Exploration of Physics Students' Mathematical Difficulties

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Supported in part by NSF DUE #1504986

The Problem

- Difficulties with very basic math skills impact performance of introductory physics students.
- The difficulties are often not resolved by students' previous mathematical training.
- Students can't effectively grapple with physics ideas when they feel overburdened in dealing with calculational issues.

Difficulty #1: Trigonometry

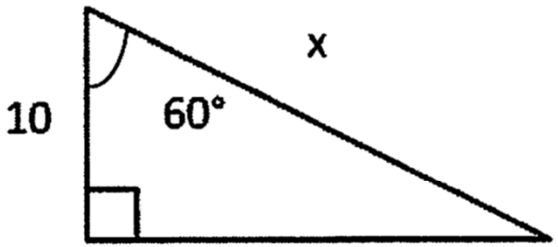
- Many students are confused or unaware (or have forgotten) about the relationships between sides and angles in a right triangle.

- *Examples:* Questions from a diagnostic math test administered at Arizona State University, 2016-2017 (Administered as no-credit quiz during first week labs and/or recitation sections; calculators allowed)

Trigonometry Questions

with samples of correct student responses

1.



What is the value of x?

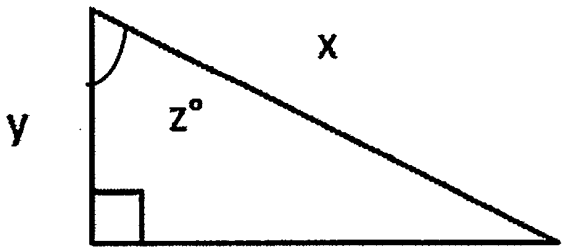
$\cos 60 = \frac{10}{x}$

$x \cos 60 = 10$

$x = \frac{10}{\cos 60}$

$= 20$

2.



$\cos z = \frac{y}{x}$

What is the value of x?

A. ycos(z)

B. ycos(z)sin(z)

C. y/sin(z)

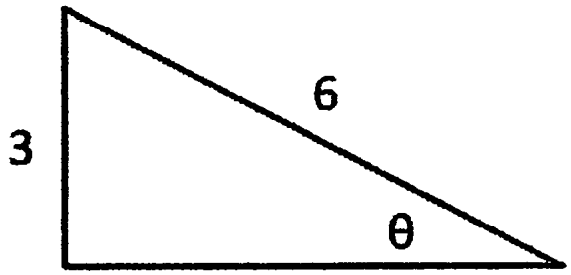
D. ysin(z)

E. ycos(z)/sin(z)

F. y/cos(z)

G. None of the above_____

3.



What is the value of θ?

$\sin^{-1}(\theta) = \sin^{-1}\left(\frac{3}{6}\right)$

$\theta = 30^{\circ}$

Trigonometry Questions:

Correct Response Rate, #1-3 combined

ASU Polytechnic campus, Spring + Fall average:

Algebra-based course, 1st semester, (*N* = 116): 37%

Algebra-based course, 2nd semester, (*N* =79): 48%

ASU Polytechnic campus, Spring (2-year average):

Calculus-based course, 1st semester, (*N* = 146): 66%

Results on Trigonometry Questions

Errors oberved: use of incorrect trigonometric function (e.g., cosine instead of sine), calculator set on radians instead of degrees, algebra errors; *unaware (or forgot) about inverse trigonometric functions, e.g., arctan, arcsin*, *arccos [tan^{-1}, sin^{-1}, cos^{-1}]*

- **How to address these problems:** It seems that students require substantial additional *practice and repetition* with basic trigonometric procedures

Trigonometry Questions:
Polytechnic/Tempe Campus Difference

Error Rate (% incorrect responses)

Algebra-based course, second semester; #1-3 combined:

ASU Polytechnic campus, Spring (*N* = 52): 59%

ASU Tempe campus, Spring (*N* = 61): 35%

Trigonometry Questions:
Spring/Fall Semester Difference

Error Rate (% incorrect responses)

Algebra-based course, first semester; #1-3 combined:

