest (N = 216, r = 0.35)



# Relevant Questions

- Can an “accurate” predictive model be created that incorporates multiple predictor variables? *No, but better than random*

## Alg-2 ASU-P, 2022-23-24, $N = 216$

Model:  $\text{Grades} = 26.363 + 0.185 \cdot \text{Lawson Pretest} + 0.295 \cdot \text{Math Pretest}$

**Q:** How accurately can this model predict students' grades?

**A:** If we randomly guess in which grade quartile each student will end up, we'd be right 25% of the time.

If instead we had applied this model to each *actual*\* student's Lawson and Math pretest scores, we would have correctly predicted whether they ended up with top- or bottom-quartile grades 45% of the time.

*\*that is, the students in Alg-2 ASU-P 2022-23-24*

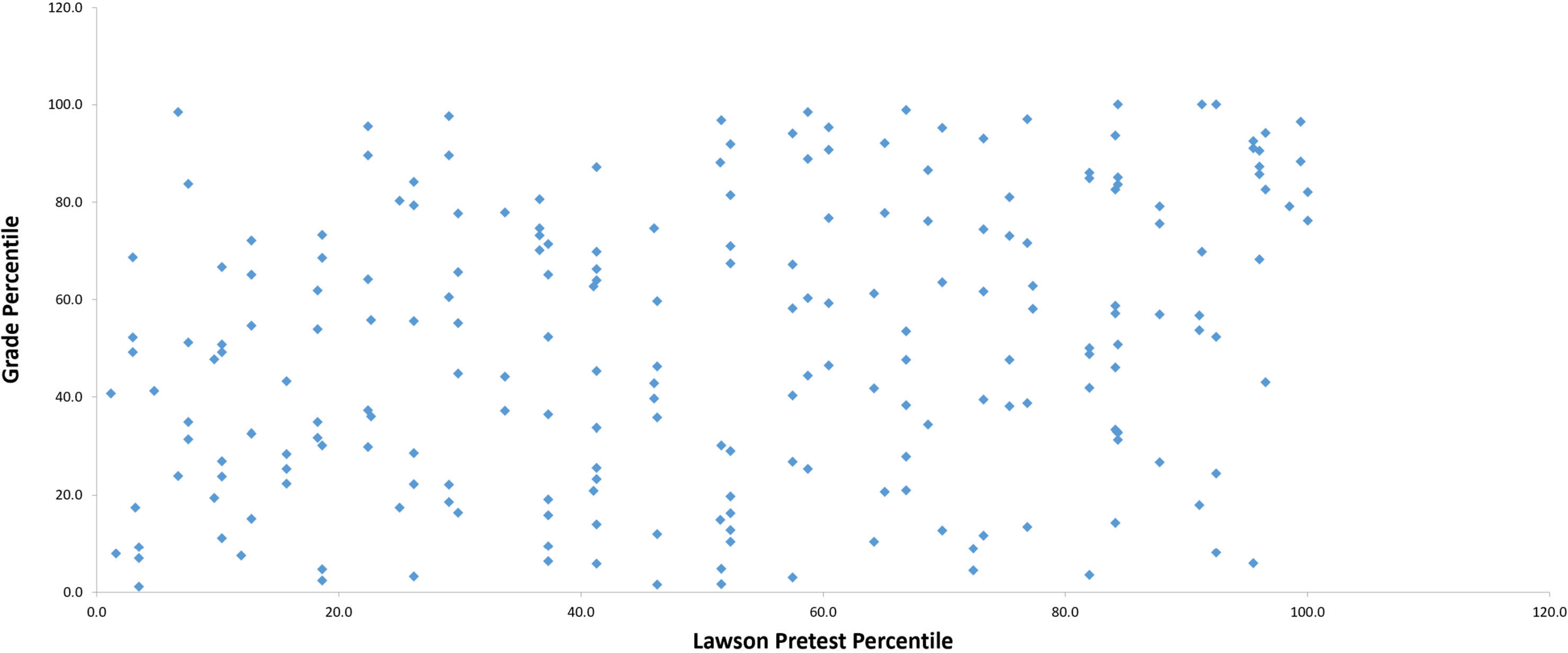
# Relevant Questions

- Does using multiple predictor variables offer greater predictive power than using just one of them? *Yes*

