Physics Students' Difficulties with Introductory Mathematics

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Outline

- Weak skills with basic pre-college mathematics can severely impact physics students' course performance
- We have explored the nature and prevalence of physics students' difficulties with elementary mathematics, using "stripped-down" problems with little or no physics context

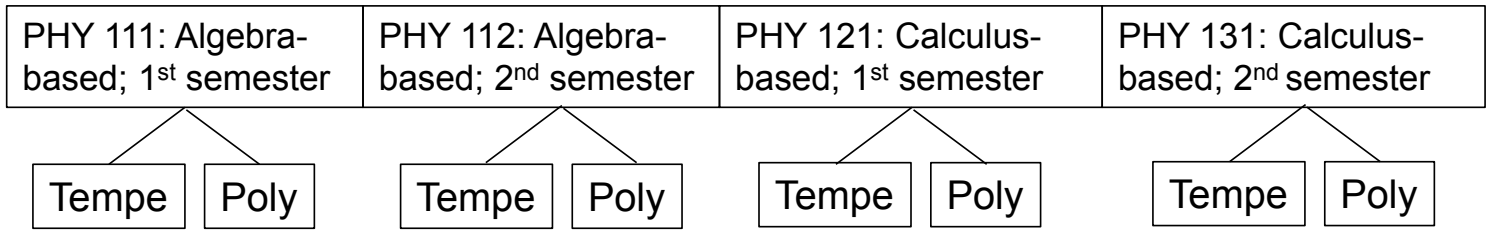
Work to Date

- Administer (and analyze) written diagnostic quiz, given to > 4000 students in ≈ 30 algebra- and calculus-based physics classes over seven semesters at Arizona State University during 2016-2019; calculators *are* allowed
- Carry out individual interviews with 75 students enrolled in those or similar courses during same period (Primary interviewer: Matt Jones)
- Topics: trigonometry, algebra, vectors, graphing, geometry
- Comparison data: University of Colorado, algebra-based course

Our 4 Sample Populations

PHY 111: Algebra-based; 1st semester	PHY 112: Algebra-based; 2nd semester	PHY 121: Calculus-based; 1st semester	PHY 131: Calculus-based; 2nd semester

Our 8 Sample Populations



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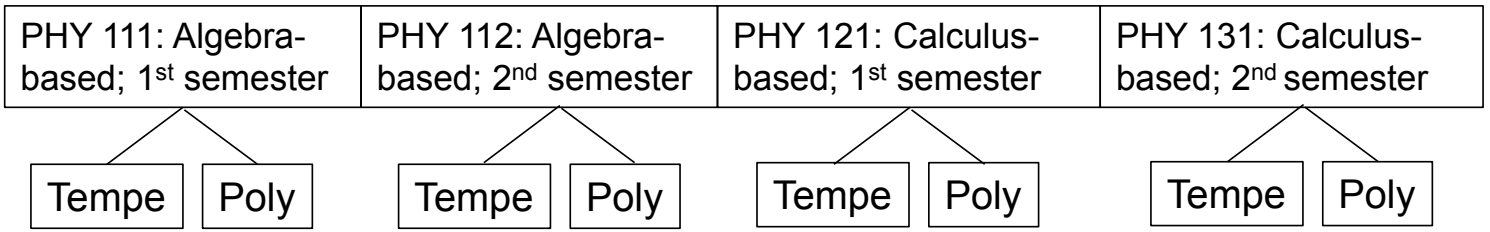
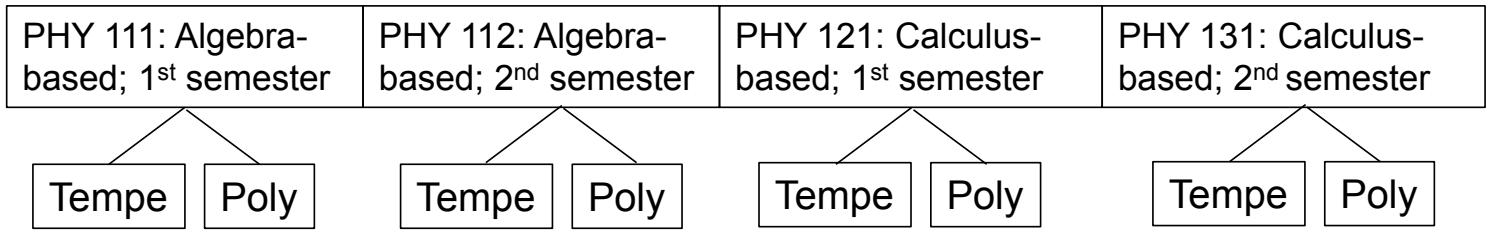
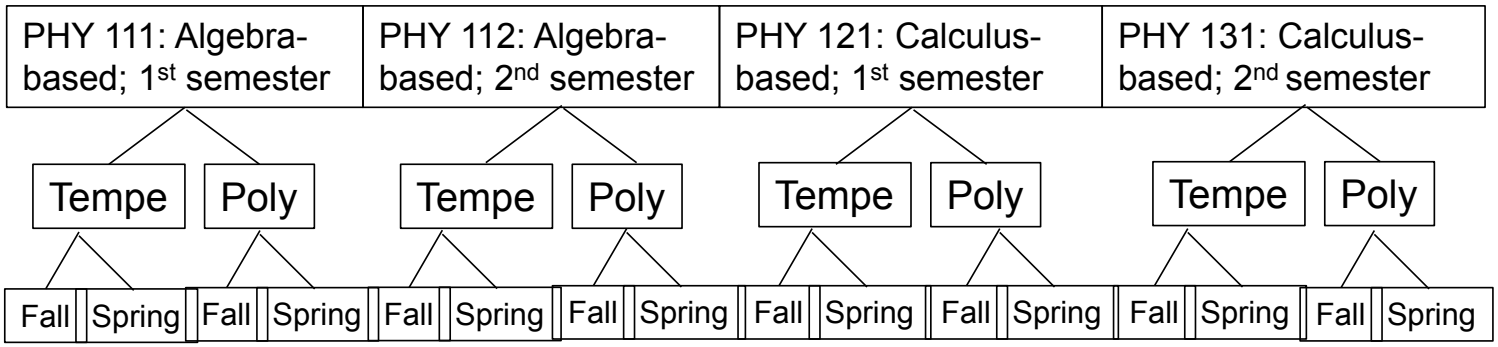


On average, students in the Tempe courses have more extensive background and preparation (and different majors) than those in the corresponding Poly courses.

