Developing a Strategy to Address Physics Students' Mathematical Difficulties

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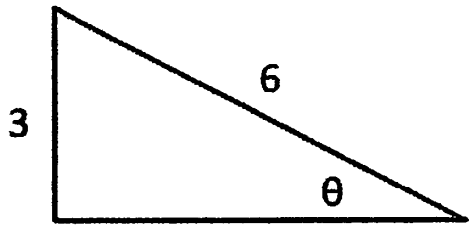
Outline

- Weak skills with basic pre-college mathematics can severely impact physics students' course performance
- We have explored the nature and prevalence of physics students' difficulties with elementary mathematics, using "stripped-down" problems with little or no physics context
- In collaboration with Ohio State University, we are developing and testing an online "skill-practice" tool to improve physics students' mathematical problem-solving performance

Work to Date

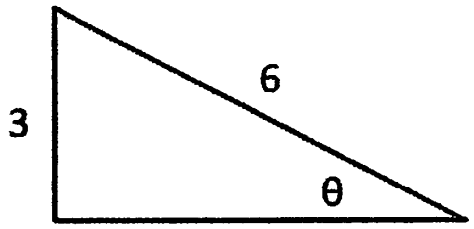
- Administer (and analyze) written diagnostic quiz, given to > 4000 students at Arizona State University; calculators *are* allowed
- Carry out individual interviews with 75 students enrolled in those or similar courses during same period
- Comparison data: University of Colorado, algebra-based course (*N* = 388)

“Find Unknown Angle”



What is the value of θ?

“Find Unknown Angle”



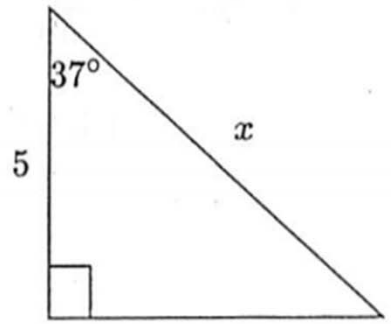
What is the value of θ?

$\sin^{-1}(\theta) = \sin^{-1}\left(\frac{3}{6}\right)$

$\theta = 30^{\circ}$

“Find Unknown Side”

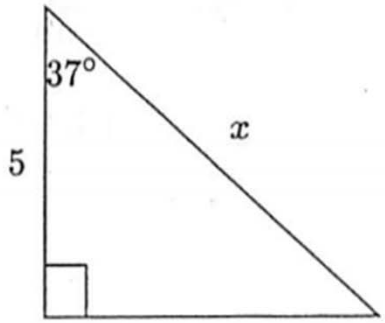
1. What is the length of side x?



“Find Unknown Side”

