Nature of Students' Mathematical Difficulties in Introductory Physics Courses

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Outline

- Weak skills with basic pre-college mathematics can severely impact physics students' course performance
- We have explored the nature and prevalence of physics students' difficulties with elementary mathematics, using "stripped-down" problems with little or no physics context

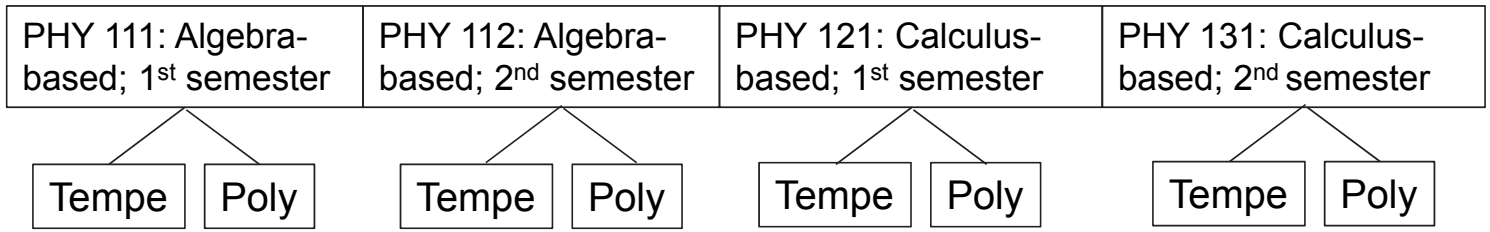
Work to Date

- Administer (and analyze) written diagnostic quiz, given to > 4000 students in ≈ 30 algebra- and calculus-based physics classes over seven semesters at Arizona State University during 2016-2019; calculators *are* allowed
- Carry out individual interviews with 75 students enrolled in those or similar courses during same period (Primary interviewer: Matt Jones)
- Topics: trigonometry, algebra, vectors, graphing, geometry

Our 4 Sample Populations

PHY 111: Algebra-based; 1st semester	PHY 112: Algebra-based; 2nd semester	PHY 121: Calculus-based; 1st semester	PHY 131: Calculus-based; 2nd semester

Our 8 Sample Populations



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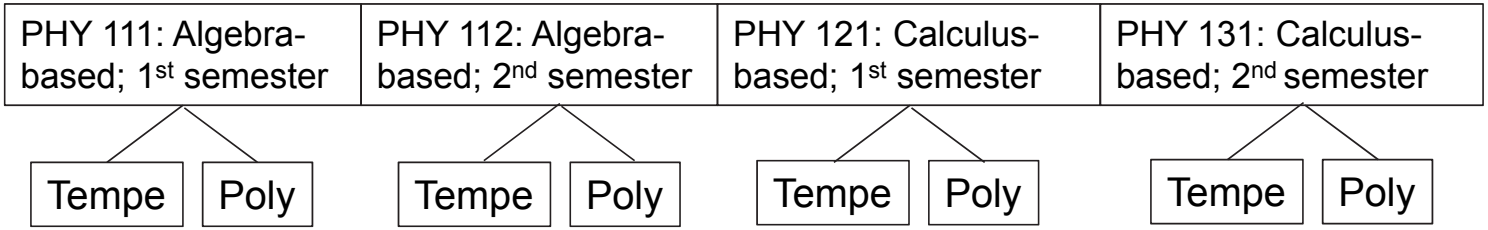
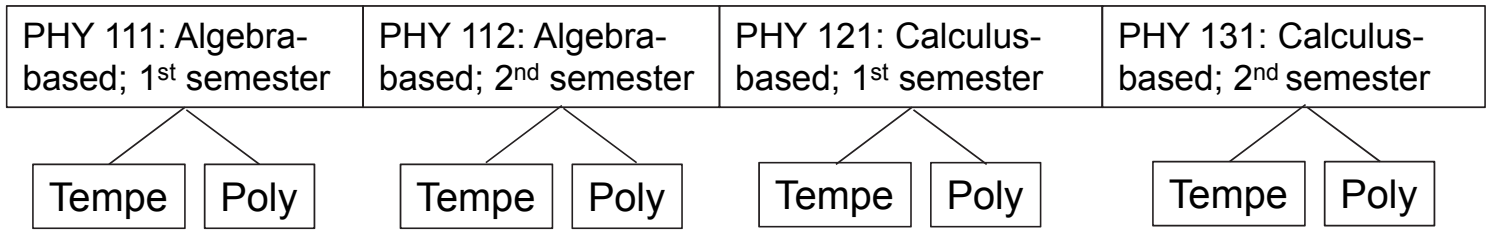
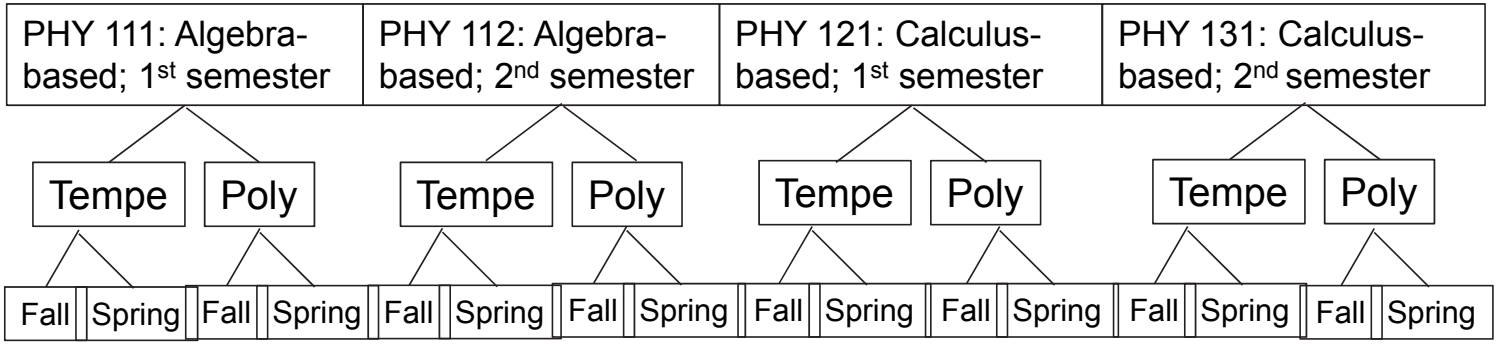