ASU Polytechnic campus, Spring (*N* = 72): 67% ⬅

ASU Polytechnic campus, Fall (*N* = 44): 58%

Algebra-based course, second semester; #1-3 combined:

ASU Polytechnic campus, Spring (*N* = 52): 59% ⬅

ASU Polytechnic campus, Fall (*N* = 27): 44%

Calculus-based course, first semester; #1 only:

ASU Polytechnic campus, Spring (*N* = 104): 40%

ASU Polytechnic campus, Fall (*N* = 98): 56% ⬅

Trigonometry Questions:
Multiple-Choice vs. Non-Multiple-Choice
(Higher Error Rate on Non-Multiple-Choice [Non-MC])

Error Rate Difference (% incorrect responses), Non-MC−MC

Course #1, Problem #2: +15
Course #1, Problem #3: +18

Course #2, Problem #2: +9
Course #2, Problem #3: +9

Course #3, Problem #2: +5
Course #3, Problem #3: +34

Course #4, Problem #2: +10
Course #4, Problem #3: +5

Difficulty #2: Algebra

- Students have difficulties in solving two simultaneous equations, and those difficulties are much greater when the equations are in symbolic form.

Algebra: Simultaneous Equations

$3x = 2y$
$5x + y = 26$

What are the values of x and y? Show all your steps. For example, $x = 2, y = 5$ (These are NOT the correct answers).

Correct Response Rate, ASU (% correct responses)

Algebra-based course, second semester (*N* = 123): 70%

Algebra: Simultaneous Equations

$$x \cdot \cos(20°) = y \cdot \cos(70°)$$
$$x \cdot \cos(70°) + y \cdot \cos(20°) = 10$$

What are the values of x and y? Show all your steps. Note: The value for x should NOT include y, and the value for y should NOT include x.

Correct Response Rate, ASU (% correct responses)

Algebra-based course, second semester (*N* =150): 20-30%
(different campuses, slightly different versions)

Algebra: Simultaneous Equations

$a \cdot x = b \cdot y$
$b \cdot x + a \cdot y = c$

a, b, and c are constants.
What are the values of x and y in terms of a, b, and c? Show all your steps. Note: The value for x should NOT include y, and the value for y should NOT include x.

Correct Response Rate, ASU (% correct responses)

Algebra-based course, second semester (*N* =150): 10-20%
(different campuses, slightly different versions)

Only 10-20% correct responses!

Algebra: Simultaneous Equations

$a \cdot x = b \cdot y$
$b \cdot x + a \cdot y = c$

a, b, and c are constants.
What are the values of x and y in terms of a, b, and c? Show all your steps. Note: The value for x should NOT include y, and the value for y should NOT include x.

$x = \frac{by}{a}$

$b\left(\frac{by}{a}\right) + ay = c$

$\frac{b^2y}{a} + ay = c$

$y\left(\frac{b^2}{a} + a\right) = c$

$y = \frac{c}{\left(\frac{b^2}{a} + a\right)}$

$x = \frac{b\left(\frac{c}{(b^2/a + a)}\right)}{a}$

Sample of Correct Student Response

Sources of Difficulties

- Carelessness
  - Students *very frequently* self-correct errors during interviews
  - Evidence of carelessness on written diagnostic
- Skill practice deficit: Insufficient repetitive practice with learned skills
  - E.g., applying definitions of sine and cosine; factoring out variables in algebraic expressions
- Conceptual confusion
  - E.g., not realizing that sides and angles of right triangle are related by trigonometric functions

How to Address Difficulties?

- Carelessness:
  (1) review and check steps
  (2) find alternative solutions
  (3) habitual use of estimation
  (4) apply dimensional analysis (for physical problems)
- Skill deficit: Practice and repetition
- Conceptual confusion: Review and study of basic ideas

Summary:

What Options do Physics Instructors Have for Dealing with Students' Mathematics Difficulties?

- Test to assess scope of problem
- Take time to review basic math
- Assign or suggest out-of-class math review practice

  [We will be developing appropriate instructional materials]
- Reduce mathematical burden of syllabus
  - more qualitative problems, fewer problems requiring algebraic manipulation
- Nothing: Leave it up to the students