SOH CAH TOA

1. What is the length of side x?



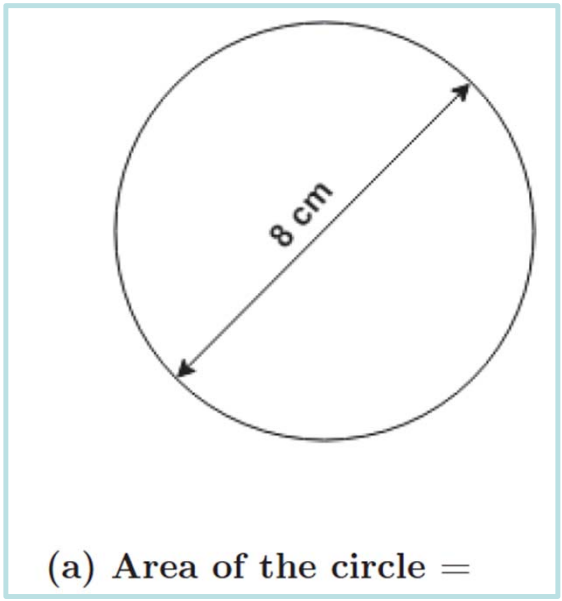
$$\cos(37^\circ) = \frac{5}{x}$$

$$x\cos(37^\circ) = 5$$

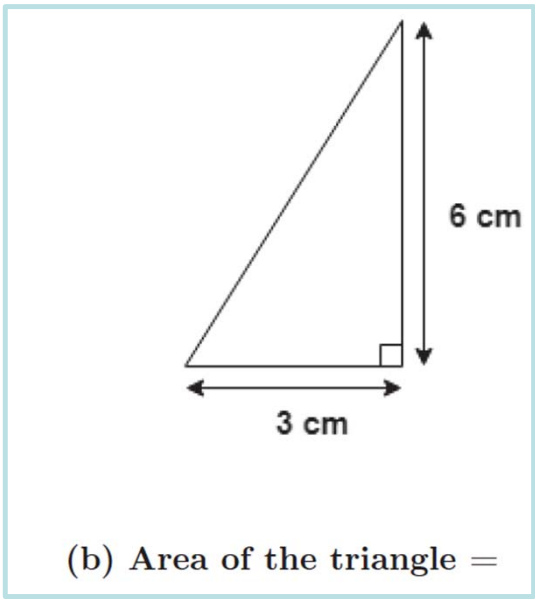
$$x = \frac{5}{\cos(37^\circ)}$$

$$x = 6.26$$

Find Area

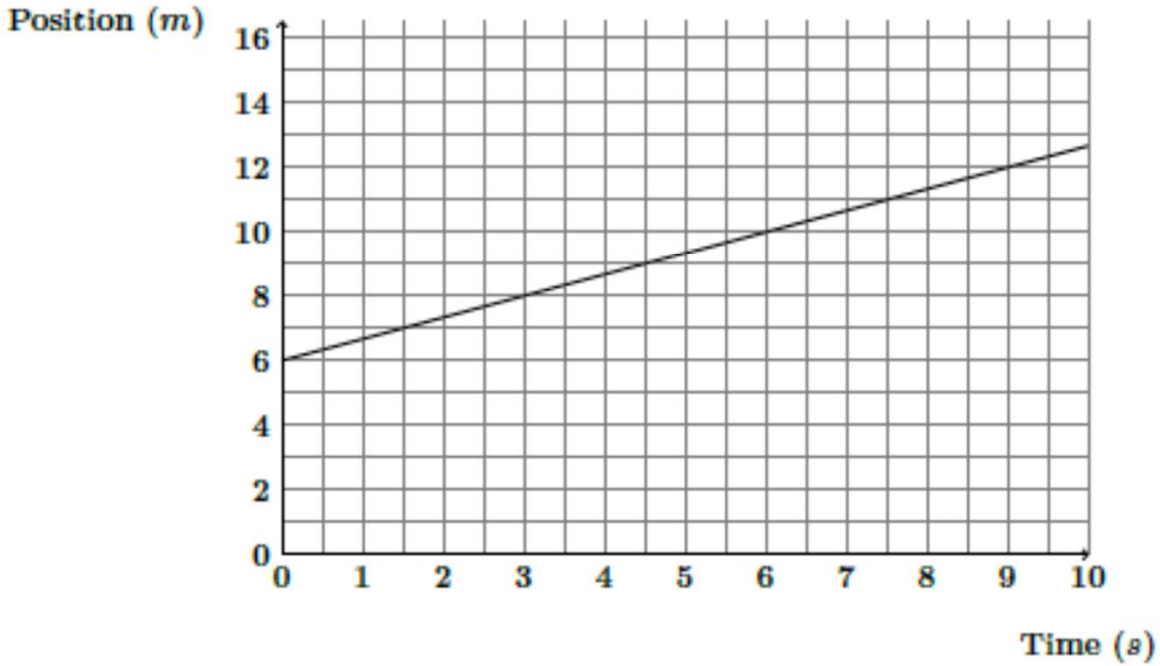


(a) Area of the circle =



(b) Area of the triangle =

What is the slope of the graph below?



Simultaneous Equations, “Numeric” Version

What is the numerical value of x?

$$0.5y = 2x$$

$$78.4 - y = 8x$$

Simultaneous Equations, “Symbolic” Version

$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

Simultaneous Equations, “Symbolic” Version

18.

$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

Simultaneous Equations, “Symbolic” Version

$cy = dx$

$a - y = bx$

$x = ?$

$y = a - bx$

$c(a - bx) = dx$

$ca - cbx = dx$

$ca = (cb + d)x$

$$x = \frac{ca}{cb + d}$$

“Multi-step” problem

14. $v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \dfrac{v_1}{t_1}$

$v = \dfrac{v_1}{2}$

$d = ?$

(Please clearly *circle* your answer and show all work.)

A. $d = v_1 t_1$ B. $d = \dfrac{v_1 t_1}{2}$ C. $d = \dfrac{v_1 t_1}{4}$ D. $d = \dfrac{v_1 t_1}{8}$ E. $d = \dfrac{v_1 t_1}{16}$

"Symbolic Multiple Choice"

4. Find the value of each of the following.

$cos(0°) = ?$

$sin(90°) = ?$

$tan(0°) = ?$

6. Solve for θ.

$\gamma\theta + \eta = \lambda\theta + \omega$

7. Solve for x.

$ax + b = cx + d$

10. $2\left(\frac{3}{4}\right) = ?$

11. $\frac{a/b}{c^2/d} = ?$

A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

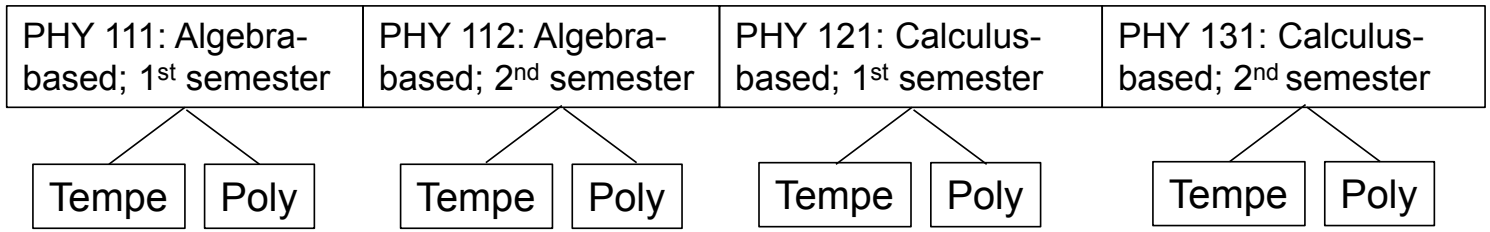
17. $ax - dx = c$

$x = ?$

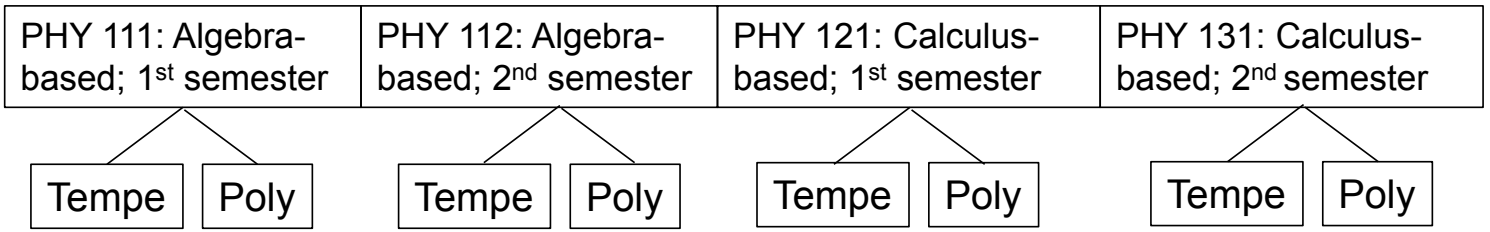
Our 4 Sample Populations

PHY 111: Algebra-based; 1st semester	PHY 112: Algebra-based; 2nd semester	PHY 121: Calculus-based; 1st semester	PHY 131: Calculus-based; 2nd semester