On average, students in the Tempe courses have more extensive background and preparation (and different majors) than those in the corresponding Poly courses.

Our 8 Sample Populations



Our 16 Sample Populations



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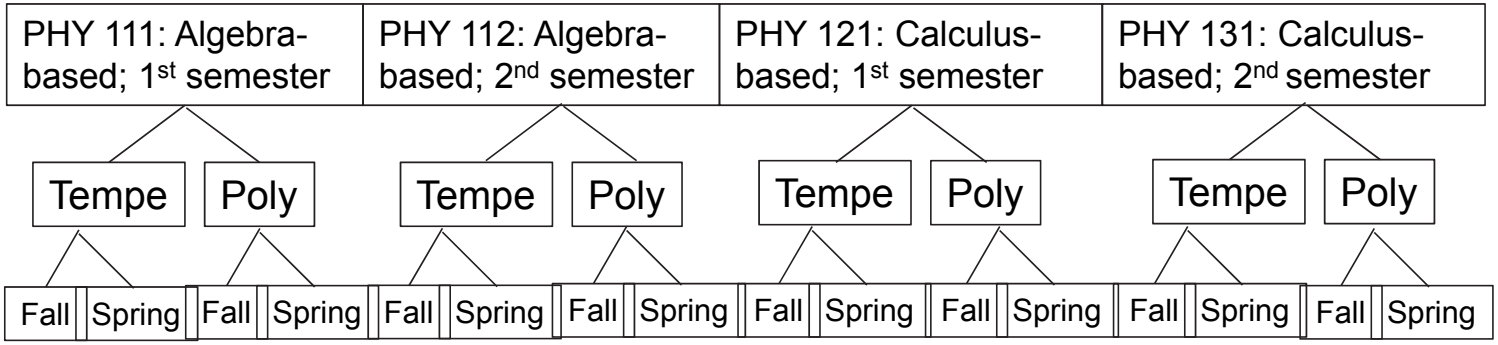


Each of the 16 sample populations has distinct and consistent differences from all of the others!

Primary Findings

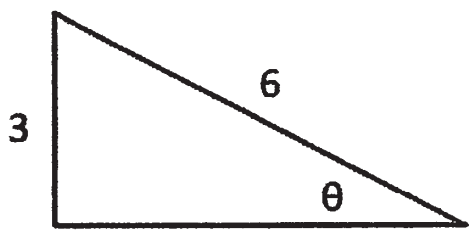
Regardless of course (algebra- or calculus-based), campus (Tempe or Poly), or semester (Spring or Fall):

- Difficulties with basic mathematical operations are widespread; average error rates range from 20-70%;
- Performance on algebraic problems using symbols for constants is significantly worse than on problems using numbers;
- During problem-solving interviews, students self-correct approximately 50% of errors following minimal prompts.

