# Nature of Students' Mathematical Difficulties and of Potentially Productive Remedies

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Supported in part by NSF DUE #1504986

# The Problem

- Difficulties with very basic math skills impact performance of introductory physics students.
- The difficulties are often not resolved by students' previous mathematical training.
- Students can't effectively grapple with physics ideas when they feel overburdened in dealing with calculational issues.

# Work to Date

- Administer written diagnostic to 1300 students in 14 algebra- and calculus-based physics classes over three semesters at Arizona State University during 2016-2017
- Carry out individual interviews with 65 students enrolled in those or similar courses during same period

# Difficulty #1: Trigonometry

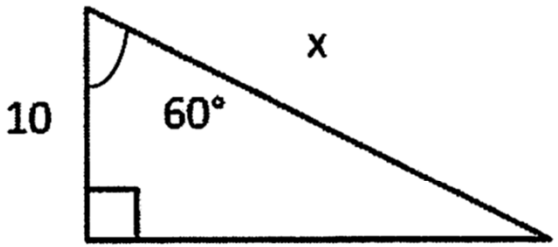
- Many students are confused or unaware (or have forgotten) about the relationships between sides and angles in a right triangle.

- *Examples:* Questions from a diagnostic math test administered at Arizona State University, 2016-2017 (Administered as no-credit quiz during first week labs and/or recitation sections; **calculators allowed**)

# Trigonometry Questions

## with samples of correct student responses

**1.**




What is the value of x?

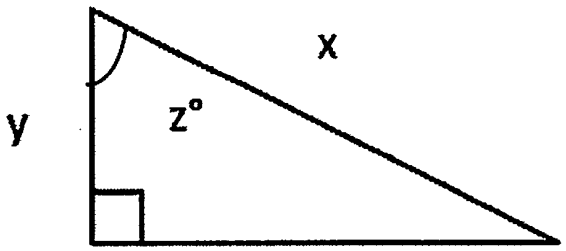
$\cos 60 = \frac{10}{x}$

$x \cos 60 = 10$

$x = \frac{10}{\cos 60}$

$= 20$

**2.**




$\cos z = \frac{y}{x}$

What is the value of x?

A. ycos(z)

B. ycos(z)sin(z)

C. y/sin(z)

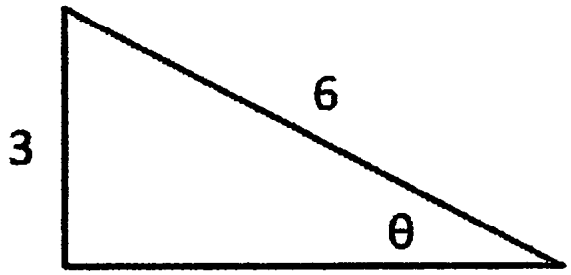
D. ysin(z)

E. ycos(z)/sin(z)

F. y/cos(z)

G. None of the above_____

3.




What is the value of θ?

$\sin^{-1}(\theta) = \sin^{-1}\left(\frac{3}{6}\right)$

$\theta = 30^{\circ}$

# Trigonometry Questions:

**Correct Response Rate, #1-3 combined**

*ASU Polytechnic campus, Spring + Fall average:*

Algebra-based course, 1st semester, (*N* = 116): 37%

Algebra-based course, 2nd semester, (*N* =79): 48%

*ASU Polytechnic campus, Spring (2-year average):*

Calculus-based course, 1st semester, (*N* = 146): 66%

# Results on Trigonometry Questions

**Errors observed:**

(i) use of incorrect trigonometric function (e.g., cosine instead of sine), or misunderstanding of definition;

(ii) unaware (or forgot) about inverse trigonometric functions, e.g., arctan, arcsin, arccos [$\tan^{-1}$, $\sin^{-1}$, $\cos^{-1}$]

- **How to address these problems:** It seems that many students require substantial additional *practice and repetition* with basic trigonometric procedures.

# Difficulty #2: Algebra

- Students have difficulties in solving two simultaneous equations, and those difficulties are much greater when the equations are in symbolic form.

# Algebra: Simultaneous Equations

$3x = 2y$
$5x + y = 26$

What are the values of $x$ and $y$? Show all your steps. For example, $x = 2, y = 5$ (These are NOT the correct answers).