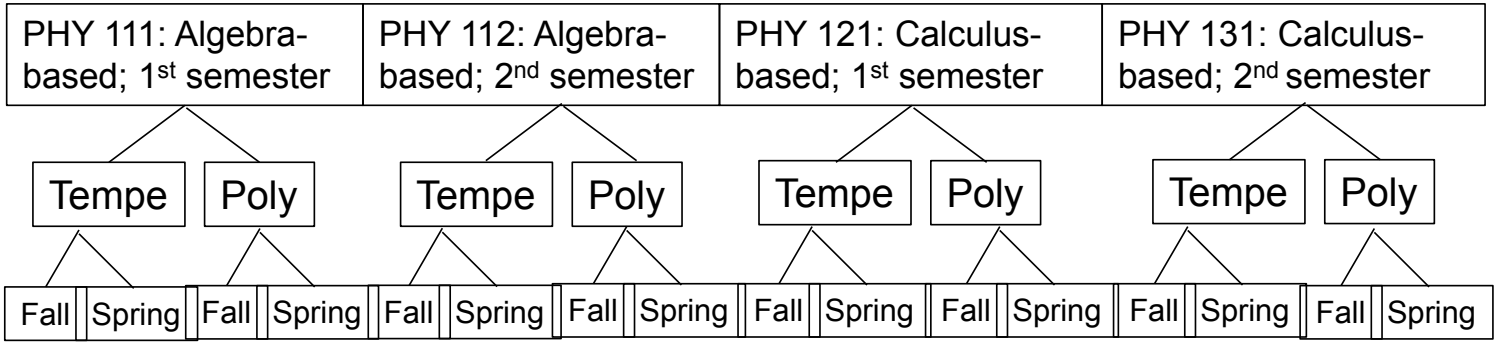
Our 8 Sample Populations



Our 16 Sample Populations

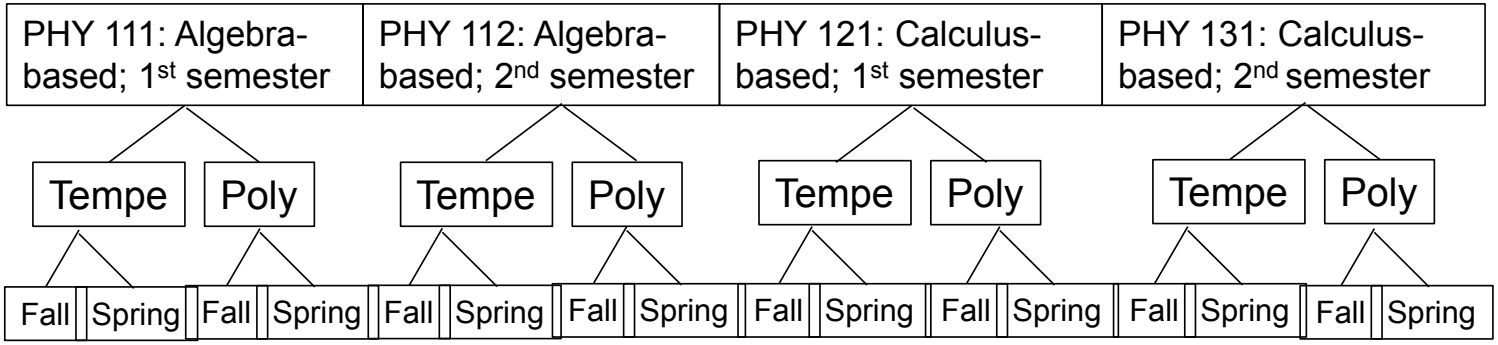


Our 16 Sample Populations



Each of the 16 sample populations has distinct and consistent differences from all of the others!

Our 16 Sample Populations



Our data-collection period (2016-19) included 4 spring semesters and 4 fall semesters, and several of the 16 populations were only sampled once or twice, some with low *N*. So, even with total *N* > 4000, confirmation of consistent patterns is challenging.

Primary Findings

Regardless of course (algebra- or calculus-based), campus (Tempe or Poly), or semester (Spring or Fall):

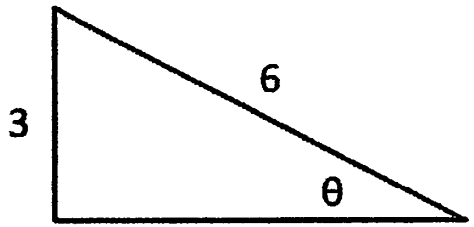
- Difficulties with basic mathematical operations are widespread; average error rates range from 20-70%;
- Performance on algebraic problems using symbols for constants is significantly worse than on problems using numbers;
- During problem-solving interviews, students self-correct approximately 50% of errors following minimal prompts.

Variations in Student Performance

- Even in the same course, diagnostic scores on individual test items can vary substantially from one year to the next
- Variations are often larger than expected based purely on statistical uncertainty

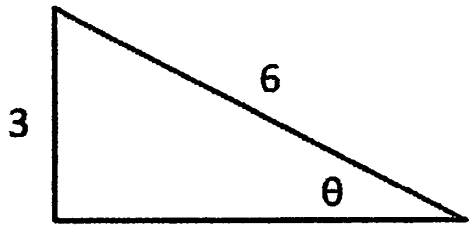
- *Example:* Class Size = 100, score = 50%, 80%-confidence interval is ±6.4% (based on binomial proportions)

“Find Unknown Angle”



What is the value of θ?

“Find Unknown Angle”



What is the value of θ?

$\sin^{-1}(\theta) = \sin^{-1}\left(\frac{3}{6}\right)$

$\theta = 30^{\circ}$

Performance Variations: *Examples*

- Same course, same campus, same semester, same question—three different years.

  (Algebra-based physics, 1st semester, Polytechnic campus, spring semester; “find unknown angle”)

2016: 36% correct (*N* = 72)

2018: 47% correct (*N* = 88)

2019: 39% correct (*N* = 54)

Performance Variations: *Examples*

- Same course, same campus, same semester, same question—three different years.

  (Calculus-based physics, 1st semester, Tempe campus, spring semester; "find unknown angle")

2017: 77% correct (*N* = 98)

2018: 87% correct (*N* = 903)

2019: 88% correct (*N* = 99)

Performance Variations: *Examples*

- Same course, same campus, same semester, same question—three different years.

  (Calculus-based physics, 1st semester, Polytechnic campus, spring semester; "find unknown angle")

2016: 70% correct (*N* = 105)

2017: 71% correct (*N* = 42)

2019: 42% correct (*N* = 69)

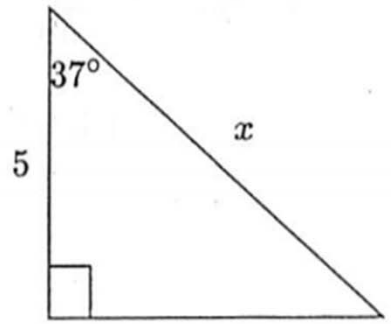
Significant Variations Between Campuses

Students at the Tempe and Polytechnic campuses:

- generally follow different majors with different course sequences
- have different average levels of preparation

“Find Unknown Side”

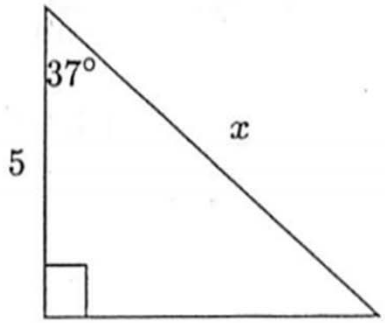
1. What is the length of side x?



“Find Unknown Side”

SOH CAH TOA

1. What is the length of side x?



$$\cos(37^\circ) = \frac{5}{x}$$

$$x\cos(37^\circ) = 5 \qquad x = 6.26$$

$$x = \frac{5}{\cos(37^\circ)}$$

Comparison: Tempe campus vs. Polytechnic campus

Take average of (fall + spring) semesters; Year 2018:

Algebra-based course, 1st semester

Example #1: Find unknown side of triangle

Polytechnic campus: 25% correct (*N* = 121)

Tempe campus: 57% correct (*N* = 376)

Example #2: Find unknown angle of triangle

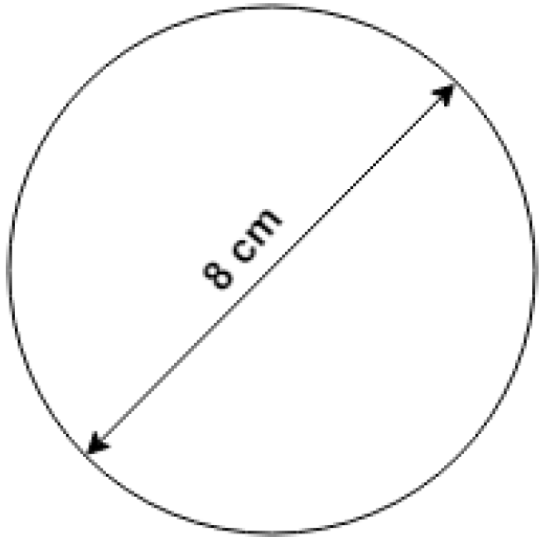
Polytechnic campus: 35% correct (*N* = 152)

Tempe campus: 52% correct (*N* = 533)

Superior performance on Tempe campus

Some Basic Operations: Area, Graphing, Algebra

- Find area of circle
- Find slope of graph
- Solve two simultaneous equations



(a) Area of the circle =

[$Area = \pi r^2$=16π cm^2]

Algebra- and Calculus-based Courses Combined
(% correct responses)

Polytechnic campus: 57% (*N* = 250)
(5 classes, range 48-61%)