# Further Analysis of Alg-2 ASU-P sample ( $N = 216$ )

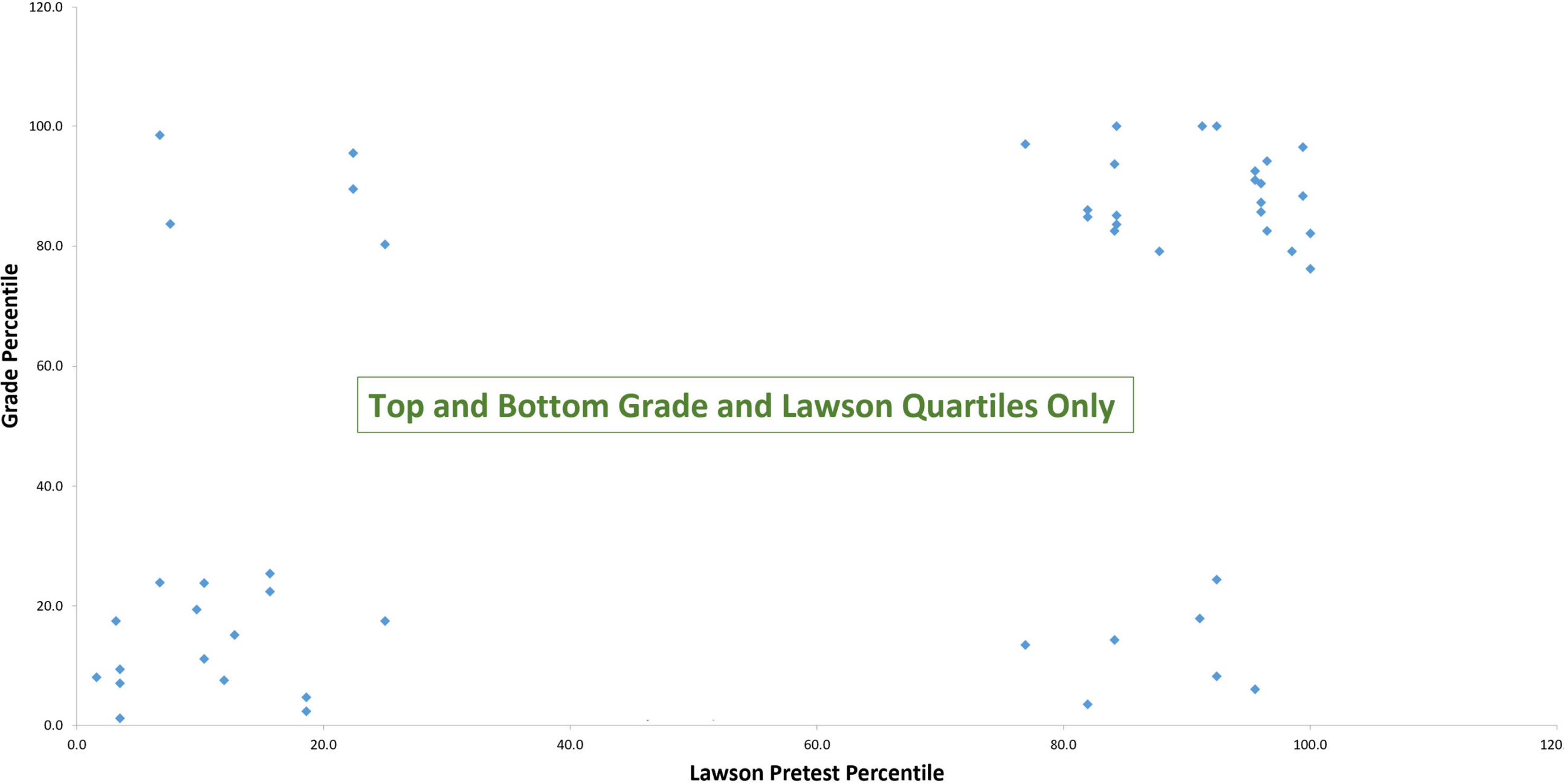
		Probability of top-quartile grade	Probability of bottom-quartile grade
Top-Quartile on Lawson Pretest	Top-half on Math pretest	54%	0%
	Bottom-half on Math pretest	24%	28%
	Ratio	2.3	"∞"

