

Introductory and advanced students' difficulties with thermodynamic work

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Introduction

Here we analyze data from administration of the conceptual diagnostic test "Survey of Thermodynamic Processes and First and Second Laws-Long" (STPFaSL-Long), a validated 78-item multiple-choice survey that focuses on concepts covered in introductory physics courses. Details of its development and validation are in Ref. 1. It was administered to 12 different traditionally taught lecture-based courses from four different universities in the US. The written data analyzed here were obtained by administering the survey in proctored in-person classes as a pre-test (before instruction) and as a post-test, after students had studied the relevant concepts but before the final exam. Students completed the survey in class on Scantrons during a 50-minute class period. Here we examine written post-test data from students taking upper-level courses and from introductory students in both algebra-based and calculus-based courses. In addition, we interviewed 11 introductory and 6 upper-level students individually using a think-aloud protocol to get a deeper insight into students' thinking. A detailed report can be found in Ref. 2.

Our focus here is on questions related to thermodynamic work; all relevant survey items are shown below, along with post-test response rates for each answer option (Table I). All but one item could have been answered by application of the mnemonic "work done by the system equals the (signed) area under the curve in a PV diagram." The other item (#46) could be answered simply by invoking the path-dependent nature of thermodynamic work.

Previous research (Refs. 3-6) has shed considerable light on students' difficulties related to work. Our results are consistent with previous findings, but broaden them considerably by extending to previously unexamined student groups and by revealing the relative degree of difficulty encountered on different thermodynamic processes. Moreover, we have been able to pinpoint specific "surface features" of PV diagrams that seem to be particularly influential in students' thinking.

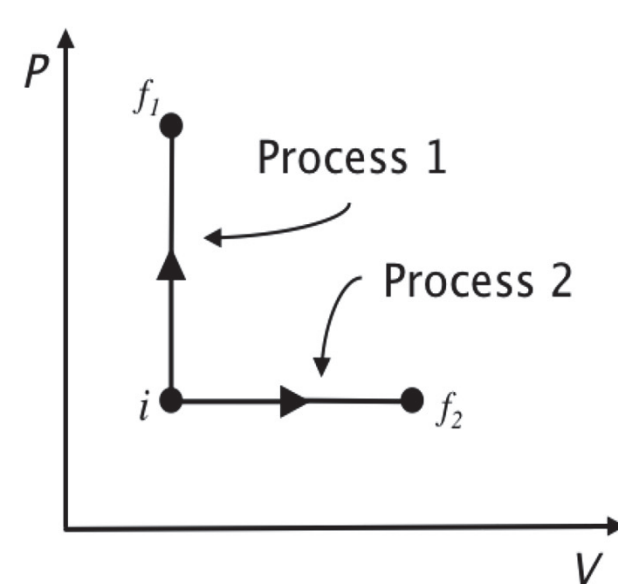
Primary Findings

(Note that all problems except #46 may be solved by using the "work equals (signed) area under the curve" mnemonic)

Upper: students enrolled in upper-level undergraduate courses or first-year, first-semester graduate courses
Int-Calc: students enrolled in the calculus-based physics course
Int-Alg: students enrolled in the algebra-based physics course

- Highest overall error rate on the cyclic-process problem (#7; correct answer: $W < 0$):
 - All three student groups had $\approx 30\%$ sign errors
 - More than half (51%) of Int-Alg answered $W = 0$
 - More than one fifth (21%) of Int-Calc answered $W = 0$
- All student groups had greater difficulty comparing the amount of work done in an isothermal process to that done in an adiabatic process (problem #59) than when comparing the isothermal process to an isobaric process (problem #10). Error rates on the isothermal/adiabatic comparison (#59) were 18-20% higher for the introductory students.
 - Students who answered $W_1 = W_2$ on these problems often argued that this was because ΔV was the same for both processes, but there were more $W_1 = W_2$ errors on #59.
 - There were more sign errors on #59. Written pretest responses suggested that students who made sign errors on #59 (26-44% of the introductory students) seemed to find the larger ΔP of the adiabatic process to be a particularly distracting feature.
 - Students may have found the isobaric process to be easier to analyze in general; see #3 below.
- Introductory students were more likely to give a correct ($W > 0$) response on the isobaric expansion (#43, 60-74% correct) than on the adiabatic expansion (#58, 44-61% correct).
- Less than half (46%) of Int-Alg students realized that $W = 0$ in the isochoric process (problem #42); more than one third thought instead that $W > 0$, perhaps misled by the upward-pointing arrow.