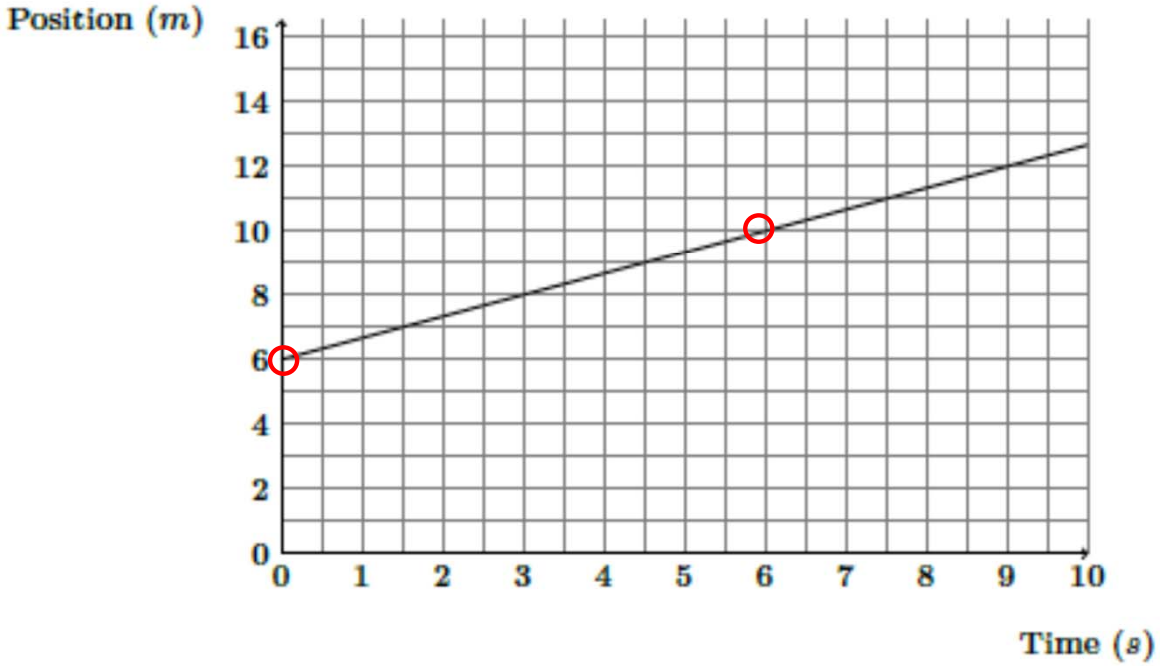
….with correct units: 29%

Tempe campus: 76% (*N* = 1086)
(5 classes, range 74-79%)

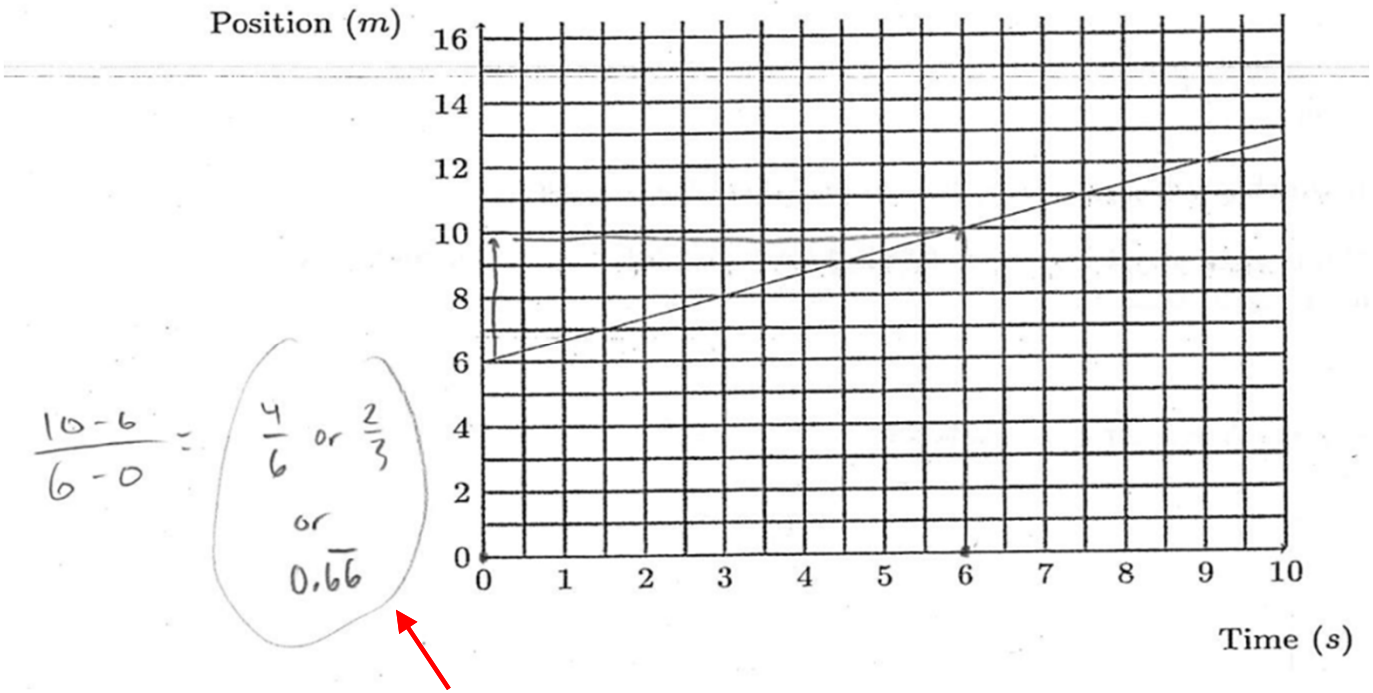
….with correct units: 45%

- **Little difference between algebra- and calculus-based courses**
- **Interchanging radius and diameter was *NOT* most-common error**

What is the slope of the graph below?



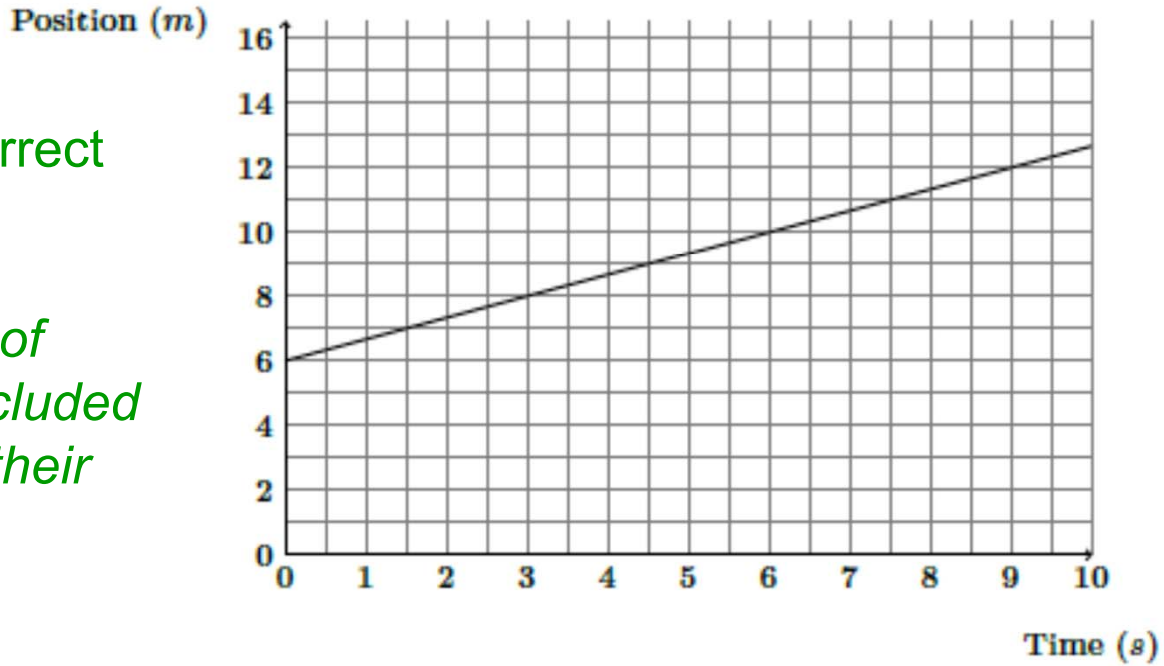
[Some physics content]



Note: Units missing

What is the slope of the graph below?