Our 8 Sample Populations



Our 8 Sample Populations



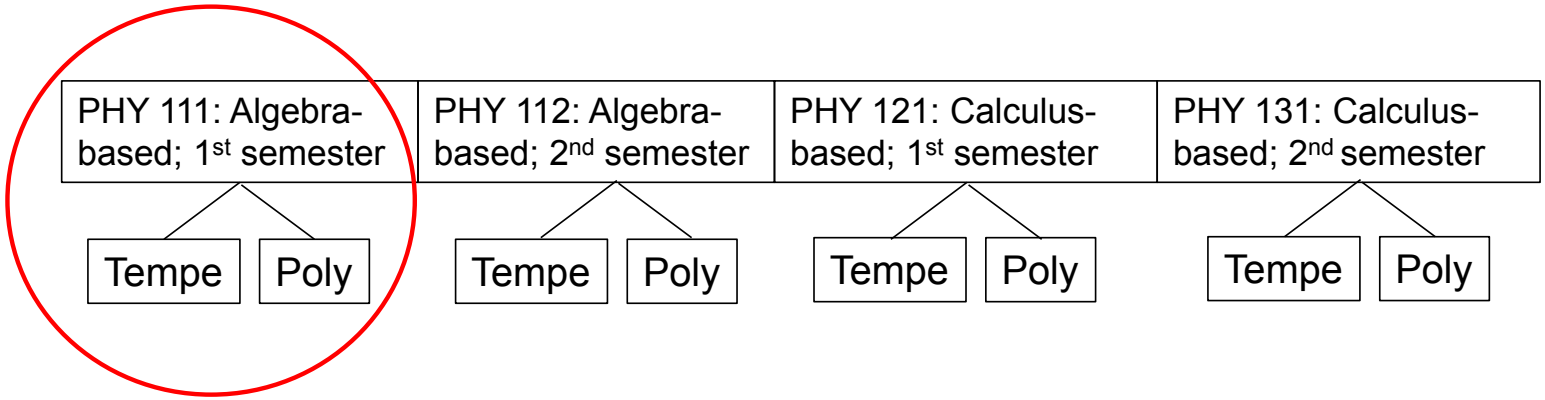
On average, students in the Tempe courses have more extensive background and preparation (and different majors) than those in the corresponding Poly courses.

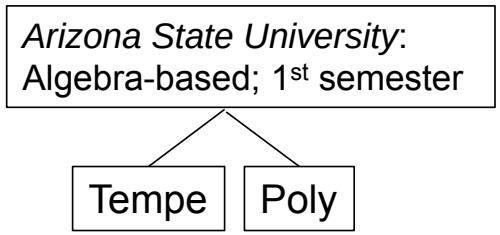
Primary Findings

Regardless of course (algebra- or calculus-based), campus (Tempe or Poly), or semester (Spring or Fall):

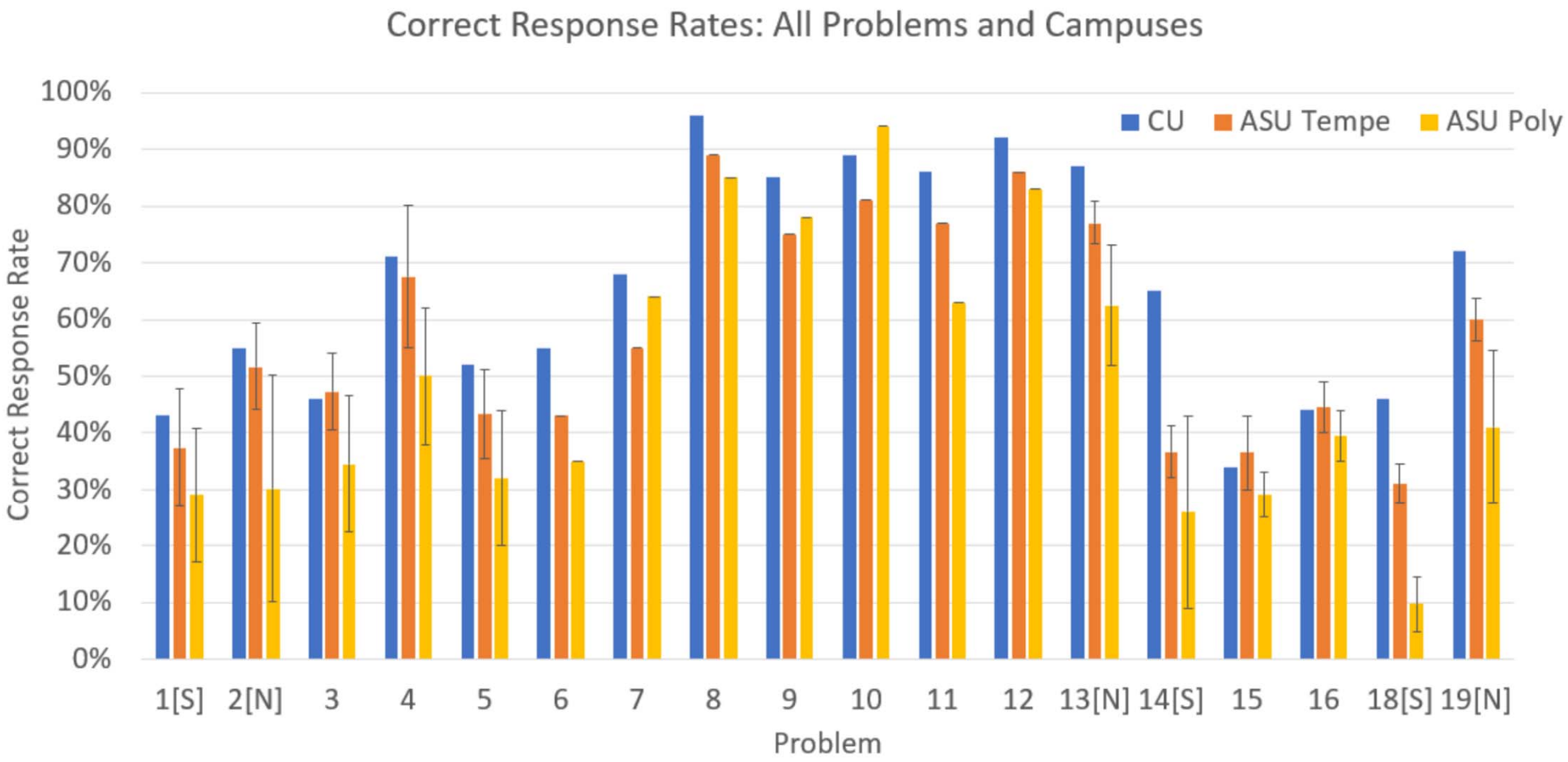
- Difficulties with basic mathematical operations are widespread; average error rates range from 20-70%;
- Performance on algebraic problems using symbols for constants is significantly worse than on problems using numbers;
- During problem-solving interviews, students self-correct approximately 50% of errors following minimal prompts.