# ASU-P Alg-2 (2022, 2023, 2024); Grade Points vs. Lawson Pretest (N = 216, r = 0.28)

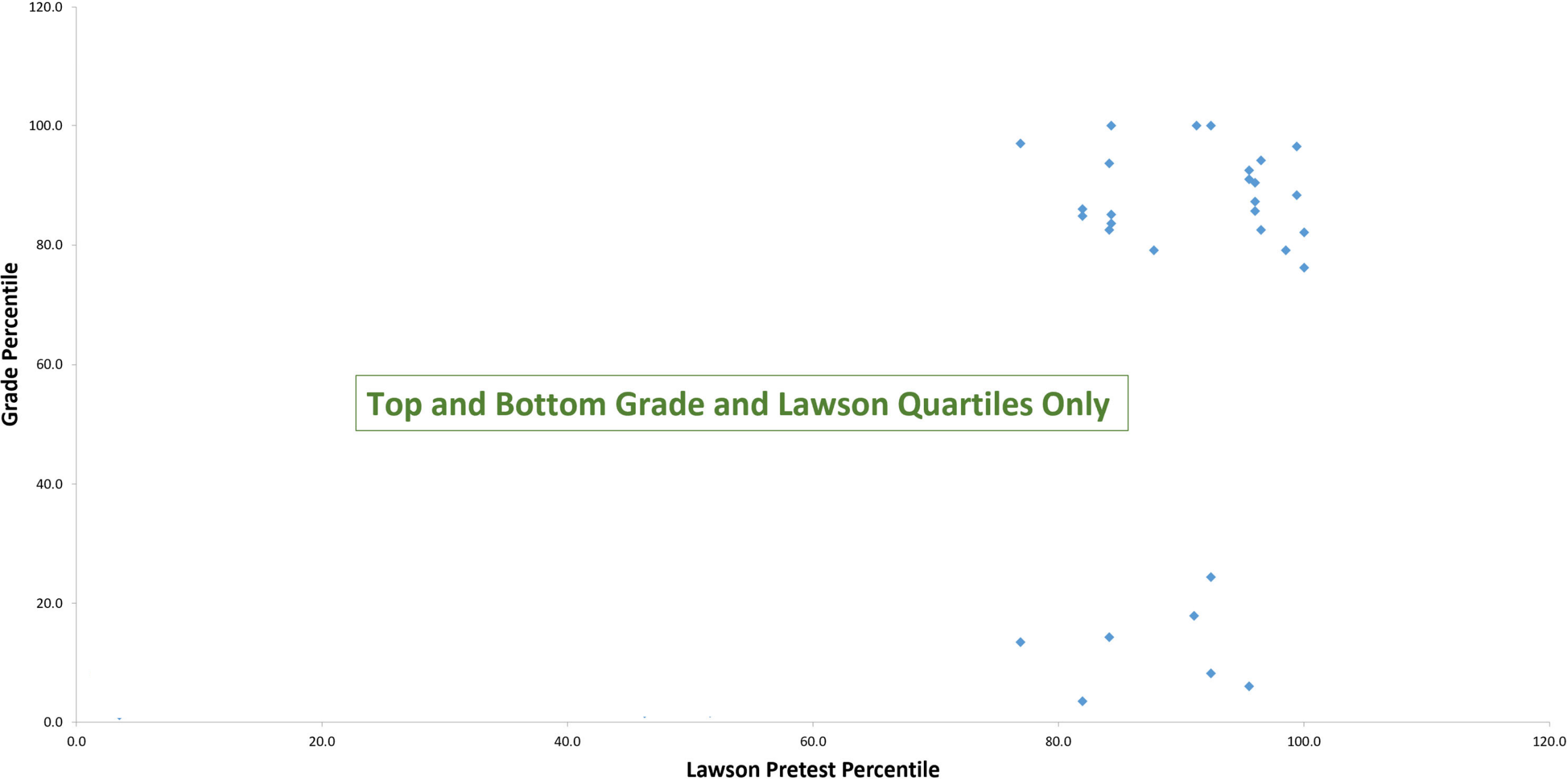