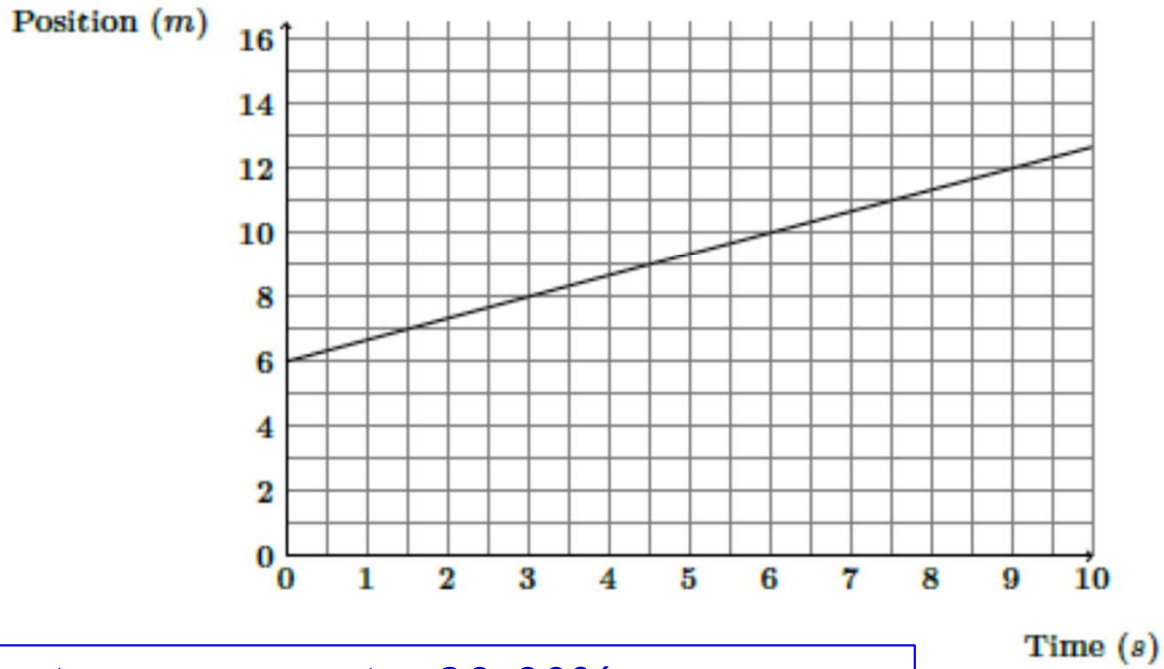
Answer: 2/3 m/s



Accepted as correct response: “2/3”

[less than 10% of respondents included proper units in their answer]

What is the slope of the graph below?



Correct-response rate: 30-60% (N > 2000); nearly independent of course or campus

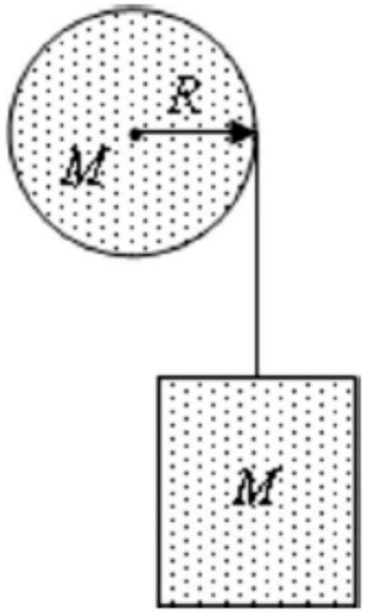
Most common error: counting grid squares and ignoring numbers on axes

Algebra: Simultaneous Equations

- Do differences in students' success rate between numerical and symbolic versions of same problem persist when simultaneous equations are involved? (E.g., two equations, two unknowns)

From Torigoe and Gladding (2011):

$F_{net} = ma$
$\tau_{net} = I\,\alpha$



$Mg - T = Ma$
$TR = I\alpha$
$[I = ½\ MR^2;\ \alpha = a/R]$

…→

$Mg - T = Ma$
$TR = [½\ MR^2][a/R]$
$a = ?$

Fig. 7. Diagram for question 10.

Question 10 (numeric). A uniform disk of mass $M=8$ kg and radius $R=0.5$ m has a string wound around its rim. The disk is free to spin about a pin through the center of the disk. A mass $M=8$ kg (same mass as the disk) is connected to the string and is dropped from rest. What is the acceleration a of the block? (See Fig. 7.)

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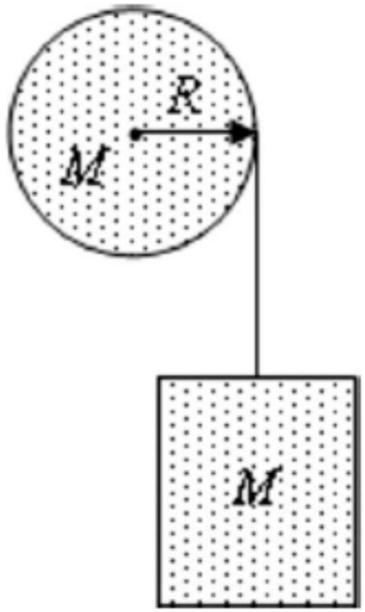

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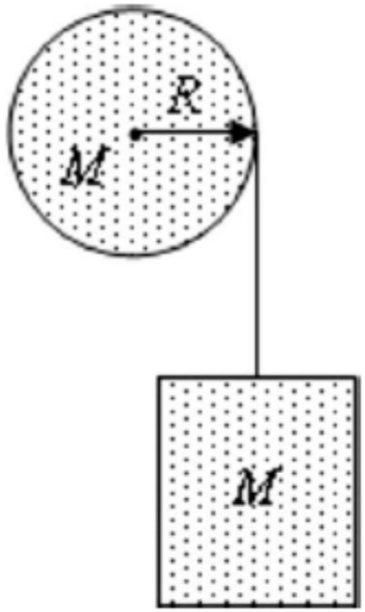
$Mg - T = Ma$
$TR = I\alpha$
$[I = \frac{1}{2} MR^2; \alpha = a/R]$

$\ldots\rightarrow$

$Mg - T = Ma$
$TR = [\frac{1}{2} MR^2][a/R]$
$a = ?$

Numeric version

From Torigoe and Gladding (2011):



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$\ldots\rightarrow$

$Mg - T = Ma$
$TR = [\frac{1}{2} MR^2][a/R]$
$a = ?$

Symbolic version

Results on #10

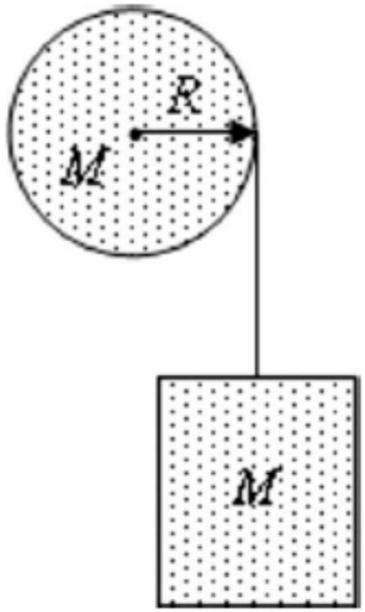
[Torigoe and Gladding, 2011]

- **Numeric version:** 49% correct ($N \approx 380$)
- **Symbolic version:** 53% correct ($N \approx 380$)

➡ *No significant difference*

("…because students are forced to use the same procedure to solve both the numeric and symbolic versions." Torigoe and Gladding, 2011)

From Torigoe and Gladding (2011):



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Symbolic version

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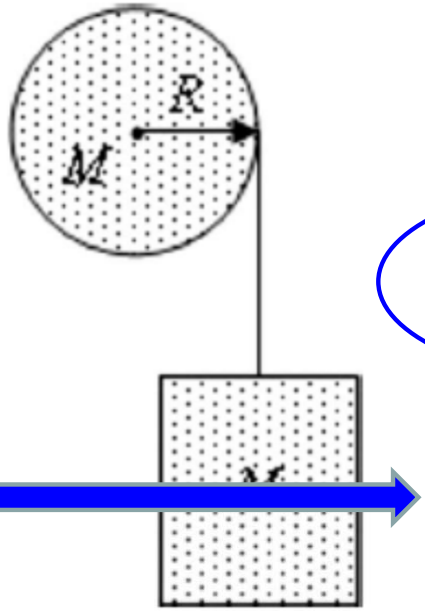
$TR = I\alpha$
$[I = \frac{1}{2} MR^2;\ \alpha = a/R]$

…→

$Mg - T = Ma$
$TR = [\frac{1}{2} MR^2][a/R]$
$a = ?$

Rename to simplify:

"Mg" → "a"
"M" → "b"
"R" → "c"
"$\frac{1}{2}MR$" → "d"
"T" → "y"
"a" → "x"



$a - y = bx$
$cy = dx$
$x = ?$

Our Symbolic version

Fig. 7. Diagram for question 10.