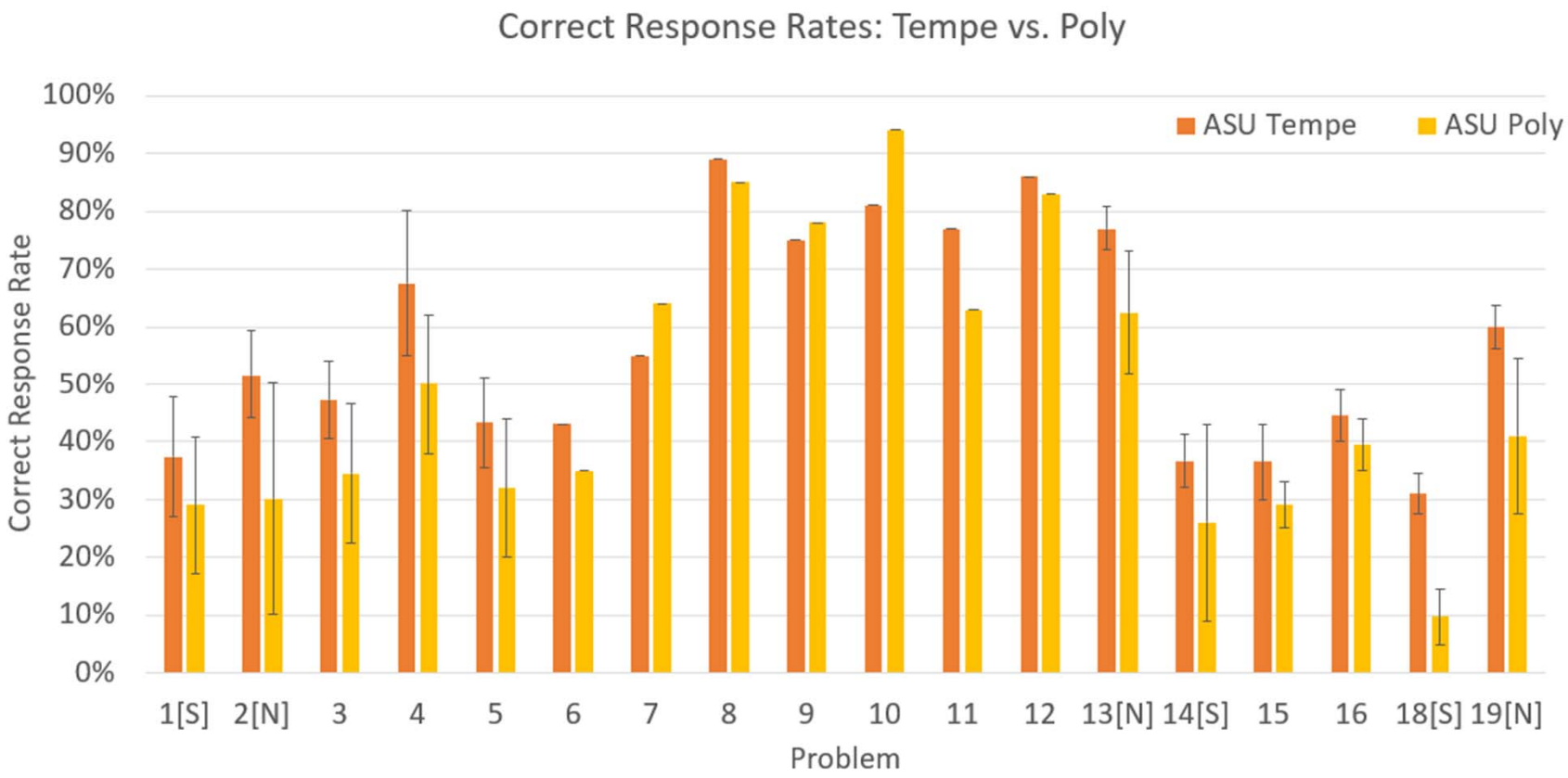
Our 8 Sample Populations




Arizona State University:
Algebra-based; 1st semester
Tempe
Poly

Our 3 Sample Populations


Correct Response Rates: All Problems and Campuses
CU
ASU Tempe
ASU Poly
Correct Response Rate
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
1[S]
2[N]
3
4
5
6
7
8
9
10
11
12
13[N]
14[S]
15
16
18[S]
19[N]
Problem