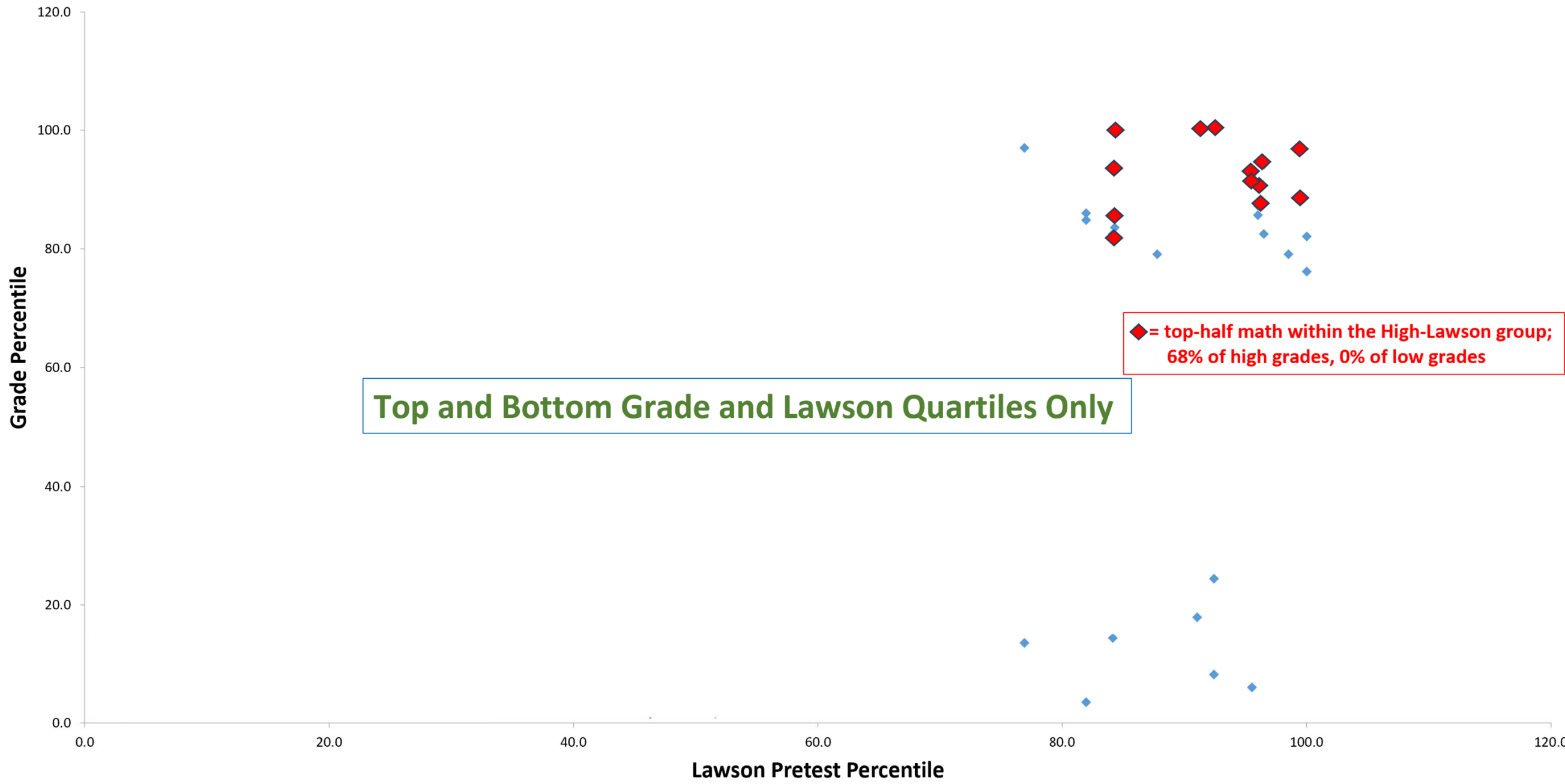
# ASU-P Alg-2 (2022, 2023, 2024); Grade Points vs. Lawson Pretest (N = 216, r = 0.28)



