Some Mathematical Aspects of Physics Students' Problem-Solving Difficulties

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The Challenge

- College physics instructors must make certain assumptions regarding their students' calculational skills
- Students' problem-solving difficulties can be hard to disentangle from weak skills with basic pre-college mathematics
- The prevalence and nature of physics students' difficulties with basic skills has not previously been investigated systematically

Our Work

- Examine prevalence and nature of physics students' difficulties with trigonometry, graphing, vectors, and algebra
  - Use "stripped-down" problems with *no* physics context

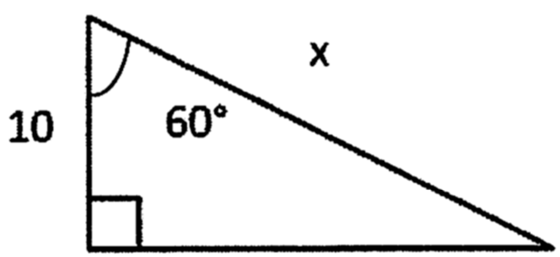
Work to Date

- Administer (and analyze) written diagnostic, given to 2700 students in 21 algebra- and calculus-based physics classes over five semesters at Arizona State University during 2016-2018; calculators *are* allowed
- Carry out individual interviews with 75 students enrolled in those or similar courses during same period

Trigonometry Questions

with samples of correct student responses

1.



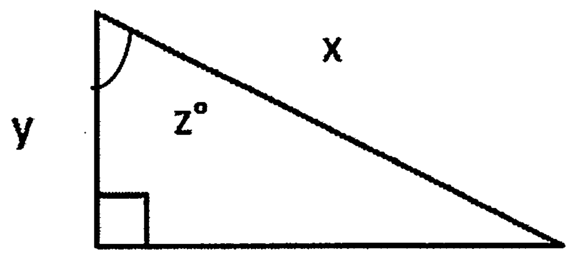
What is the value of x?

$\cos 60 = \frac{10}{x}$

$x \cos 60 = 10$

$x = \frac{10}{\cos 60} = 20$

2.



$\cos z = \frac{y}{x}$

What is the value of x?

A. ycos(z)

B. ycos(z)sin(z)

C. y/sin(z)

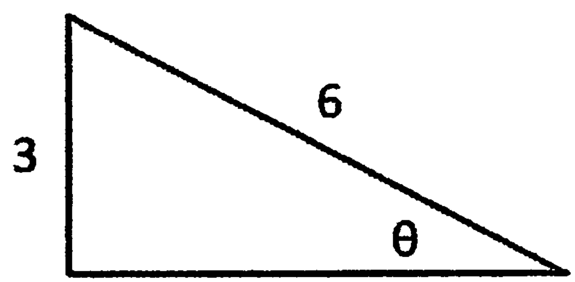
D. ysin(z)

E. ycos(z)/sin(z)

F. y/cos(z)

G. None of the above________________________

3.



What is the value of θ?

$\sin^{-1}(\theta) = \sin^{-1}\left(3/6\right)$

$\theta = 30°$

Trigonometry Questions: Representative Data

Correct Response Rate, #1-3 combined

ASU Polytechnic campus, Spring + Fall average:

Algebra-based course, 1st semester, (N = 116): **37%**

ASU Polytechnic campus, Spring (2-year average):

Calculus-based course, 1st semester, (N = 146): **66%**

➡ $\frac{1}{3}$ *to* $\frac{2}{3}$ ***of students confused on basic trigonometry relations***

Trigonometry Questions: Summary

- Regardless of course, semester, campus, or question type, between 20% and 70% of introductory physics students at ASU have significant difficulties with basic trigonometric relationships.
- Students frequently tended to self-correct errors during interviews, suggesting that many of the errors were “careless” or due to insufficient review or practice.

Physics Students' Difficulties with Algebraic Symbols and Operations

- Extensive investigations by Torigoe and Gladding (2007; 2007; 2011): Probed differences in University of Illinois students' responses to physics problems posed in numerical and symbolic form.
  - In general, students tended to have more difficulties with questions in symbolic form.
- Our investigation at Arizona State probed physics students' responses to mathematical problems stripped of all physics context

Students' Difficulties with Symbols

Confusion of symbolic meaning: Students perform worse on solving problems when symbols are used to represent common physical quantities in equations [Torigoe and Gladding, 2007; 2011)

Example [University of Illinois]:

Version #1: A car can go from 0 to 60 m/s in 8 s. At what distance *d* from the start at rest is the car traveling 30 m/s? *[93% correct]*

Version #2: A car can go from 0 to v_1 in t_1 seconds. At what distance *d* from the start at rest is the car traveling (v_1/2)? *[57% correct]*

Much worse!