Correct Response Rates: Tempe vs. Poly
ASU Tempe
ASU Poly
Correct Response Rate
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13[N]
14[S]
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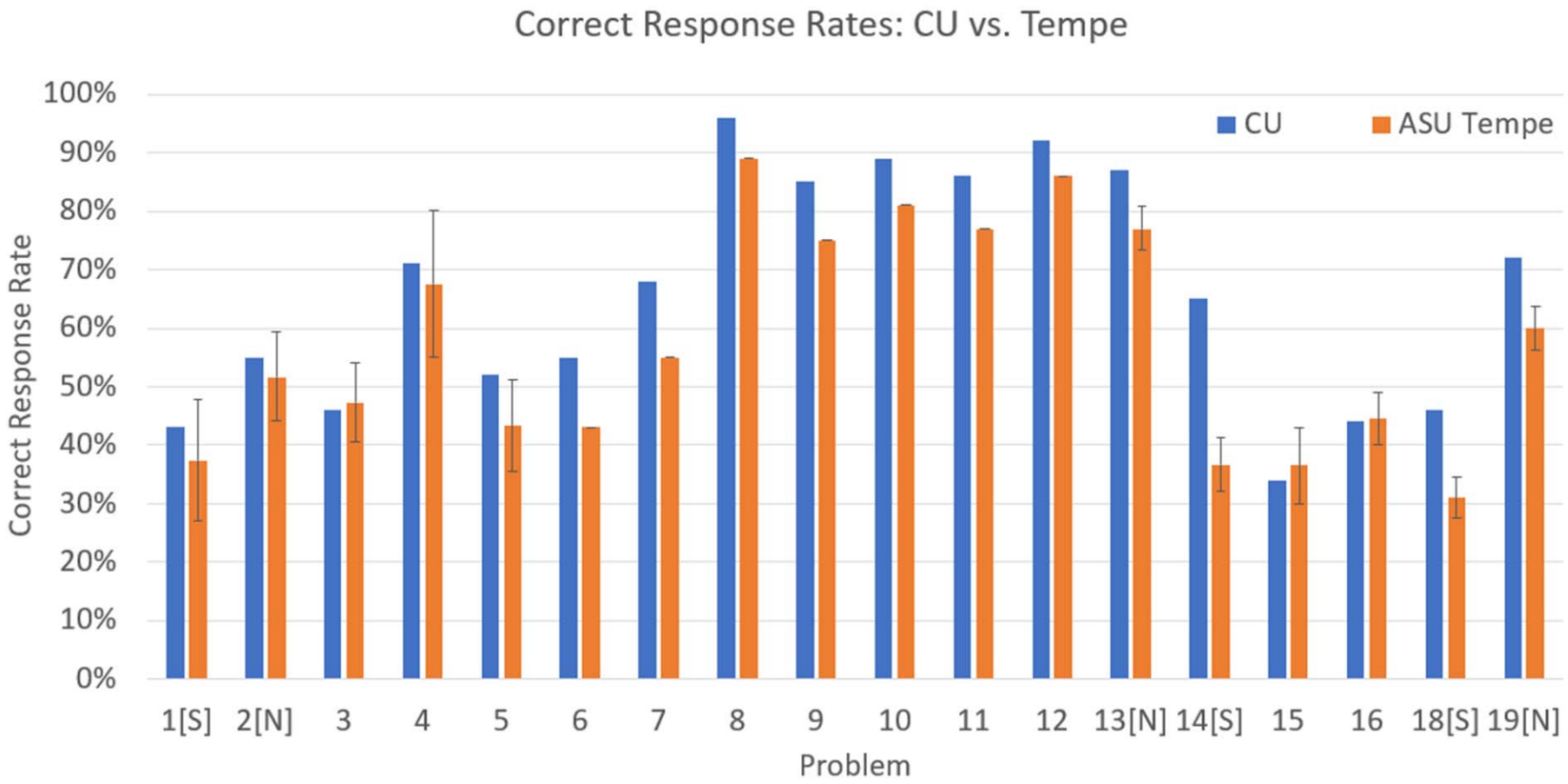
18[S]
19[N]
Problem