**Correct Response Rate, ASU (% correct responses)**

Algebra-based course, second semester (*N* = 123): 70%

*[This is the "numerical" problem]*

# Algebra: Simultaneous Equations

$x \cos(20°) = y \cos(70°)$
$x \cos(70°) + y \cos(20°) = 10$

What are the values of *x* and *y*? Show all your steps. Note: The value for *x* should NOT include *y*, and the value for *y* should NOT include *x*.

**Correct Response Rate, ASU (% correct responses)**

Algebra-based course, second semester (*N* =150): 20-30%
(different campuses, slightly different versions)

# Algebra: Simultaneous Equations

$ax = by$
$bx + ay = c$

a, b, and c are constants.
What are the values of *x* and *y* in terms of a, b, and c?
Show all your steps. Note: The value for *x* should NOT include *y*, and the value for *y* should NOT include *x*.

**Correct Response Rate, ASU (% correct responses)**

Algebra-based course, second semester (*N* =150): 10-20%
(different campuses, slightly different versions)

Only 10-20% correct responses!

# Algebra: Simultaneous Equations

$a \cdot x = b \cdot y$
$b \cdot x + a \cdot y = c$

a, b, and c are constants.
What are the values of x and y in terms of a, b, and c? Show all your steps. Note: The value for x should NOT include y, and the value for y should NOT include x.

$x = \frac{by}{a}$

$b\left(\frac{by}{a}\right) + ay = c$

$\frac{b^2y}{a} + ay = c$

$y\left(\frac{b^2}{a} + a\right) = c$

$y = \frac{c}{\left(\frac{b^2}{a} + a\right)}$

$x = \frac{b\left(\frac{c}{(b^2/a + a)}\right)}{a}$

Sample of Correct Student Response

# Difficulties with Equations

- Interviews indicate that students who missed the first (numerical) problem had fundamental difficulties with arithmetic and/or algebra (e.g., failing to isolate variables, failing to substitute expression from first equation into the second equation).
- Many students who could solve the first (numerical) problem failed on one or both of the other two.

# Why the Difficulties with Symbols?

## Some Suggestions Arising from the Interviews

- In elementary math courses, “simplified forms” of equations are emphasized (i.e., few messy symbols and functions)
- Students get “overloaded” by seeing all the variables, and are unable to carry out procedures (e.g., multiplying each term in an expression by a constant [symbol]) that they do successfully with numbers (e.g., multiply through by a number)
- Other procedural failures that occur more often with symbols: cancellation, factoring out a constant, retaining coefficients from one line to the next

# Students' Difficulties with Symbols

**Confusion of symbolic meaning:** Students perform worse on solving problems when symbols are used to represent common physical quantities in equations, e.g., "*m*" instead of "1.5 kg" [Torigoe and Gladding, 2007; 2011)

***Example [University of Illinois]:***

*Version #1*: A car can go from 0 to 60 m/s in 8 s. At what distance *d* from the start at rest is the car traveling 30 m/s? *[93% correct]*

*Version #2*: A car can go from 0 to $v_1$ in $t_1$ seconds. At what distance *d* from the start at rest is the car traveling ($v_1$/2)? *[57% correct]*

Much worse!

# Other Difficulties: Vectors

- Diagnostic also indicates widespread difficulties with:
  - Graphical addition of vectors (25-80% correct)
  - Meaning of vector “direction” (40-70% correct)

# Sources of Difficulties

- Carelessness
  - Students *very frequently* self-correct errors during interviews
  - Evidence of carelessness on written diagnostic
- Skill practice deficit: Insufficient repetitive practice with learned skills
  - E.g., applying definitions of sine and cosine; factoring out variables in algebraic expressions
- Inability to efficiently access previous learning

# Summary: What to Do?

- I have to teach the first-semester algebra-based course next fall: What can I do?
  - Administer first-week diagnostic test (anonymous) to inform instructor and students about prevalence of difficulties
  - Develop small sets of practice exercises with mathematical operations similar to those required in the course
  - Have students complete exercise sets periodically, on-line or on paper, for (small amount of) course credit
  - **See also: Mikula and Heckler, PRPER (2017)**