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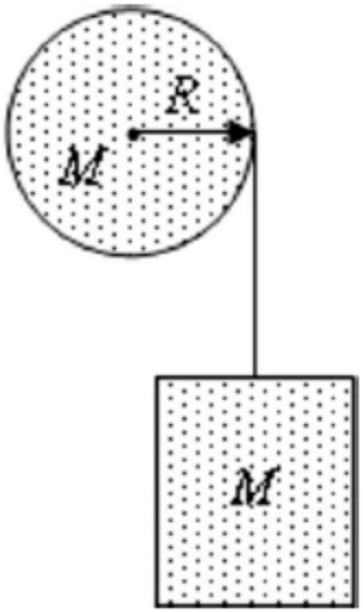

$Mg - T = Ma$

$TR = I\alpha$

$[I = \frac{1}{2} MR^2;\ \alpha = a/R]$

…→

$Mg - T = Ma$

$TR = [\frac{1}{2} MR^2][a/R]$

$a = ?$

$78.4 - y = 8x$

$0.5y = 2x$

$x = ?$

Our Numeric version

Fig. 7. Diagram for question 10.

Question 10 (numeric). A uniform disk of mass $M=8$ kg and radius $R=0.5$ m has a string wound around its rim. The disk is free to spin about a pin through the center of the disk. A mass $M=8$ kg (same mass as the disk) is connected to the string and is dropped from rest. What is the acceleration a of the block? (See Fig. 7.)

Results on Our Versions (Tempe)

Calculus-based course, 1st semester:

- **Numeric version:** 87% correct (*N* = 733)
- **Symbolic version:** 63% correct (*N* = 733)

➡ *Large and highly significant difference*

(Because [?] many of the students who can't do the physics, can do the math—but only when posed in numerical form)

“Symbolic” Version

$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

“Symbolic” Version

$cy = dx$

$a - y = bx$

$x = ?$

$y = a - bx$

$c(a - bx) = dx$

$ca - cbx = dx$

$ca = (cb + d)x$

$$x = \frac{ca}{cb + d}$$

Algebra: Simultaneous Equations

Algebra-based course, 1st semester (% correct; 2018 fall + spring average)

What is the numerical value of x?

$78.4 - y = 8x$

$0.5y = 2x$

Numeric version

Polytechnic campus: 40% (*N* = 104)

Tempe campus: 61% (*N* = 335)

≈20% higher correct-response-rate at Tempe on both versions;

$cy = dx$

$a - y = bx$

$x = ?$

Symbolic version

Polytechnic campus: 10% (*N* = 63)

Tempe campus: 32% (*N* = 241)

≈30% lower correct-response-rate on symbolic version on both campuses

Algebra: Simultaneous Equations

Calculus-based course, 1st semester (% correct; 2018 fall + spring average)

What is the numerical value of x?

$78.4 - y = 8x$

$0.5y = 2x$

Numeric version

Tempe campus: 79% (*N* = 1043)

18% higher correct-response-rate than in algebra-based course

$cy = dx$

$a - y = bx$

$x = ?$

Symbolic version

Tempe campus: 55% (*N* = 862)

24% lower correct-response-rate on symbolic version

Why the Difficulties with Symbols?

Some Suggestions Arising from the Interviews

- In elementary math courses, “simplified forms” of equations are emphasized (i.e., few messy symbols and functions).
- Many students get “overloaded” by seeing all the variables, and are unable to carry out procedures that they do successfully with numbers.
- Many students have had *insufficient practice* with algebraic operations to avoid being overwhelmed by standard algebraic manipulations.
  - Students tend to become *careless*

Possible Origins of Errors

- We assume several different possible sources for students' errors:
  - Difficulty with operations: Inadequate learning or expertise with fundamental operations
  - Difficulty accessing knowledge: Students don't connect context of problem to context in which operations were learned
  - "Careless" errors, due to simple inattention, lack of checking, etc.; can be corrected (in principle) by greater attentiveness.
    - (*Note:* ≈50% of errors were "self-corrected" during interviews)
- Through interviews and diagnostic items, we probe these items.

Diagnostics for Operations with Fractions

Diagnostics for Operations with Fractions

$2\left(\frac{a}{b}\right) = ?$

A. $\frac{2a}{b}$ B. $\frac{2a}{2b}$ C. $\frac{a}{2b}$

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$2\left(\frac{a}{b}\right) = ?$

A. $\frac{2a}{b}$ B. $\frac{2a}{2b}$ C. $\frac{a}{2b}$

$\frac{a/b}{c^2/d} = ?$ $\frac{ad}{bc^2}$

A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

Diagnostics for Operations with Fractions

$2\left(\frac{a}{b}\right) = ?$

A. $\frac{2a}{b}$ B. $\frac{2a}{2b}$ C. $\frac{a}{2b}$

$\frac{a/b}{c^2/d} = ?$ $\frac{ad}{bc^2}$

A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

$\left(\frac{a}{3}\right)^3 = ?$

A. $\frac{a^3}{3}$ B. $\frac{a}{27}$ C. $\frac{a^3}{27}$

“Symbolic” Versus “Numeric”

- Torigoe and Gladding (2007; 2011) investigated differences in students’ responses to physics problems in both numeric and symbolic form
  - “**Numeric**” and “**symbolic**” refer to the nature of the constant coefficients

Students' Difficulties with Symbols

Confusion of symbolic meaning: Students perform worse on solving problems when symbols are used to represent common physical quantities in equations, e.g., "*m*" instead of "1.5 kg" [Torigoe and Gladding, 2007; 2011)

Example [University of Illinois]:

Version #1: A car can go from 0 to 60 m/s in 8 s. At what distance *d* from the start at rest is the car traveling 30 m/s?

[93% correct]

Version #2: A car can go from 0 to v_1 in t_1 seconds. At what distance *d* from the start at rest is the car traveling (v_1/2)?

[57% correct]

Much worse!

From Torigoe & Gladding (2011):

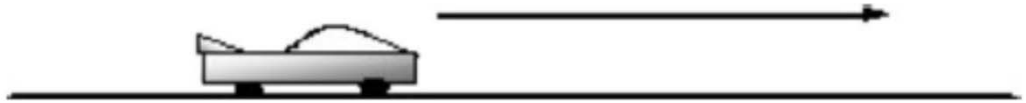


Fig. 5. Diagram for question 2.

Question 2 (numeric). A car can go from 0 to 60 m/s in 8 s. At what distance d from the start (at rest) is the car traveling 30 m/s? [Assume a constant acceleration (see Fig. 5).]

Question 2 (symbolic). A car can go from 0 to v_1 in t_1 seconds. At what distance d from the start (at rest) is the car traveling $(v_1/2)$? [Assume a constant acceleration (see Fig. 5).]

(numeric)

(a) 30 m
(b*) 60 m
(c) 120 m
(d) 240 m
(e) 480 m

(symbolic).

(a) $d=v_1t_1$
(b) $d=v_1t_1/2$
(c) $d=v_1t_1/4$
(d*) $d=v_1t_1/8$
(e) $d=v_1t_1/16$

Results on #2

[Torigoe and Gladding, 2007; 2011]

- **Numeric version:** 93% correct ($N \approx 380$)
- **Symbolic version*:** 57% correct ($N \approx 380$)

  *numerical values of v_1 and t_1 *not* provided

➡ *Highly significant difference*

Torigoe and Gladding (2011), Findings

1. Significantly higher proportion of correct responses on *some* types of numerical questions, not on others
2. Difference was greater for students in bottom quarter of class
3. Larger difference linked to difficulties with multiple and simultaneous equations, symbol confusion, and misuse of compound expressions.

“Symbolic” Versus “Numeric”

- Torigoe and Gladding (2007; 2011) investigated differences in students’ responses to physics problems in both numeric and symbolic form
  - “**Numeric**” and “**symbolic**” refer to the nature of the constant coefficients
- Starting in 2018, we asked paired problems in both symbolic and numeric form
  - Our problems were stripped of physics context and asked as pure math problems
  - Focused on trigonometry and algebra

Students' Difficulties with Symbols

Torigoe and Gladding (2011):

Numeric version:

A car can go from 0 to 60 m/s in 8 s. At what distance *d* from the start at rest is the car traveling 30 m/s?

Symbolic version:

A car can go from 0 to v_1 in t_1 seconds. At what distance *d* from the start at rest is the car traveling (v_1/2)?