- *Our results on "stripped-down" versions are analogous, although differences are smaller*

Algebra: Simultaneous Equations

- Do differences in students' success rate between numerical and symbolic versions of same problem persist when simultaneous equations are involved? (E.g., two equations, two unknowns)

From Torigoe and Gladding (2011):

$$Mg - T = Ma$$
$$TR = I\alpha$$
$$[I = \tfrac{1}{2} MR^2;\ \alpha = a/R]$$

$$\ldots\rightarrow$$

$$Mg - T = Ma$$
$$TR = [\tfrac{1}{2} MR^2][a/R]$$
$$a = ?$$

$$F_{net} = ma$$
$$\tau_{net} = I\,\alpha$$

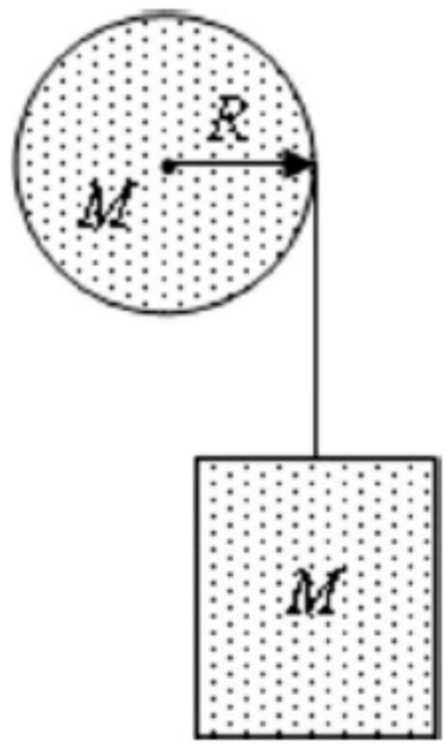


Fig. 7. Diagram for question 10.

Question 10 (numeric). A uniform disk of mass $M = 8$ kg and radius $R = 0.5$ m has a string wound around its rim. The disk is free to spin about a pin through the center of the disk. A mass $M = 8$ kg (same mass as the disk) is connected to the string and is dropped from rest. What is the acceleration a of the block? (See Fig. 7.)

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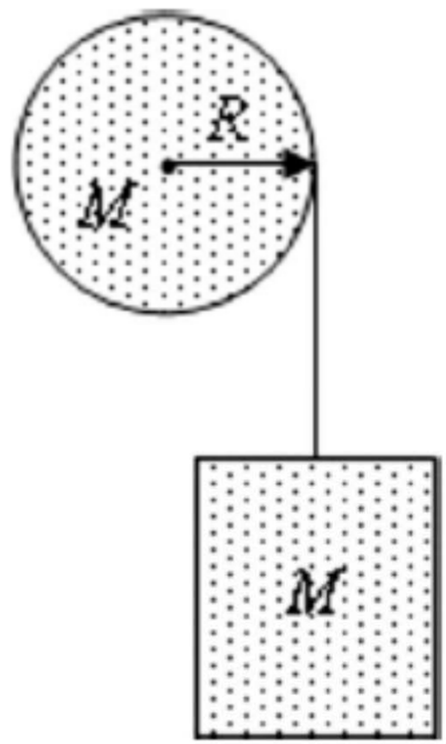

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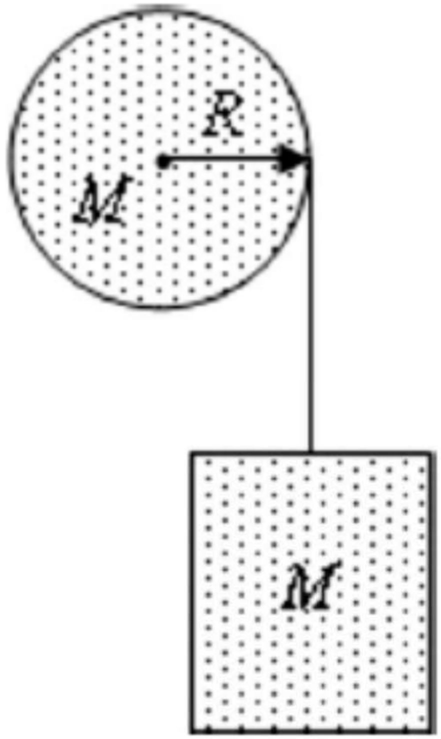

Fig. 7. Diagram for question 10.

