College physics students' mathematical difficulties suggest need for awareness and action at the high school level

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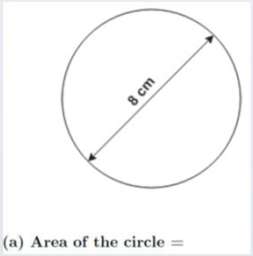
SUMMARY

- We have administered written diagnostic tests to over 5000 students enrolled in introductory physics courses at three state universities.
- Students' responses to high-school-level questions on trigonometry, algebra, geometry, and graphing consistently reflected a large number of operational errors, to a degree that could significantly interfere with success in an introductory physics course.
- Use of symbols to replace numbers significantly lowered students' correct-response rate.
- During interviews, students tended to self-correct approximately 60% of their initial errors, suggesting many errors are "careless."
- Students with better overall performance had *higher* rates of "careless" errors not traceable to inability to perform basic algebra.
- Results from Arizona State University, University of Colorado, and Ohio State University were consistent with each other.

STUDENT PERFORMANCE ON CALCULATING AREA OF CIRCLE



Correct-Response Rate, Algebra- and Calculus-Based Courses Combined (% correct responses)

ASU Polytechnic campus (*N* = 287): 56% [with units: 24%]

ASU Tempe campus (*N* = 1767): 77% [with units: 42%]

Univ. of Colorado [algebra-based] (*N* = 383): 82% [with units: 34%]

Notes:

- Most students provide incorrect units, or no units
- Little difference between algebra- and calculus-based courses
- Confusion between radius and diameter was ***NOT*** most-common error

STUDENT PERFORMANCE ON TRIGONOMETRY PROBLEMS

Correct-Response Rate, #1 and #4 combined

ASU Polytechnic campus

Algebra-based course, 1st semester, (*N* ~ 380): 34%

Calculus-based course, 1st semester, (*N* ~ 340): 56%

ASU Tempe campus

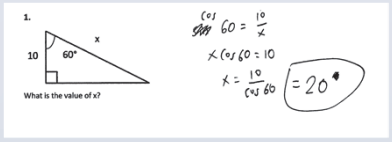
Calculus-based course, 1st semester, (*N* ~ 1500): 78%

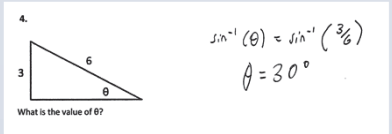
University of Colorado

Algebra-based course, 1st semester, (*N* ~ 290): 51%

Trigonometry Questions

with samples of correct student responses



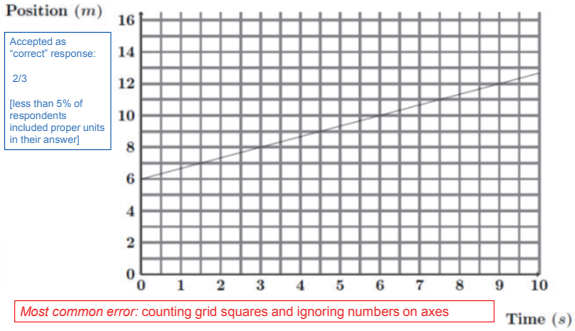


20-60% *of students confused on basic trigonometry relations*

STUDENT PERFORMANCE ON GRAPHING PROBLEMS

What is the slope of the graph below?

Correct-response rate (*N* > 2000): 30-60%, nearly independent of course or campus



Accepted as "correct" response:

2/3

[less than 5% of respondents included proper units in their answer]

Most common error: counting grid squares and ignoring numbers on axes

STUDENT PERFORMANCE ON ALGEBRA PROBLEMS

Our Findings: Significantly worse performance on "symbolic" versions of single-equation and simultaneous-equations problems, compared to "numeric" versions

Summary of the Simultaneous-Equations Data (three campuses)

- *Algebra-based course:* ~30-70% correct on numeric versions, ~10-55% on symbolic versions;
- *Calculus-based course:* ~55-90% correct on numeric versions, ~30-65% correct on symbolic versions.

Algebra: Simultaneous Equations

$0.5y = 2x$
$78.4 - y = 8x$ [Solve for x] Numeric Version

Correct-Response Rate (% correct responses)

Algebra-based course, 1st semester,
ASU Polytechnic campus (*N* = 179): 43%
ASU Tempe campus (*N* = 423): 60%
University of Colorado, Boulder (*N* = 180): 72%

Calculus-based course, 1st semester,
ASU Tempe campus (*N* =1205): 79%

$cy = dx$
$a - y = bx$ [Solve for x] Symbolic Version

Correct-Response Rate (% correct responses)

Algebra-based course, 1st semester,
ASU Polytechnic campus (*N* = 136): 13%
ASU Tempe campus (*N* = 326): 31%
University of Colorado, Boulder (*N* = 167): 46%

Calculus-based course, 1st semester,
ASU Tempe campus (*N* =1029): 55%

Why the Difficulties with Symbols? Some Hints From the Interviews

- In elementary math courses, simplified forms of equations are emphasized (i.e., few messy symbols and functions)
- Students get "overloaded" by seeing all the variables, and are unable to carry out procedures that they can do successfully with numbers.
- Results indicate that mathematical difficulties and confusion with symbols can pose significant obstacles to physics students' problem-solving performance.

Algebra: Math problem stripped of physics context

14. $v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{v_1}{t_1}$

$v = \frac{v_1}{2}$

$d = ?$

(Please clearly *circle* your answer and show all work.)

A. $d = v_1 t_1$ B. $d = \frac{v_1 t_1}{2}$ C. $d = \frac{v_1 t_1}{4}$ D. $d = \frac{v_1 t_1}{8}$ E. $d = \frac{v_1 t_1}{16}$

Adapted from Torigoe and Gladding, Am. J. Phys. **79**, 133 (2011)

Correct-Response Rate (% correct responses)

ASU Tempe campus,
Algebra-based course: 37% (*N* = 461)
Calculus-based course: 67% (*N* = 1227)

University of Colorado,
Algebra-based course: 65% (*N* = 191)

SUMMARY: IMPLICATIONS OF FINDINGS

- High consistency among four different campuses (see three below) suggests our findings are representative of much larger population
- Current high school preparation often not sufficient; we are testing an online instructional tool to provide skill practice in physics courses