You carry out two experiments each with one mole of an ideal monatomic gas such that both processes start in the same state i as shown on the PV diagram below. Process 1 is a constant volume (isochoric) process and process 2 is a constant pressure (isobaric) process. Answer the following six questions about the two processes:

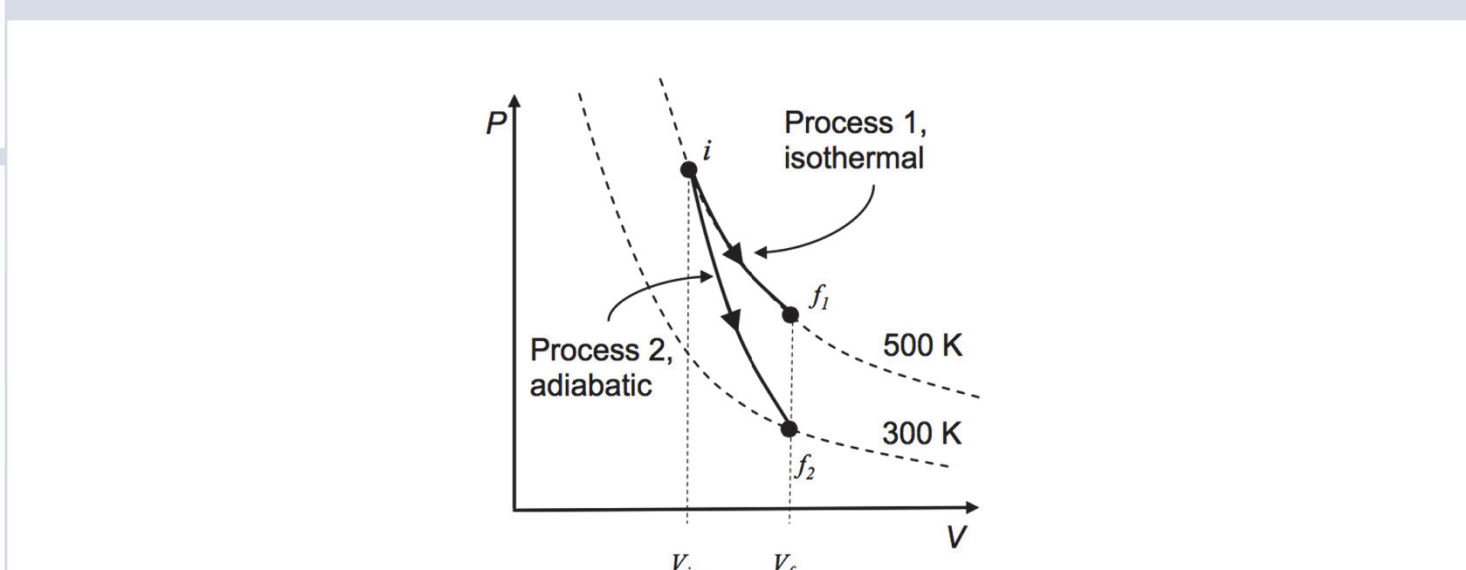


(42) Which one of the following statements is correct about the work done by the gas in process 1?

- The work done by the gas is positive in process 1.
- The work done by the gas is negative in process 1.
- The work done by the gas is zero in process 1.
- Not enough information.

(43) Which one of the following statements is correct about the work done by the gas in process 2?

- The work done by the gas is positive in process 2.
- The work done by the gas is negative in process 2.
- The work done by the gas is zero in process 2.
- Not enough information.



(57) Which one of the following statements is true about the isothermal process (process 1)?

- No work is done by the gas in the isothermal process.
- The work done by the gas in the isothermal process is positive.
- The work done by the gas in the isothermal process is negative.
- Not enough information.

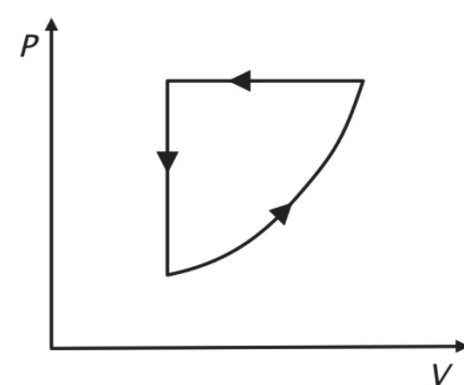
(58) Which one of the following statements is true about the adiabatic process (process 2)?

- No work is done by the gas in the adiabatic process.
- The work done by the gas in the adiabatic process is positive.
- The work done by the gas in the adiabatic process is negative.
- Not enough information.

(59) Which one of the following statements is true about the two processes?