$Mg - T = Ma$
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$[I = ½ MR^2; \alpha = a/R]$

$\ldots \rightarrow$

$Mg - T = Ma$
$TR = [½ MR^2][a/R]$
$a = ?$

Symbolic version

Question 10 (numeric). A uniform disk of mass *M* [redacted] and radius *R* [redacted] has a string wound around its rim. The disk is free to spin about a pin through the center of the disk. A mass *M* [redacted] (same mass as the disk) is connected to the string and is dropped from rest. What is the acceleration *a* of the block? (See Fig. 7.)

Results on #10
[Torigoe and Gladding, 2011]

- **Numeric version:** 49% correct ($N \approx 380$)
- **Symbolic version:** 53% correct ($N \approx 380$)

➡ *No significant difference*

("…because students are forced to use the same procedure to solve both the numeric and symbolic versions." Torigoe and Gladding, 2011)

From Torigoe and Gladding (2011):

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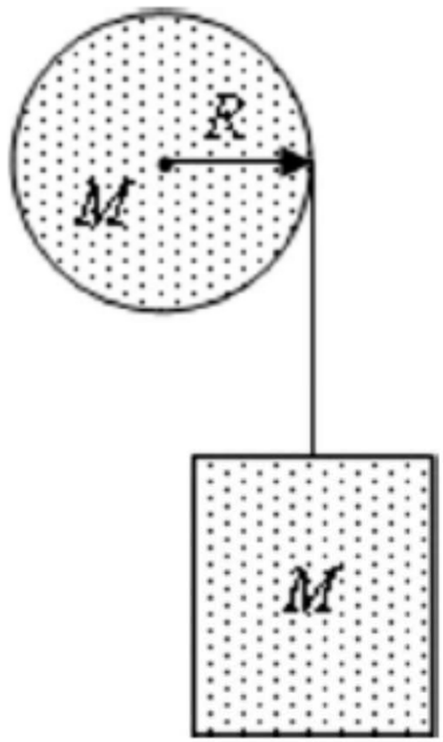

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Symbolic version

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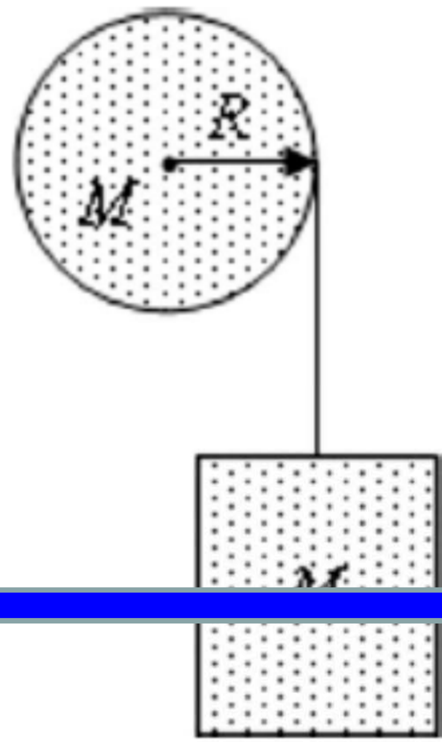
$a = ?$

Rename to simplify:

"Mg" → "a"
"M" → "b"
"R" → "c"
"½MR" → "d"
"T" → "y"
"a" → "x"

$a - y = bx$
$cy = dx$
$x = ?$

Our Symbolic version



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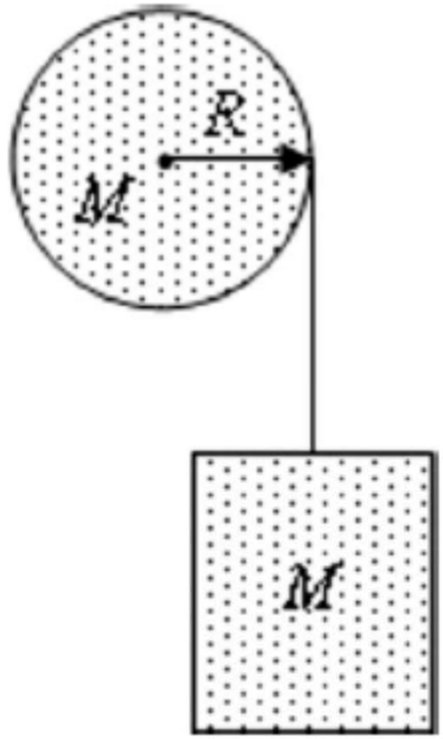


Fig. 7. Diagram for question 10.