Variations in Student Performance

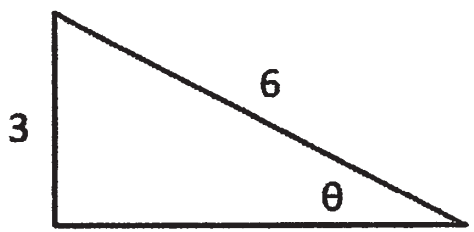
- Even in the same course, diagnostic scores on individual test items can vary substantially from one year to the next
- Variations are often larger than expected based purely on statistical uncertainty
- *Example:* Class Size = 100, score = 50%, 80%-confidence interval is ±6.4% (based on binomial proportions)

“Find Unknown Angle”



What is the value of θ?

“Find Unknown Angle”



What is the value of θ?

$$\sin^{-1}(\theta) = \sin^{-1}\left(\frac{3}{6}\right)$$

$$\theta = 30^\circ$$

Performance Variations: *Examples*

- Same course, same campus, same semester, same question—three different years.

(Algebra-based physics, 1st semester, Polytechnic campus, spring semester; “find unknown angle”)

2016: 36% correct (N = 72)

2018: 47% correct (N = 88)

2019: 39% correct (N = 54)

Performance Variations: *Examples*

- Same course, same campus, same semester, same question—three different years.

(Calculus-based physics, 1st semester, Polytechnic campus, spring semester; “find unknown angle”)

2016: 70% correct (N = 105)

2017: 71% correct (N = 42)

2019: 42% correct (N = 69)

Performance Variations: *Examples*

- Same course, same campus, same semester, same question—three different years.

(Calculus-based physics, 1st semester, Tempe campus, spring semester; “find unknown angle”)

2017: 77% correct (N = 98)

2018: 87% correct (N = 903)

2019: 88% correct (N = 99)

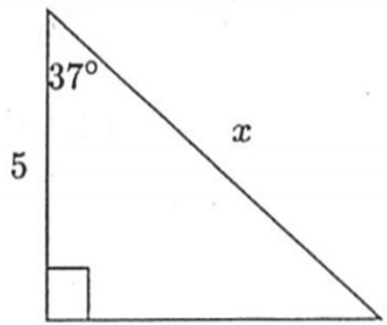
Significant Variations Between Campuses

Students at the Tempe and Polytechnic campuses:

- generally follow different majors with different course sequences
- have different average levels of preparation

“Find Unknown Side”

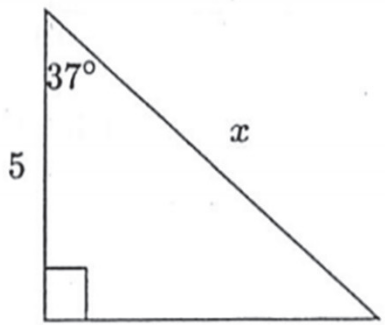
1. What is the length of side x?



“Find Unknown Side”

SOH CAH TOA

1. What is the length of side x?



$$\cos(37^\circ) = \frac{5}{x}$$

$$x\cos(37^\circ) = 5$$

$$x = \frac{5}{\cos(37^\circ)}$$

$$x = 6.26$$

Comparison: Tempe campus vs. Polytechnic campus

Take average of (fall + spring) semesters; Year 2018:

Algebra-based course, 1st semester

Example #1: Find unknown side of triangle

Polytechnic campus: 25% correct (N = 121)

Tempe campus: 57% correct (N = 376)

Example #2: Find unknown angle of triangle

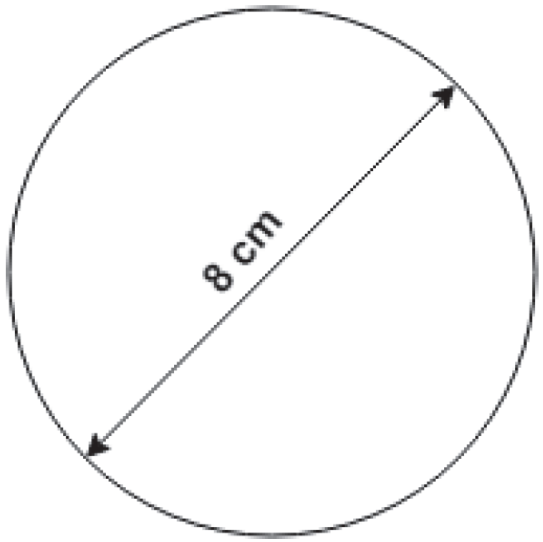
Polytechnic campus: 35% correct (N = 152)