Correct Response Rates: CU vs. Tempe
CU
ASU Tempe
Correct Response Rate
100%
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80%
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13[N]
14[S]
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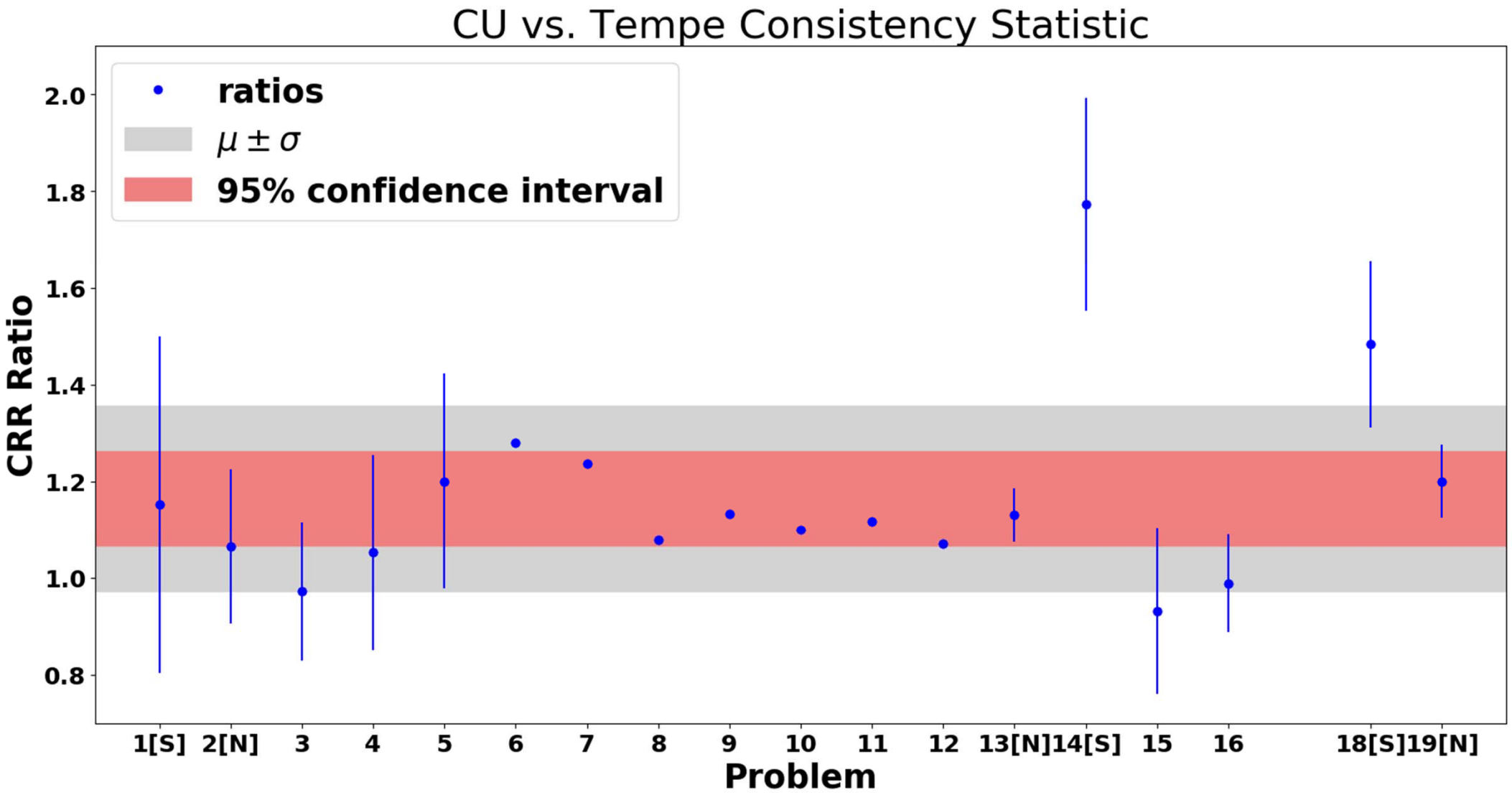
18[S]
19[N]
Problem