- The work done by the gas in the isothermal process (process 1) is equal to the work done by the gas in the adiabatic process (process 2) but is not zero.
- The work done by the gas in the isothermal process is larger than the work done by the gas in the adiabatic process.
- The work done by the gas in the isothermal process is less than the work done by the gas in the adiabatic process.
- The work done by the gas in both processes is zero.

You perform an experiment with a gas in a container involving a piston and obtain the cycle shown on the PV diagram below in which the cycle is traversed counterclockwise. Use this PV diagram to answer questions (6), (7), (8), and (9).



(7) Which one of the following statements is true regarding the net work done by the gas for one complete cycle shown?

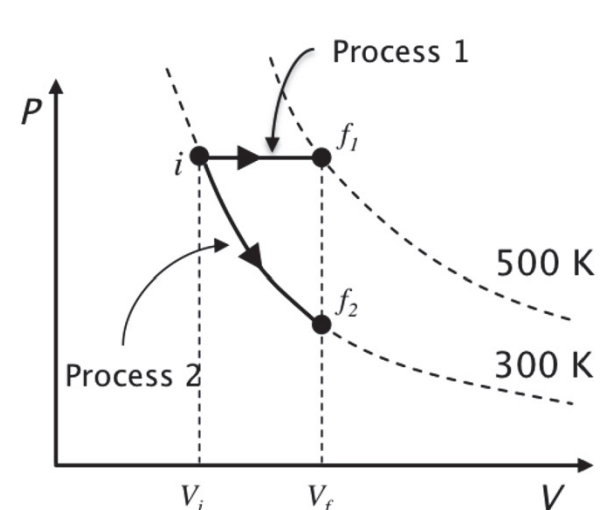
- The net work done by the gas is positive.
- The net work done by the gas is negative.
- The net work done by the gas is zero.
- Not enough information.

For a given thermodynamic system, choose all of the following quantities whose values are determined by the state of the system and not by the process that led to that state.

(46) Work done by the system.

- Yes.
- No.

You carry out two experiments each with one mole of an ideal monatomic gas starting at the same point i shown on the PV diagram below. The two curved dashed lines on the PV diagram show two different isotherms at 300K and 500K. Process 1 is a constant pressure process starting at 300K and ending at point f_1 at 500K whereas process 2 (which starts at the same point as process 1) proceeds along the 300K isotherm to point f_2 as shown below. Both processes end with the same final volume V_f . Answer questions (10) and (11) about these experiments.



(10) Which one of the following is true about the relation between the work done by the gas for the two processes?

- $W_1 < W_2$.
- $W_1 > W_2$.
- $W_1 = W_2$.
- Not enough information.

Quotations from the Interviews

On #10: "...ok, we have the same expansion though, so the work might be the same. 'Cause like the volume, the start volume is the same and the end volume is the same, so the work done is the same."

On #42: I would assume that the work done will be positive for process 1. Since the initial here is going up to the pressure to the final point 1."

On #58: "I think the work would be zero because in adiabatic processes, there's no heat transfer so I don't think it would require work to reach the final state."

On #59: "So the volume changes the same amount in both of them so they are equal, but not zero."

TABLE I. Response rates for items related to work for upper-level (Upper), and introductory calculus-based (Int-calc) and algebra-based (Int-alg) physics students. Correct responses are boldfaced and underlined. Item 46 is a true/false question.

Item #	A	B	C	D	Level
7	31%	51%	16%	2%	Upper
	31%	46%	21%	2%	Int-calc
	28%	18%	51%	3%	Int-alg
46	10%	90%	-	-	Upper
	26%	74%	-	-	Int-calc
	32%	68%	-	-	Int-alg
10	9%	88%	3%	0%	Upper
	18%	72%	9%	1%	Int-calc
	30%	58%	10%	2%	Int-alg
42	9%	7%	84%	0%	Upper
	15%	10%	74%	1%	Int-calc
	38%	14%	46%	2%	Int-alg
43	83%	12%	4%	0%	Upper
	74%	16%	9%	1%	Int-calc
	60%	21%	17%	2%	Int-alg
57	3%	89%	7%	1%	Upper
	10%	64%	24%	1%	Int-calc
	15%	52%	33%	1%	Int-alg
58	6%	87%	7%	1%	Upper
	10%	61%	27%	2%	Int-calc
	13%	44%	41%	2%	Int-alg
59	9%	78%	12%	1%	Upper
	17%	54%	26%	3%	Int-calc
	14%	38%	44%	3%	Int-alg

Sample Sizes

Upper: 89
 Int-calc: 492
 Int-alg: 550

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