Tempe campus: 52% correct (N = 533)

Superior performance on Tempe campus

Some Basic Operations: Area, Graphing, Algebra

- Find area of circle
- Find slope of graph
- Solve two simultaneous equations



(a) Area of the circle =

[$Area = \pi r^2$=16π cm^2]

Algebra- and Calculus-based Courses Combined
(% correct responses)

Polytechnic campus: 57% (*N* = 250)
(5 classes, range 48-61%)

….with correct units: 29%

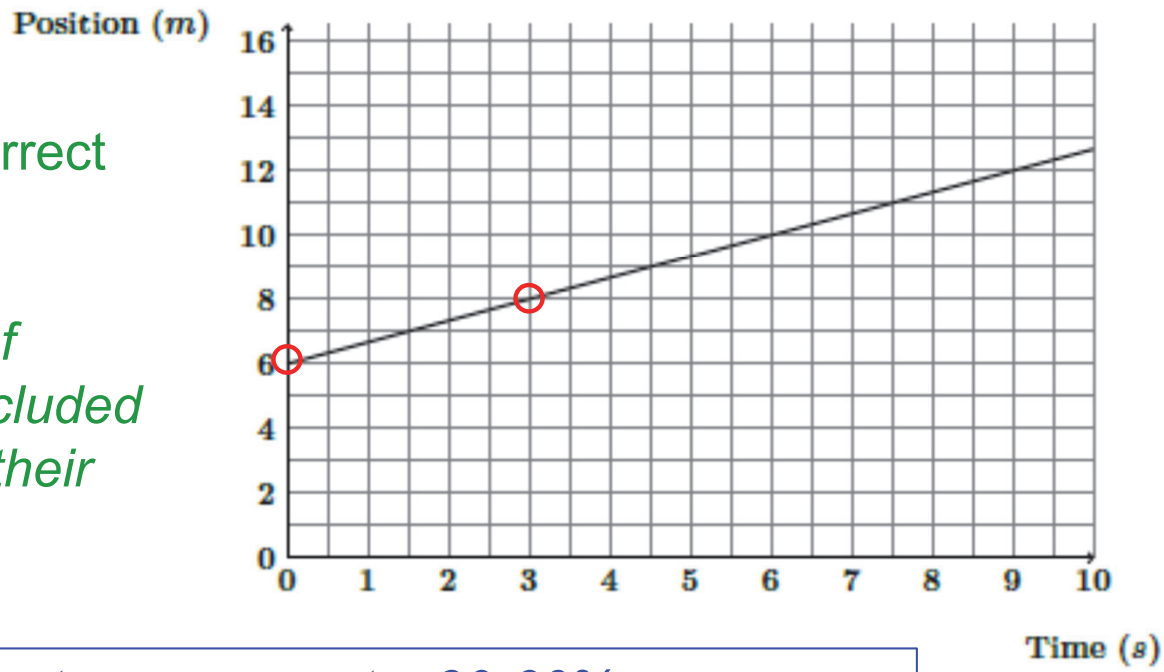
Tempe campus: 76% (*N* = 1086)
(5 classes, range 74-79%)

….with correct units: 45%

- **Little difference between algebra- and calculus-based courses**
- **Interchanging radius and diameter was *NOT* most-common error**

What is the slope of the graph below?

Answer: 2/3 m/s



Accepted as correct response: “2/3”

[less than 5% of respondents included proper units in their answer]

Correct-response rate: 30-60% (N > 2000); nearly independent of course or campus

Most common error: counting grid squares and ignoring numbers on axes

From Torigoe and Gladding (2011):

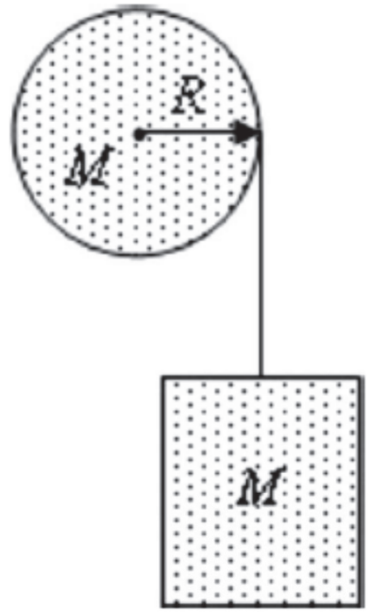


Fig. 7. Diagram for question 10.