Kinematic Equation Problem

Numeric:

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{\Delta v}{\Delta t}$

$\Delta v = 60$

$\Delta t = 8$

$v = 30$

$d = ?$

(Multiple Choice)

Correct answer: $d = 60$

Symbolic:

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{v_1}{t_1}$

$v = \frac{v_1}{2}$

$d = ?$

(Multiple Choice)

Correct answer: $d = \frac{v_1 t_1}{8}$

Correct-Response-Rate Differences

First Semester algebra-based (PHY 111) Tempe Campus:

Semester	Numeric Correct	Symbolic Correct	Difference
Spring 2018	81% (N=223)	37% (N=215)	**44%**
Fall 2018	78% (N=145)	31% (N=140)	**47%**

First Semester calculus-based (PHY 121) Tempe Campus:

Semester	Numeric Correct	Symbolic Correct	Difference
Spring 2018	89% (*N* =902)	72% (*N* =889)	**17%**
Fall 2018	85% (*N* =157)	64% (*N* =165)	**21%**
Spring 2019	90% (*N* =50)	71% (*N* =45)	**19%**

Differences in Procedure

Numeric: Example of student work

5. What is the numerical value of d?

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{\Delta v}{\Delta t}$

$\Delta v = 60$

$\Delta t = 8$

$v = 30$

$30^2 = 0^2 + 2\left(\frac{\Delta v}{\Delta t}\right)d$

$900 = 0 + 2\left(\frac{60}{8}\right)d$

$900 = 15d$

$d = 60$

Arithmetic used on both sides of equation

Differences in Procedure

Symbolic:

Squaring Fraction

Dividing Fraction

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{v_1}{t_1}$

$v = \frac{v_1}{2}$

$d = ?$

$v^2 = 2ad$

$\left(\frac{v_1}{2}\right)^2 = 2\left(\frac{v_1}{t_1}\right)d$

$\frac{v_1^2}{4} = \frac{2v_1}{t_1}d$

$d = \frac{v_1^2}{4} \cdot \frac{t_1}{2v_1}$

$= \frac{v_1 t_1}{8}$

(Please clearly *circle* your answer and show all work.)

A. $d = v_1 t_1$ B. $d = \frac{v_1 t_1}{2}$ C. $d = \frac{v_1 t_1}{4}$ D. $d = \frac{v_1 t_1}{8}$ E. $d = \frac{v_1 t_1}{16}$

Errors Observed

- **Numeric version:** substituting the wrong value into original equation, e.g., Δv for v.

- **Symbolic version:** incorrectly squaring and multiplying/dividing fractions.

Students seem to struggle with the additional steps and complexity in the symbolic version.

Example of Multiplication Error

10. In the equations below, v_1, t_1, a, and v represent (unknown) numbers, for example, 3, 8, 9, 14.

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{v_1}{t_1}$

$v = \frac{v_1}{2}$

$d = ?$

$\left(\frac{v_1}{2}\right)^2 = 0^2 + 2\left(\frac{v_1}{t_1}\right)d$

$\frac{\left(\frac{v_1}{2}\right)^2}{2\left(\frac{v_1}{t_1}\right)} = d$

Correct expression

Error

$\frac{\frac{v_1^2}{4}}{\frac{2v_1}{2t_1}} = \frac{v_1^2 \, 2t_1}{4 \, 2 \, v_1} = \frac{v_1 t_1}{4}$

(Please clearly circle your answer and show all work.)

A. $d = v_1 t_1$ B. $d = \frac{v_1 t_1}{2}$ C. $d = \frac{v_1 t_1}{4}$ D. $d = \frac{v_1 t_1}{8}$ E. $d = \frac{v_1 t_1}{16}$

Example of Squaring Error

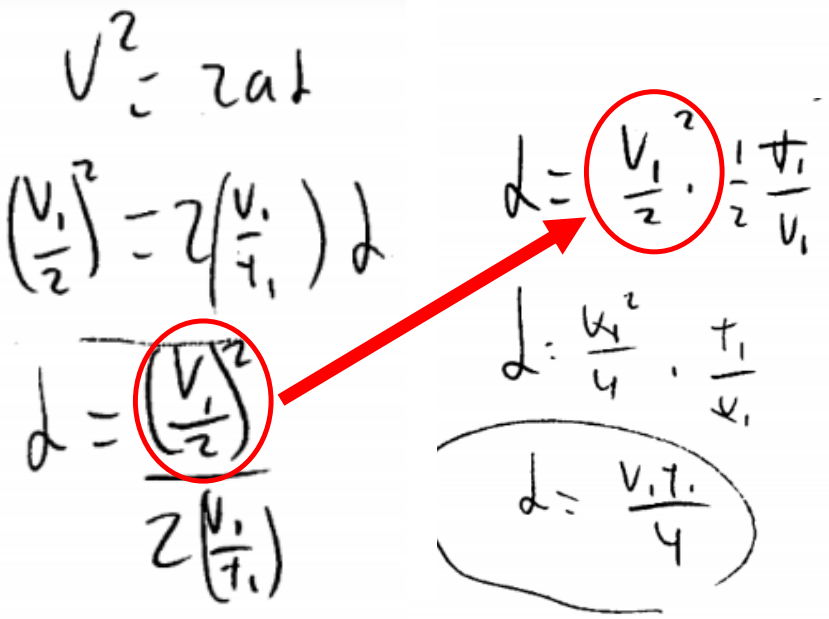
$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{v_1}{t_1}$

$v = \frac{v_1}{2}$

$d = ?$



Not squaring the denominator

(Please clearly *circle* your answer and show all work.)

A. $d = v_1 t_1$ B. $d = \frac{v_1 t_1}{2}$ C. $d = \frac{v_1 t_1}{4}$ D. $d = \frac{v_1 t_1}{8}$ E. $d = \frac{v_1 t_1}{16}$

Example of Division Error

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{v_1}{t_1}$

$v = \frac{v_1}{2}$

$d = ?$

$V^2 = 2ad$

$\frac{\left(\frac{v_1}{2}\right)^2}{2\left(\frac{v_1}{t_1}\right)} = \frac{2\left(\frac{v_1}{t_1}\right)d}{2\left(\frac{v_1}{t_1}\right)}$

$\frac{v_1^2}{4} \cdot \frac{2\left(\frac{v_1}{t_1}\right)}{} = \left(\frac{v_1 t_1}{2}\right)$

Division of fractions error

(Please clearly *circle* your answer and show all work.)

A. $d = v_1 t_1$ B. $d = \frac{v_1 t_1}{2}$ C. $d = \frac{v_1 t_1}{4}$ D. $d = \frac{v_1 t_1}{8}$ E. $d = \frac{v_1 t_1}{16}$

Testing Operational Skills

- To help better understand the errors occurring on the symbolic problem, we asked basic questions to test math skills at the middle-school level
- We wanted to reduce the complexity of the problem in order to isolate errors
  - ➢ "Complexity" implies, among other things, the number of steps involved in the problem
- We administered three problems on the manipulation of fractions which were directly related to the kinematic equation algebra problem

Fractions – 3 Problems

Symbolic Multiplication

$2\left(\frac{a}{b}\right) = ?$ **(1)**

A. $\frac{2a}{b}$ B. $\frac{2a}{2b}$ C. $\frac{a}{2b}$

Symbolic Division

$\frac{a/b}{c^2/d} = ?$ $\frac{ad}{bc^2}$ **(2)**

A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

Exponent

$\left(\frac{a}{3}\right)^3 = ?$ **(3)**

A. $\frac{a^3}{3}$ B. $\frac{a}{27}$ C. $\frac{a^3}{27}$

(1) Correct Response Rates (multiplication)

$$2\left(\frac{a}{b}\right) = ?$$

- ASU *Tempe* campus averages:

  1st semester calculus-based course, (*N*=95): **96%**

- ASU *Polytechnic* campus averages:

  1st semester calculus-based course, (*N*=69): **75%**

(2) Correct Response Rates (division)

$$\frac{a/b}{c^2/d} = ?$$

- ASU *Tempe* campus averages:

  1st semester calculus-based course, (*N*=95): **92%**

- ASU *Polytechnic* campus averages:

  1st semester calculus-based course, (*N*=69): **68%**

(3) Correct Response Rates (exponent)

$$\left(\frac{a}{3}\right)^3 = ?$$

- ASU *Tempe* campus averages:

  1st semester calculus-based course, (*N*=95): **100%**

- ASU *Polytechnic* campus averages:

  1st semester calculus-based course, (*N*=69): **91%**

Weak Operational Skills, or Carelessness?

