What should physics teachers know about students' math difficulties?

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SUMMARY

- We have administered written and online diagnostic tests covering topics in pre-college-level mathematics to over 6700 students enrolled in introductory physics courses at four state universities.
- Students' responses consistently reflected a large number of operational errors, to a degree that could significantly interfere with success in an introductory physics course.
- Results from Arizona State University, University of Colorado, Ohio State University, and the University of West Florida were consistent with each other.
- We have adjusted our own instruction based on our findings, and we offer some ideas that prospective physics teachers may find useful.

POTENTIALLY ACTIONABLE FINDINGS

- Use of symbols to replace numbers in otherwise identical algebraic equations significantly lowered students' correct-response rate.

  Implication: Instructors may choose to be much more cautious in using symbolic manipulation to explain or demonstrate concepts.
- Virtually no physics students tended to solve algebraic equations by "isolating the unknown variable"; (semi-arithmetic methods were favored instead)

  Implication: Physics instructors' standard and habitual approach to algebraic manipulation may appear foreign and confusing to their introductory students.
- Students' errors on specific topics were highly correlated with errors on other, disparate topics (e.g., trigonometry, geometry, graphing, algebra).

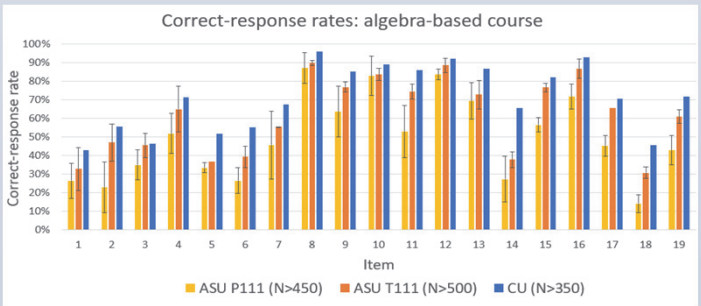
  Implication: Even a successful intervention to address students' difficulties in one specific mathematical area may be unlikely to have much impact on their difficulties in other areas.
- Class-average scores on even a *single* diagnostic test item—regardless of which item was chosen—were highly predictive of average scores on 13 other diagnostic items covering varied topics.

  Implication: It may be possible to diagnose the level of students' difficulties with only one or very few mathematics pretest items.
- Many students in both algebra- and calculus-based physics courses were extremely weak in handling units: they ignored units on graph-axis labels, and provided no or incorrect units for area and velocity.

  Implication: Instructors may not fully appreciate the degree to which many physics students are challenged in handling units.
- During interviews, students tended to self-correct approximately 60% of their initial errors, suggesting many errors are "careless."

  Implication: Instruction on error-detecting, checking, and self-correcting strategies may offer disproportionately high returns in helping students address their mathematical difficulties.

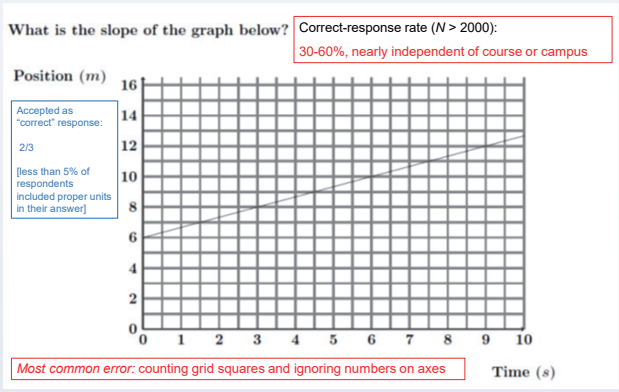
High consistency of results among five campuses at four different universities (three campuses shown below) suggests findings are generalizable.

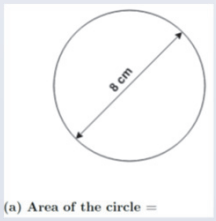


Algebra: Simultaneous Equations

Correct response rates ≈25% lower on "symbolic" versions.

$0.5y = 2x$ $78.4 - y = 8x$	[Solve for x]	Numeric Version	79% correct (N = 1043)
$cy = dx$ $a - y = bx$	[Solve for x]	Symbolic Version	55% correct (N = 862)

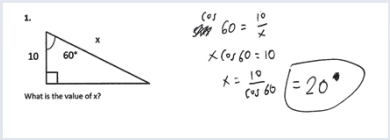


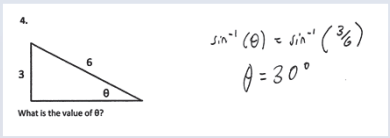


Area of Circle: Algebra- and Calculus-based courses combined, 2018

ASU-Poly: 57% correct (N = 250)
ASU-Tempe: 76% correct (N = 1086)

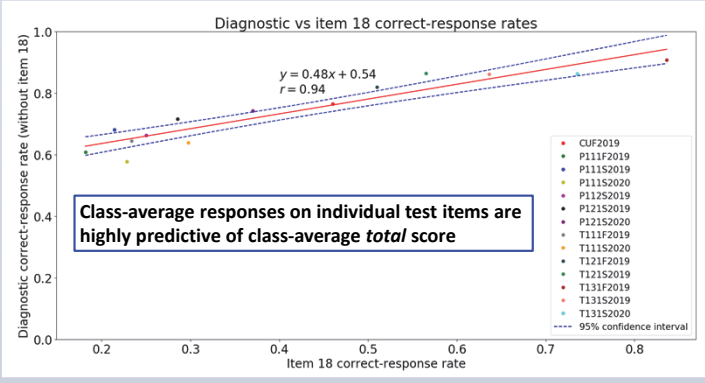
…with correct units: 29% and 45% correct, respectively





Trigonometry Problems:
Algebra-based course: 20-55% correct

18. $cy = dx$
$a - y = bx$
$x = ?$



Class-average responses on individual test items are highly predictive of class-average *total* score