CU vs. Tempe Consistency Statistic
ratios
$\mu \pm \sigma$
95% confidence interval
CRR Ratio
2.0
1.8
1.6
1.4
1.2
1.0
0.8
1[S]
2[N]
3
4
5
6
7
8
9
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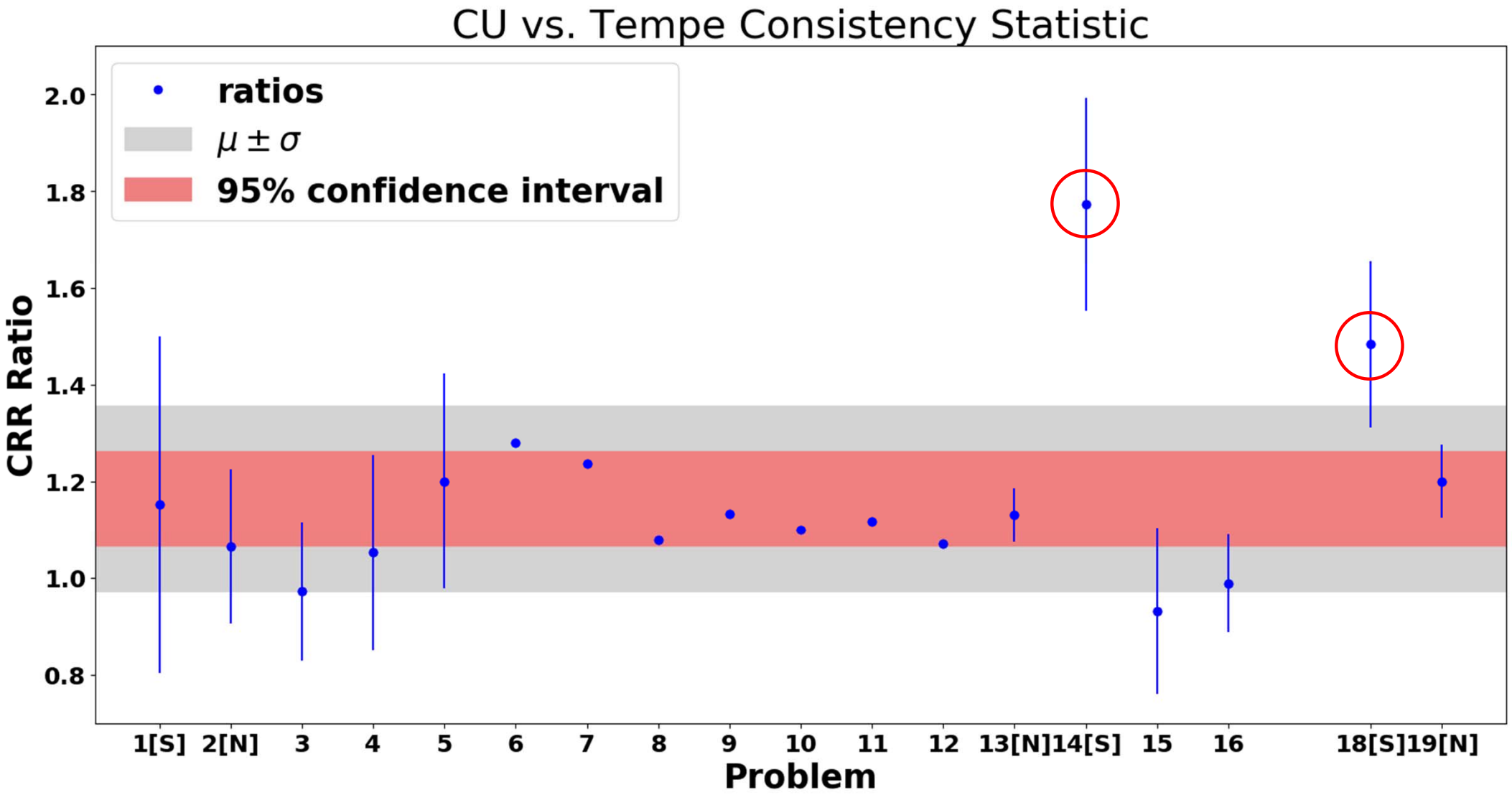
11
12
13[N]
14[S]
15
16
18[S]
19[N]
Problem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13[N]
14[S]
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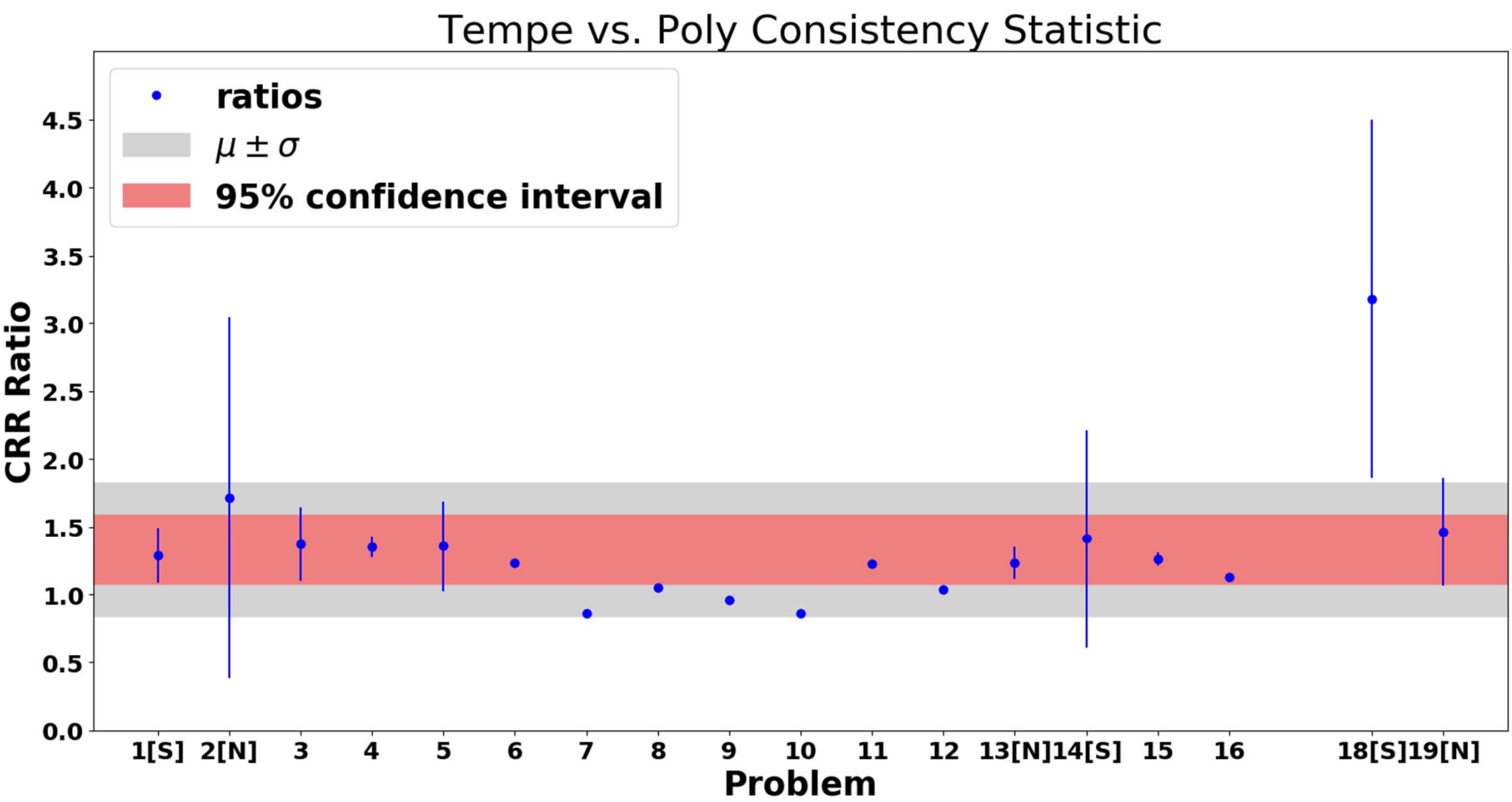
16
18[S]
19[N]
Problem