- We define “non-operational errors” as all errors that occur when the student apparently has knowledge of the mathematical operations needed to solve individual steps of a multi-step problem, but fails to solve the problem correctly, e.g., from not accessing previously learned skills, or not exercising sufficient care.
- Knowledge of how to solve each fraction problem is essential to work the kinematic equation problem correctly
- With certain assumptions, we can then estimate the percentage of students that solved the kinematic equation problem incorrectly because of “non-operational errors”

Measuring “Non-Operational Errors”

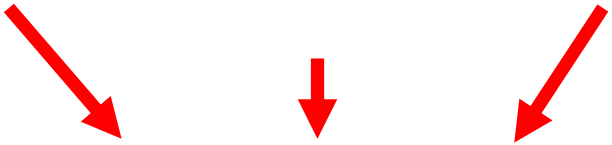
- We hypothesize, based on analysis of thousands of written diagnostics, that the majority of errors on the symbolic version were due to errors on one or more of the three “fraction” operations
- Therefore, we assume that, if a student responds incorrectly to any of the three fraction problems, they would probably give an incorrect response to the kinematics problem; we call this an “operational” error
- We define any other error as a “non-operational” error

Calculation Example

Percentage of errors that are "operational" errors

Percentage of errors that are "non-operational" errors

Total incorrect response rate (kinematics problem)



$$40\% + X = 60\% \Rightarrow \boxed{X = 20\%}$$

Calculation Example

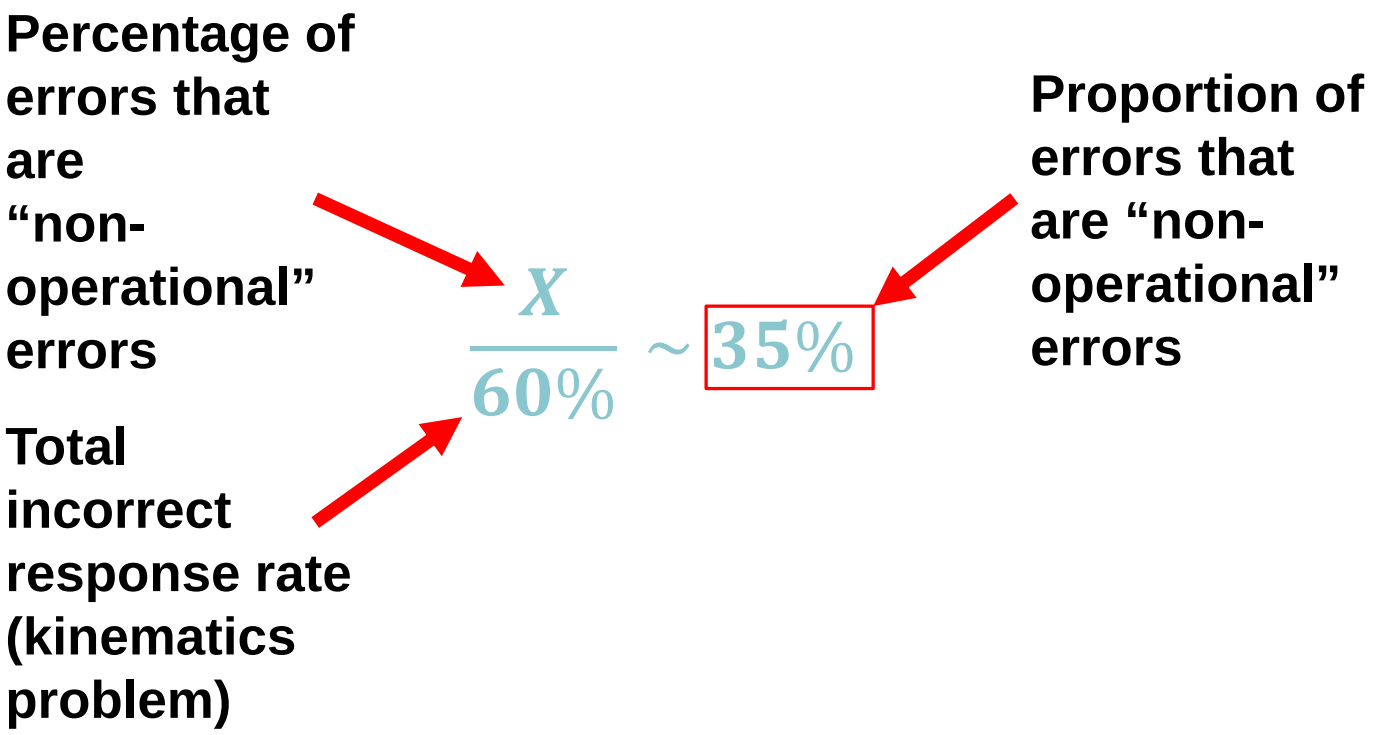


Table of Results: “Non-Operational Errors”

1st semester algebra-based course:

Campus	Semester	Number of Students	Error on any of the three fraction problems	All errors on kinematics problem	Total "non-operational error"	Proportion of errors that are "non-operational"
Polytechnic	Spring 2019	28	20%	55%	35%	~60%

1st semester calculus-based course:

Campus	Semester	Number of Students	Error on any of the three fraction problems	All errors on kinematics problem	Total "non-operational error"	Proportion of errors that are "non-operational"
Polytechnic	Spring 2019	36	50%	60%	10%	~15%
Tempe	Spring 2019	45	10%	30%	20%	~65%

2nd semester calculus-based course:

Campus	Semester	Number of Students	Error on any of the three fraction problems	All errors on kinematics problem	Total "non-operational error"	Proportion of errors that are "non-operational"
Tempe	Spring 2019	98	10%	25%	15%	~60%

Summary and Relation to Interviews

- Students' tend to self-correct their errors during problem-solving interviews approximately 50% of the time, consistent with findings on the written diagnostics
- Therefore, we conclude that students often posses the operational tools necessary to solve a problem, but make non-operational mistakes due to "carelessness", inability to access relevant knowledge, etc.
- We see the possibility of significant improvement through implementation and improvement of work-checking strategies

Primary Findings

Regardless of course (algebra- or calculus-based), campus (Tempe or Poly), or semester (Spring or Fall):

- Difficulties with basic mathematical operations are widespread; average error rates range from 20-70%;
- Performance on problems using symbols for constants is significantly worse than on problems using numbers;
- During problem-solving interviews, students self-correct approximately 50% of errors following minimal prompts;

Student Self-Correction of Errors

Our Interview Findings: Almost half of students' errors on algebra problems were self-corrected by students during interviews, as a consequence of interviewer prompts or unprompted auto-correction.

Prompts Leading to Self-Correction

- *"Explain that step"*
- *"Clarify what you mean*."
- "*What does the problem ask you to do?"*
- *[No specific prompt: Students asked to explain all work]*

Interview Results: *N* = 53

$3x = 2y$
$5x + y = 26$

What are the values of x and y? Show all your steps. For example, $x = 2, y = 5$ (These are NOT the correct answers).

Correct:	**83%**
Error, Self-corrected:	**9%**
Error, Uncorrected:	**8%**

Interview Results: *N* = 53

$x \cos(20°) = y \cos(70°)$
$x \cos(70°) + y \cos(20°) = 10$

What are the values of *x* and *y*? Show all your steps. Note: The value for *x* should NOT include *y*, and the value for *y* should NOT include *x*.

Correct:	**57%**
Error, Self-corrected:	**19%**
Error, Uncorrected:	**25%**

Interview Results: *N* = 53

a*x* = b*y*
b*x* + a*y* = c

a, b, and c are constants.
What are the values of *x* and *y* in terms of a, b, and c? Show all your steps. Note: The value for *x* should NOT include *y*, and the value for *y* should NOT include *x*.

