Physics Students' Difficulties with Mathematical Symbols and Operations

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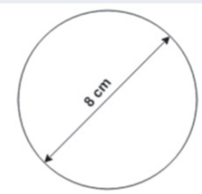
ABSTRACT

For an investigation into physics students' mathematical difficulties, we have administered written diagnostic tests to over 4000 students enrolled in introductory algebra- and calculus-based physics courses on two different campuses of Arizona State University, and carried out over 75 individual interviews. Students' responses to elementary questions on trigonometry, algebra, geometry, and graphing consistently reflected a large number of operational errors, to a degree that could significantly interfere with success in an introductory physics course. Moreover, we find that (i) merely replacing numbers (e.g., 0.5, 8, 30) by symbols (e.g., *c*, *b*, $v_1/2$) in equations and problem statements significantly lowers students' correct-response rate (cf. Torigoe and Gladding, 2011; reference below), and (ii) during problem-solving interviews, students tend to "self-correct" approximately 60% of their initial errors.

STUDENT PERFORMANCE ON CALCULATING AREA OF CIRCLE



(a) Area of the circle =

Correct-Response Rate, Algebra- and Calculus-Based Courses Combined (% correct responses)

Polytechnic campus (N = 250): 57% (5 classes; range: 48-61%) (with correct units: 29%)

Tempe campus (N =1086): 76% (5 classes; range: 74-79%) (with correct units: 45%)

- Little difference between algebra- and calculus-based courses
- Interchanging radius and diameter was ***NOT*** most-common error

STUDENT PERFORMANCE ON TRIGONOMETRY PROBLEMS

Correct-Response Rate, #1 + #4, Arizona State University campuses

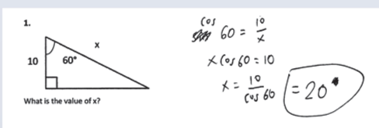
Algebra-based course, 1st semester,
- Tempe campus, (N ~ 420): 55%
- Polytechnic campus, (N ~ 350): 38%

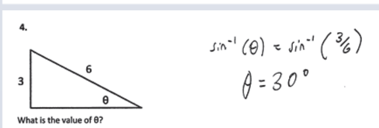
Algebra-based course, 2nd semester,
- Polytechnic campus (N ~ 210): 46%

Calculus-based course, 1st semester,
- Polytechnic campus, spring (N ~ 340): 56%
- Tempe campus, spring (N ~ 1070): 83%

Trigonometry Questions

with samples of correct student responses



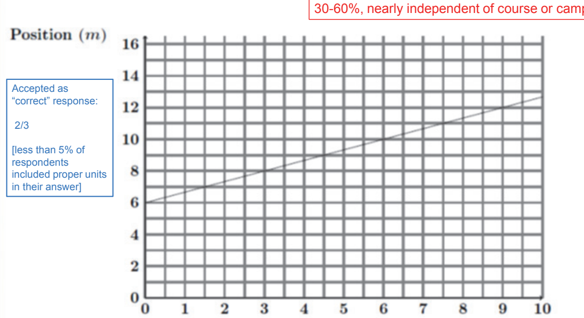


20-60% *of students confused on basic trigonometry relations*

STUDENT PERFORMANCE ON GRAPHING PROBLEMS

What is the slope of the graph below?

Correct-response rate (N > 2000): 30-60%, nearly independent of course or campus



Accepted as "correct" response: 2/3 [less than 5% of respondents included proper units in their answer]

Most common error: counting grid squares and ignoring numbers on axes

STUDENT PERFORMANCE ON ALGEBRA PROBLEMS

Our Findings: Significantly worse performance on "symbolic" versions of single-equation and simultaneous-equations problems, compared to "numeric" versions

Summary of the Simultaneous-Equations Data (both campuses)

- *Algebra-based course:* ~30-70% correct on numeric versions, ~10-35% on symbolic versions;
- *Calculus-based course:* ~55-90% correct on numeric versions, ~30-65% correct on symbolic versions.

Algebra: Simultaneous Equations

$0.5y = 2x$
$78.4 - y = 8x$ [Solve for x] **Numeric Version**

Correct-Response Rate (% correct responses, spring semester)

Algebra-based course, 1st semester, Polytechnic campus (N = 108): 53% (2 classes; range: 47-60%)

Calculus-based course, 1st semester, Tempe campus (N = 941): 86% (2 classes; range: 83-89%)

$cy = dx$
$a - y = bx$ [Solve for x] **Symbolic Version**

Correct-Response Rate (% correct responses, spring semester)

Algebra-based course, 1st semester, Polytechnic campus (N = 70): 15% (2 classes; range: 13-18%)

Calculus-based course, 1st semester, Tempe campus (N =780): 60% (2 classes; range: 57-63%)

$x \cos(20°) = y \cos(70°)$
$x \cos(70°) + y \cos(20°) = 10$ **Trigonometric Version**
[Solve for x and y]

Correct-Response Rate (% correct responses, spring semester)

Calculus-based course, 1st semester, Tempe campus (N =197): 36%

Why the Difficulties with Symbols? Some Hints From the Interviews

- In elementary math courses, "simplified forms" of equations are emphasized (i.e., few messy symbols and functions)
- Students get "overloaded" by seeing all the variables, and are unable to carry out procedures (e.g., multiplying each term in an expression by a constant [symbol]) that they do successfully with numbers (e.g., multiply through by a number)
- Other procedural failures that occur more often with symbols: cancellation, factoring out a constant, retaining coefficients from one line to the next

Algebra: "Numeric" vs. "Symbolic" Problems

Numeric Version

5. What is the numerical value of d?

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{\Delta v}{\Delta t}$

$\Delta v = 60$

$\Delta t = 8$

$v = 30$

$d = ?$

(Please clearly *circle* your answer and show all work.)

A. $d = 30$ B. $d = 60$ C. $d = 120$ D. $d = 240$ E. $d = 480$

Symbolic Version

10. In the equations below, v_1, t_1, a, and v represent (unknown) numbers, for example, 3, 8, 9, 14.

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{v_1}{t_1}$

$v = \frac{v_1}{2}$

$d = ?$

(Please clearly *circle* your answer and show all work.)

A. $d = v_1 t_1$ B. $d = \frac{v_1 t_1}{2}$ C. $d = \frac{v_1 t_1}{4}$ D. $d = \frac{v_1 t_1}{8}$ E. $d = \frac{v_1 t_1}{16}$

Correct-Response Rate (% correct responses, Tempe campus)

Numeric Version
Calculus- and algebra-based courses: 80-90% (N = 1678)

Symbolic Version
Algebra-based course: 35% (N = 355)
Calculus-based course: 70% (N = 1303)

Adapted from Torigoe and Gladding, Am. J. Phys. **79**, 133 (2011)

SUMMARY: SOURCES OF DIFFICULTIES

- Carelessness
  - Students *very frequently* self-correct errors during interviews (~60% of errors are self-corrected with no or minimal prompting)
- Skill practice deficit: Insufficient repetitive practice with learned skills
- Inability to efficiently access previous learning

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