Tempe vs. Poly Consistency Statistic
ratios
$\mu \pm \sigma$
95% confidence interval
CRR Ratio
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
1[S]
2[N]
3
4
5
6
7
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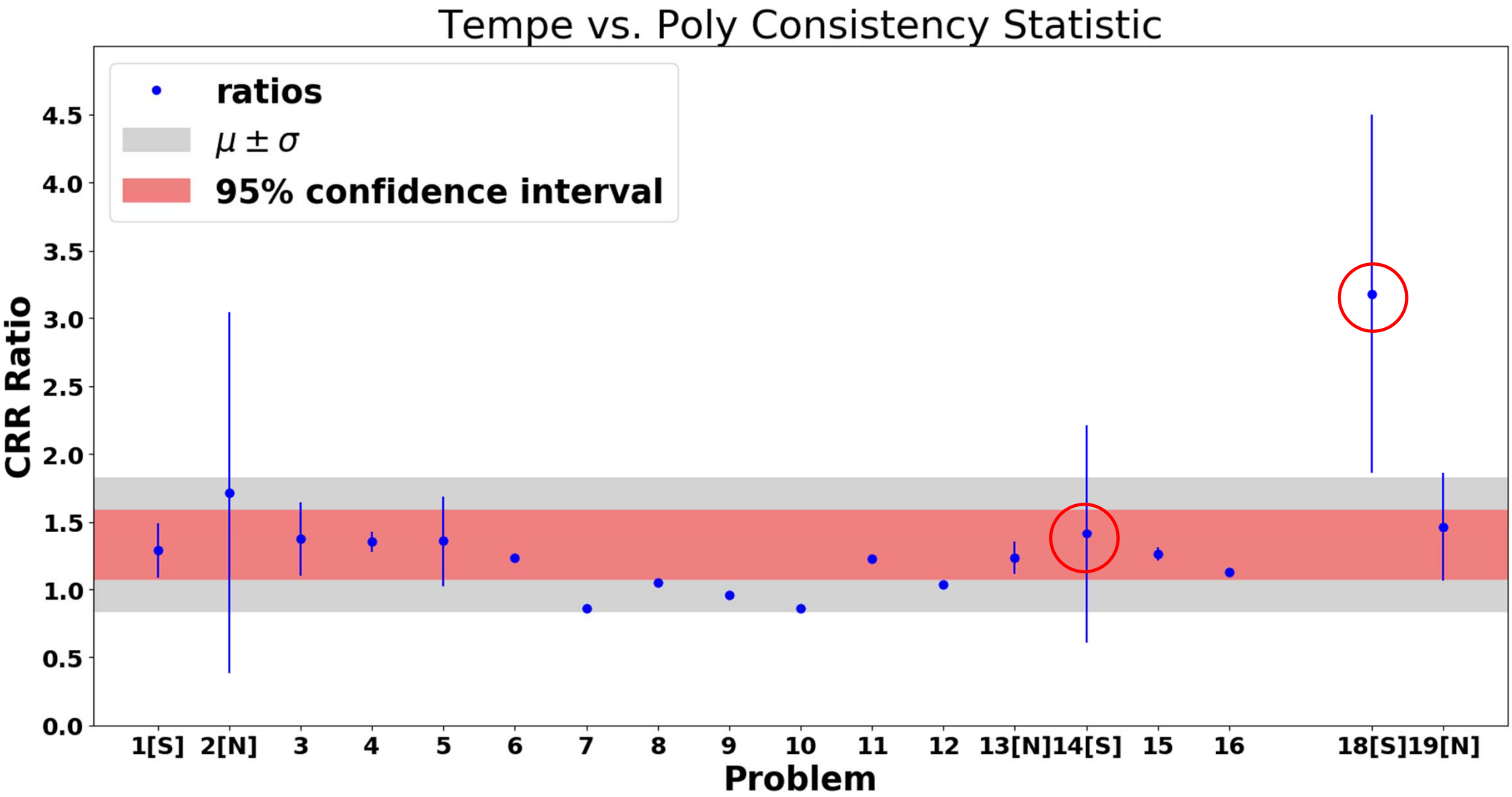
12
13[N]
14[S]
15
16
18[S]
19[N]
Problem


Tempe vs. Poly Consistency Statistic
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95% confidence interval
CRR Ratio
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0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
1[S]
2[N]
3
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7
8
9
10
11
12
13[N]
14[S]
15
16
18[S]
19[N]
Problem

Item Responses Reflect Institutional Differences

- The correct-response rate (CRR) for CU on the 19 test items averages 16% higher than those at ASU-Tempe, while Tempe averages 33% higher than Poly, with ratios of all but two test items falling within fairly narrow bands (mean +/− 1 sd).
- Conjecture #1: The differences in mean CRRs reflect differences among the institutions' student populations
- Conjecture #2: Most of the (otherwise diverse) test items probe operational ability to similar "degrees"
- Conjecture #3: Another "level" of operational-ability difference is probed by the multi-step symbolic test items

Error Types

- "Operational" Errors: Inadequate learning or expertise with fundamental operations
  - ***Conceptual*** confusion, e.g., What is an inverse sine? What is slope?
  - Weak ***skills*** with numerical and/or algebraic operations (e.g., factoring)
  - Inadequate ***practice*** with symbolic operations (e.g., dividing fractions)
- "Non-operational" Errors: Difficulties connecting context of problem to context in which operations were learned, or "carelessness"
  - Physics context, e.g., position-time graph with appropriate units
  - Problems involving multiple linked steps, each involving basic operations
  - Inattention to detail; failure to check work

Possible Instructional Strategies

- Difficulties due to **skill-practice deficits** might be addressed by short-term, in- and out-of-class tutorials and assignments, designed to refresh students' previously learned knowledge and skills (e.g., Mikula and Heckler, 2017)
  - Current project, OSU + ASU, NSF DUE #1914709/1914712
  - Regular low-stakes on-line homework assignments requiring multiple consecutive correct answers
- Inclusion of multi-step contexts in these assignments ***may*** reduce the prevalence of non-operational errors as well.

Possible Instructional Strategies

- Difficulties due to **“carelessness”** might be addressed by guiding students to (1) carefully check and re-check key steps in their calculation; (2) slow down, review problem statements, and re-solve when possible

- Other studies (e.g., G. White) have shown that much practice and repetition is needed to induce students to adopt consistent error-checking habits

Summary

- Physics students' mathematical errors have a variety of causes
- Errors due to skill-practice deficits with basic operations may (perhaps) be addressable through regular, brief online assignments
- Errors due to "carelessness," or difficulties in matching operational skills to physics or mathematics context, may require other corrective measures