# ASU-P Alg-2 (2022, 2023, 2024); Grade Points vs. Lawson Pretest (N = 216, r = 0.28)



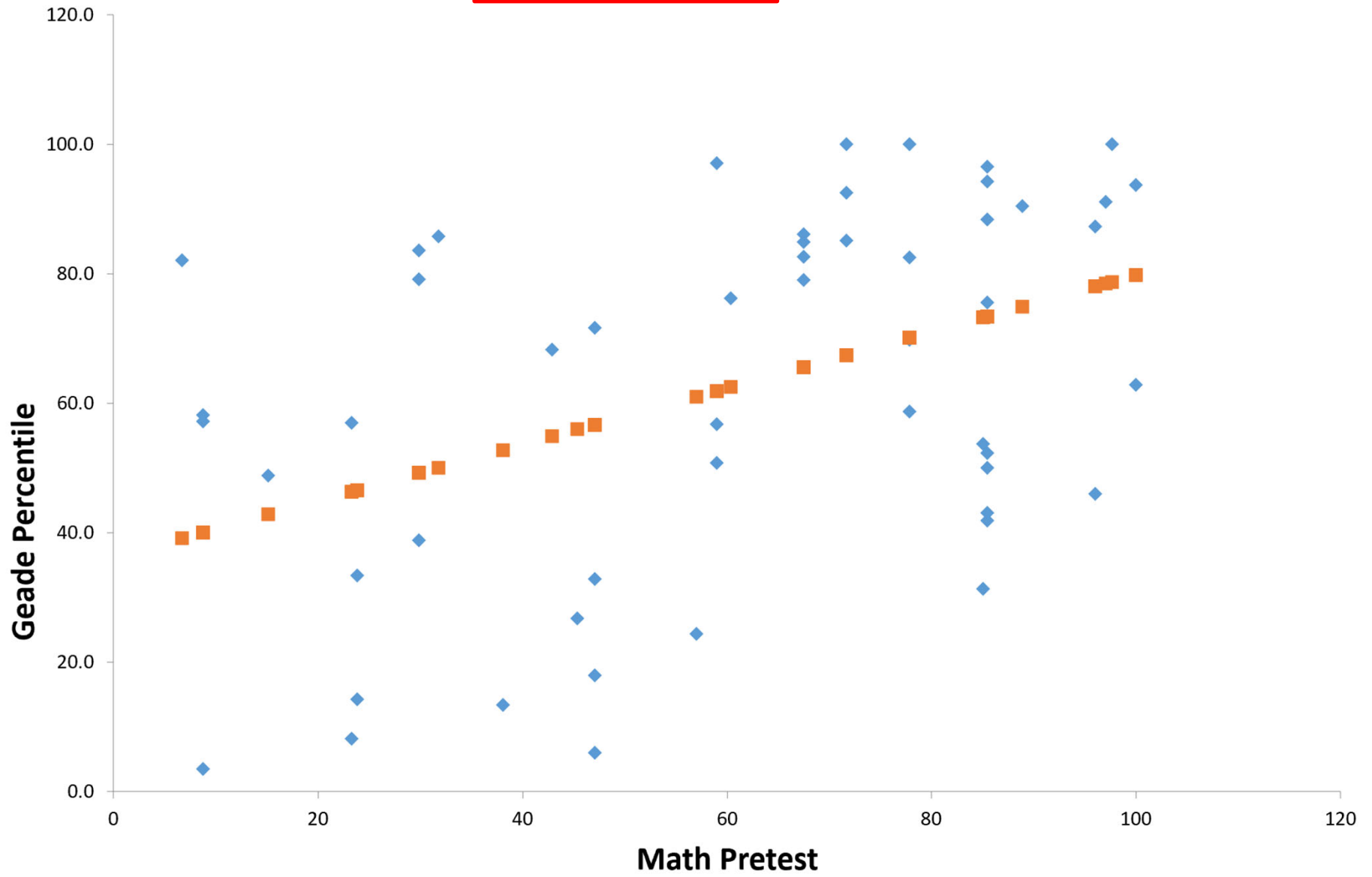
# ASU-P Alg-2 (2022, 2023, 2024); Grade Points vs. Lawson Pretest (N = 216, r = 0.28)