Question 10 (numeric). A uniform disk of mass $M=8$ kg and radius $R=0.5$ m has a string wound around its rim. The disk is free to spin about a pin through the center of the disk. A mass $M=8$ kg (same mass as the disk) is connected to the string and is dropped from rest. What is the acceleration a of the block? (See Fig. 7.)

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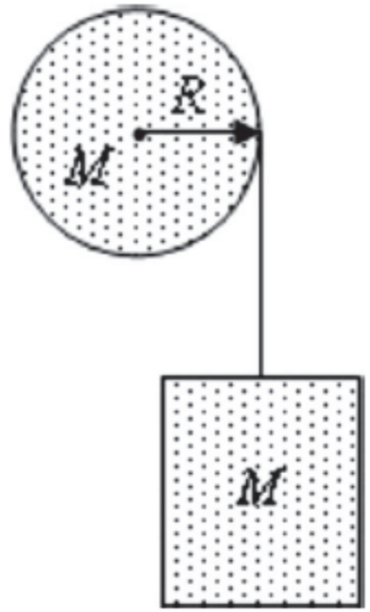

Fig. 7. Diagram for question 10.

$$Mg - T = Ma$$
$$TR = I\alpha$$
$$[I = \tfrac{1}{2} MR^2;\ \alpha = a/R]$$

…→

$$Mg - T = Ma$$
$$TR = [\tfrac{1}{2} MR^2][a/R]$$
$$a = ?$$

$$78.4 - y = 8x$$
$$0.5y = 2x$$
$$x = ?$$

Our Numeric version

Question 10 (numeric). A uniform disk of mass $M=8$ kg and radius $R=0.5$ m has a string wound around its rim. The disk is free to spin about a pin through the center of the disk. A mass $M=8$ kg (same mass as the disk) is connected to the string and is dropped from rest. What is the acceleration a of the block? (See Fig. 7.)

“Symbolic” Version

$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

“Symbolic” Version

$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

$$y = a - bx$$

$$c(a - bx) = dx$$

$$ca - cbx = dx$$

$$ca = (cb + d)x$$

$$x = \frac{ca}{cb + d}$$

Algebra: Simultaneous Equations

Algebra-based course, 1st semester (% correct; 2018 fall + spring average)

What is the numerical value of x?

$$78.4 - y = 8x$$

$$0.5y = 2x$$

Numeric version

Polytechnic campus: 40% (N = 104)

Tempe campus: 61% (N = 335)

≈20% higher correct-response-rate at Tempe on both versions;

$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

Symbolic version

Polytechnic campus: 10% (N = 63)

Tempe campus: 32% (N = 241)

≈30% lower correct-response-rate on symbolic version on both campuses

Algebra: Simultaneous Equations

Calculus-based course, 1st semester (% correct; 2018 fall + spring average)

What is the numerical value of x?

$$78.4 - y = 8x$$

$$0.5y = 2x$$

Numeric version

Tempe campus: 79% (N = 1043)

18% higher correct-response-rate than in algebra-based course

$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

Symbolic version

Tempe campus: 55% (N = 862)

24% lower correct-response-rate on symbolic version

Possible Origins of Errors

- We assume several different possible sources for students' errors:
  - Difficulty with operations: Inadequate learning or expertise with fundamental operations
  - Difficulty accessing knowledge: Students don't connect context of problem to context in which operations were learned
  - "Careless" errors, due to simple inattention, lack of checking, etc.; can be corrected (in principle) by greater attentiveness.
    - (*Note:* ≈50% of errors were "self-corrected" during interviews)
- Through interviews and diagnostic items, we probe these items.

Diagnostics for Operations with Fractions

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$$2\left(\frac{a}{b}\right) = ?$$

A. $\frac{2a}{b}$ B. $\frac{2a}{2b}$ C. $\frac{a}{2b}$

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$$\frac{a/b}{c^2/d} = ?$$

$\frac{ad}{bc^2}$

A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

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$$\left(\frac{a}{3}\right)^3 = ?$$

A. $\frac{a^3}{3}$ B. $\frac{a}{27}$ C. $\frac{a^3}{27}$

Summary: Implications for Instruction

- Difficulties due to **skill-practice deficits** might be addressed by brief out-of-class assignments (e.g., Mikula and Heckler, 2017)

- Difficulties due to **“carelessness”** might be addressed by, e.g., guiding students to carefully check and re-check key steps in their calculation.