Question 10 (numeric). A uniform disk of mass $M=8$ kg and radius $R=0.5$ m has a string wound around its rim. The disk is free to spin about a pin through the center of the disk. A mass $M=8$ kg (same mass as the disk) is connected to the string and is dropped from rest. What is the acceleration a of the block? (See Fig. 7.)

$Mg - T = Ma$

$TR = I\alpha$

$[I = \frac{1}{2} MR^2;\ \alpha = a/R]$

…→

$Mg - T = Ma$

$TR = [\frac{1}{2} MR^2][a/R]$

$a = ?$

$78.4 - y = 8x$

$0.5y = 2x$

$x = ?$

Our Numeric version

Results on Our Versions

Calculus-based course, 1st semester:

- **Numeric version:** 87% correct (*N* = 733)
- **Symbolic version:** 63% correct (*N* = 733)

➡ *Large and highly significant difference*

(Because [?] many of the students who can't do the physics, can do the math—but only when posed in numerical form)

Other Difficulties with Symbols

- Possible confusion due merely to replacing numbers by symbols
- Is this a real difficulty for physics students?

Confusion due to replacing numbers by symbols

$$\cos 60° = \frac{10}{x}$$

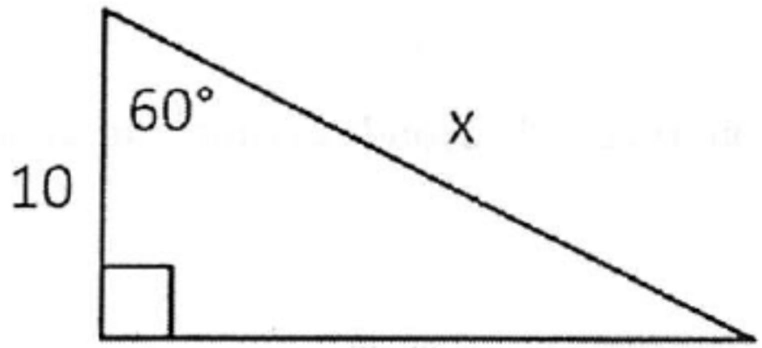
$$x = ?$$

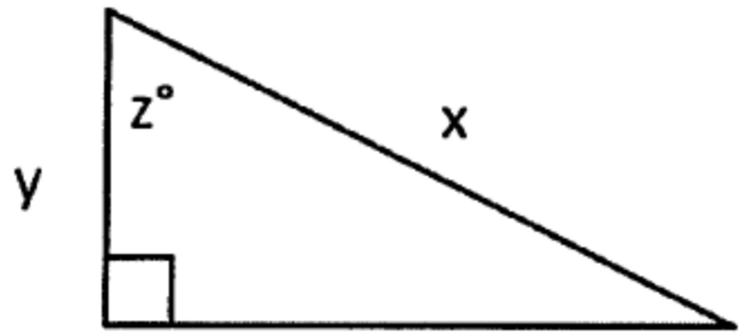
$$\cos z° = \frac{y}{x}$$

$$x = ?$$

“Level 0”: Confusion due to replacing numbers by symbols

What is the value of *x*?





49% correct

41% correct

[First-semester (Fall 2017), calculus-based; N = 91]

McNemar Test for Correlated Proportions: p = 0.10

New results, N = 903: 3% difference, p = 0.03

Why the Difficulties with Symbols?

Some Suggestions Arising from the Interviews

- In elementary math courses, "simplified forms" of equations are emphasized (i.e., few messy symbols and functions).
- Many students get "overloaded" by seeing all the variables, and are unable to carry out procedures that they do successfully with numbers.
- Many students have had *insufficient practice* with algebraic operations to avoid being overwhelmed by standard algebraic manipulations.
  - Students tend to become *careless*

Summary: Implications for Instruction

Difficulties might be addressed by:

- short-term, in- and out-of-class tutorials and assignments, designed to refresh students' previously learned knowledge and skills (e.g., Mikula and Heckler, 2017)
- guiding students to (1) explicitly identify known and unknown variables; (2) carefully check and re-check key steps in calculation; (3) slow down, review, and re-solve when possible