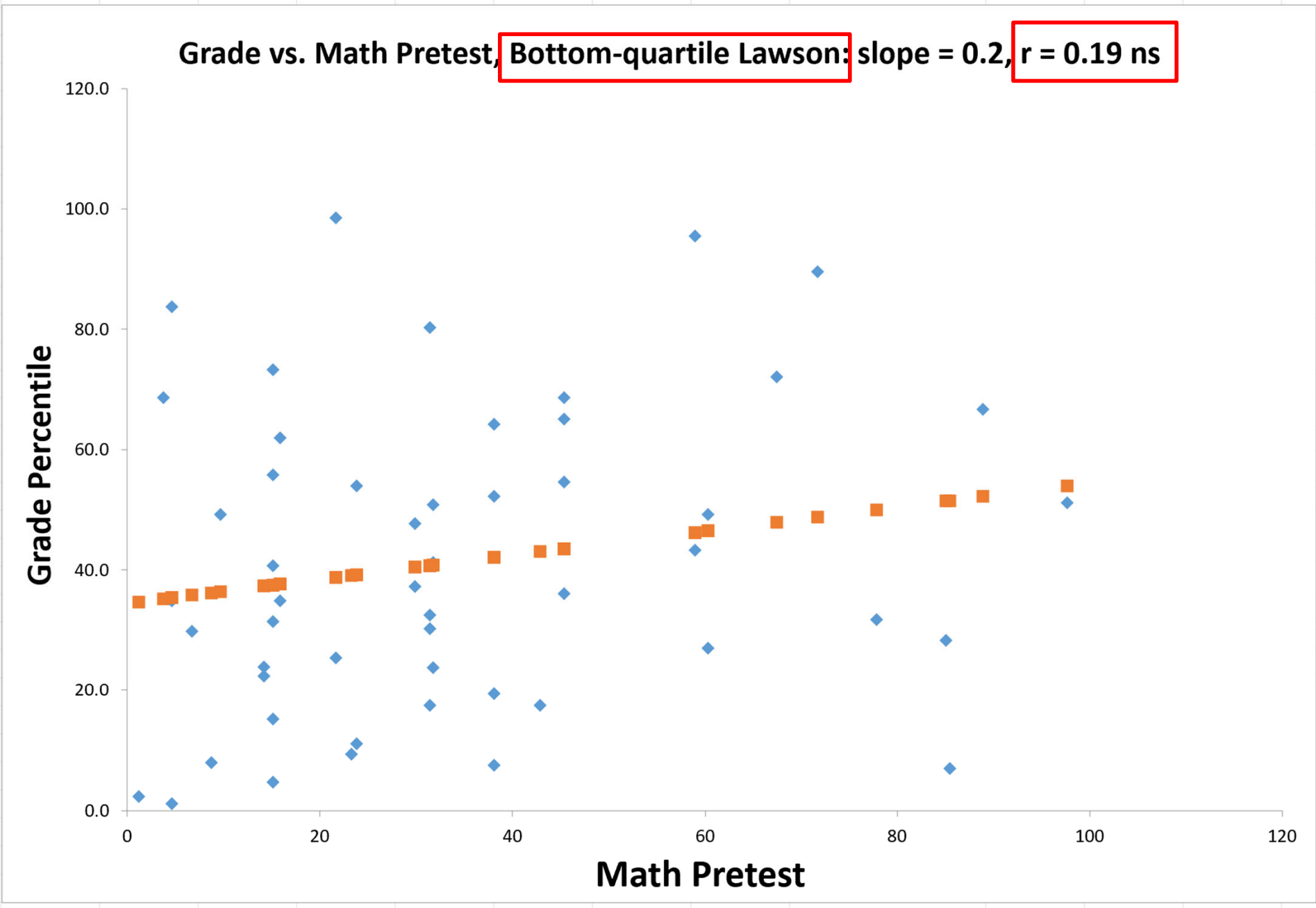


# Relevant Questions

- Does better performance on one pretest indicate that another pretest is more (or less) predictive? (This would be an “interaction” effect.) *Maybe*

Grade vs. Math Pretest, **Top-quartile Lawson:** slope = 0.44, r = 0.43, p = 0.001





# Important Note

- Anecdotal evidence shows:
  - Students with low pretest scores but high grades are often highly engaged and regular participants in class activity
  - High scores but low grades are often associated with missing many classes and assignments

# Summary

- High and low pretest scores on diagnostic tests are consistently predictive of students' probability of attaining high or low grades
- High pre-instruction scores on a math diagnostic, a test of scientific reasoning, and the FCI are all *independently* associated with higher probability of getting high grades (and avoiding low grades)

Our results are consistent with findings reported by:

- L. Ding, PRPER **10**, 023101 (2014)]
- Salehi et al., PRPER **15**, 020114 (2019)
- Stewart et al., PRPER **17**, 010107 (2021)