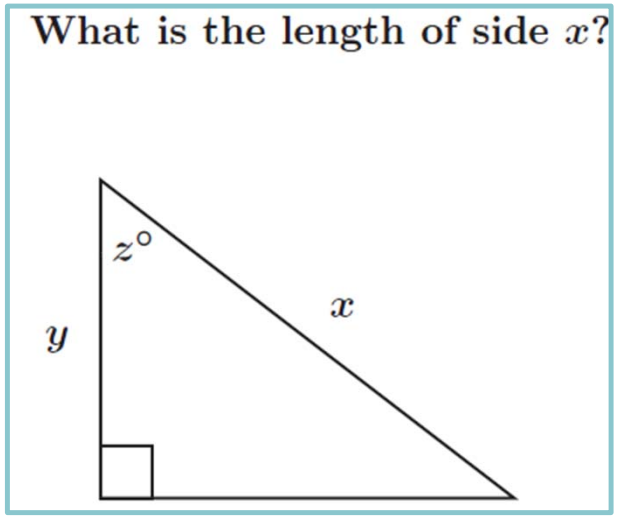
Correct:	**55%**
Error, Self-corrected:	**21%**
Error, Uncorrected:	**25%**

Comparison Population: University of Colorado

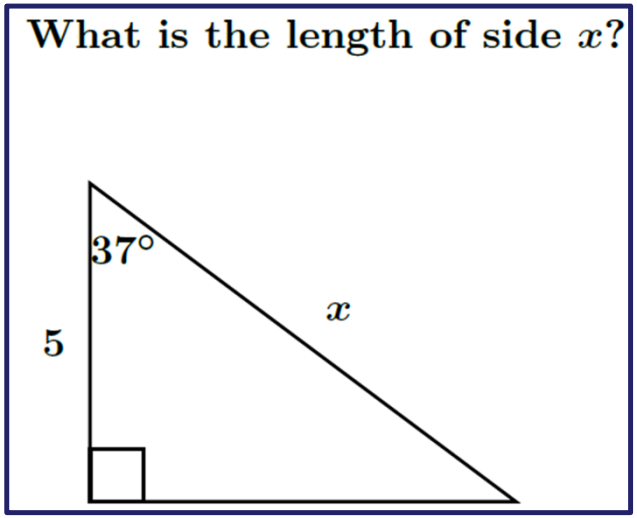
- In Fall 2019, our diagnostic was administered in the algebra-based general physics course at the University of Colorado (N = 388). Results were broadly consistent with ASU results from the Tempe campus.

Fall 2019 Diagnostic Results
with Comparisons to Colorado University

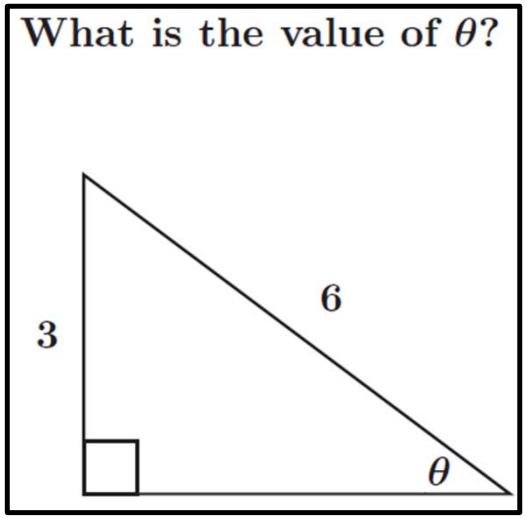
Trigonometry Problems



“symbolic”
hypotenuse problem

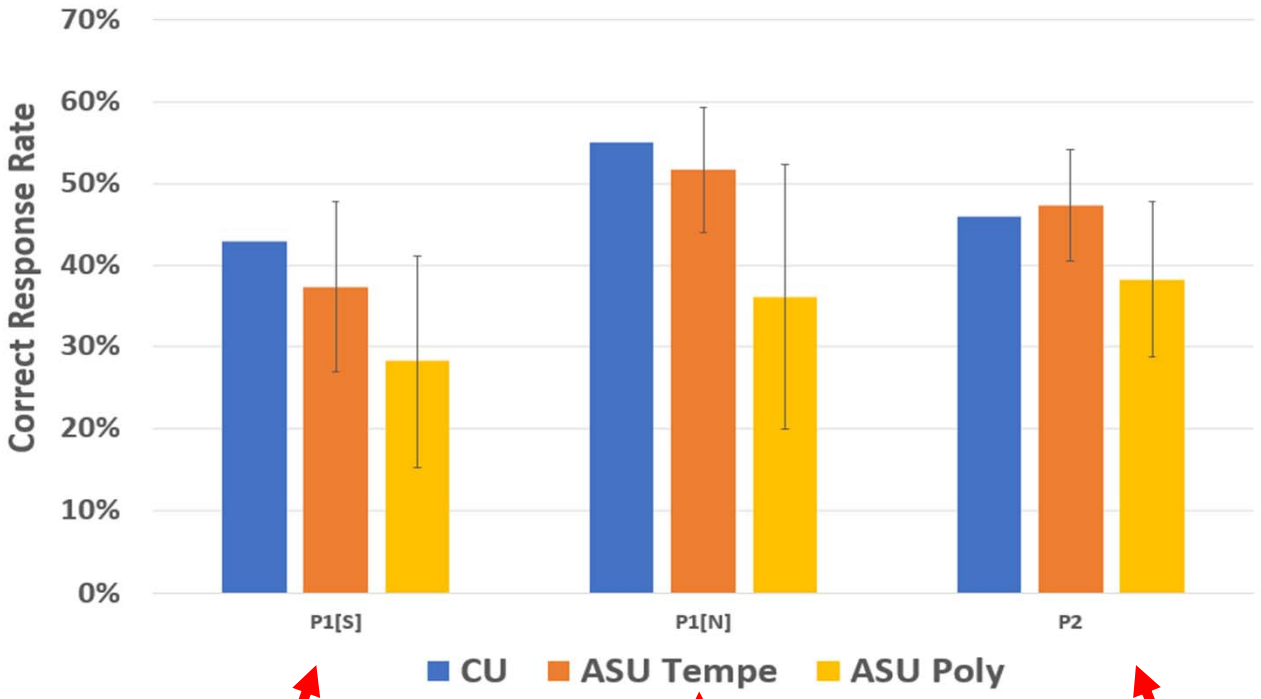


“numeric”
hypotenuse problem

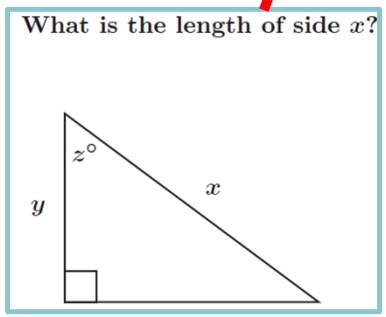


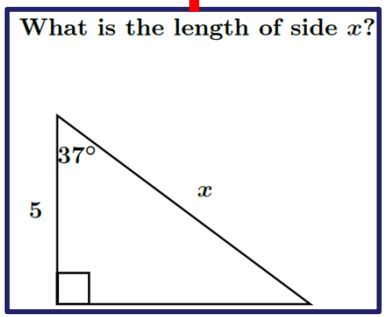
Solving for the angle problem

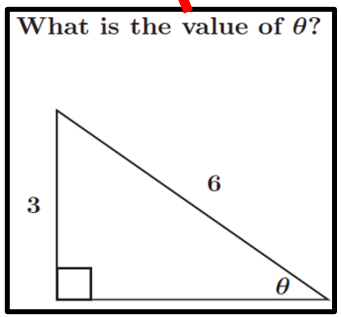
1st semester algebra-based; CRR Trigonometry Problems CU vs ASU



- CU: *N*=197, 191, 388
- ASU Tempe: *N≈500* (average from 3 semesters)
- ASU Poly: *N≈300* (average from 5 semesters)
- Error bars are ±σ

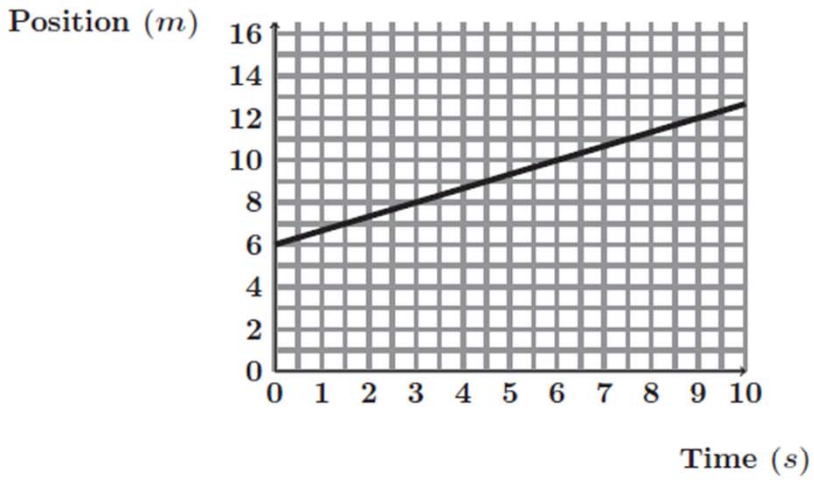






Slope Problem

4. What is the slope of the graph below?

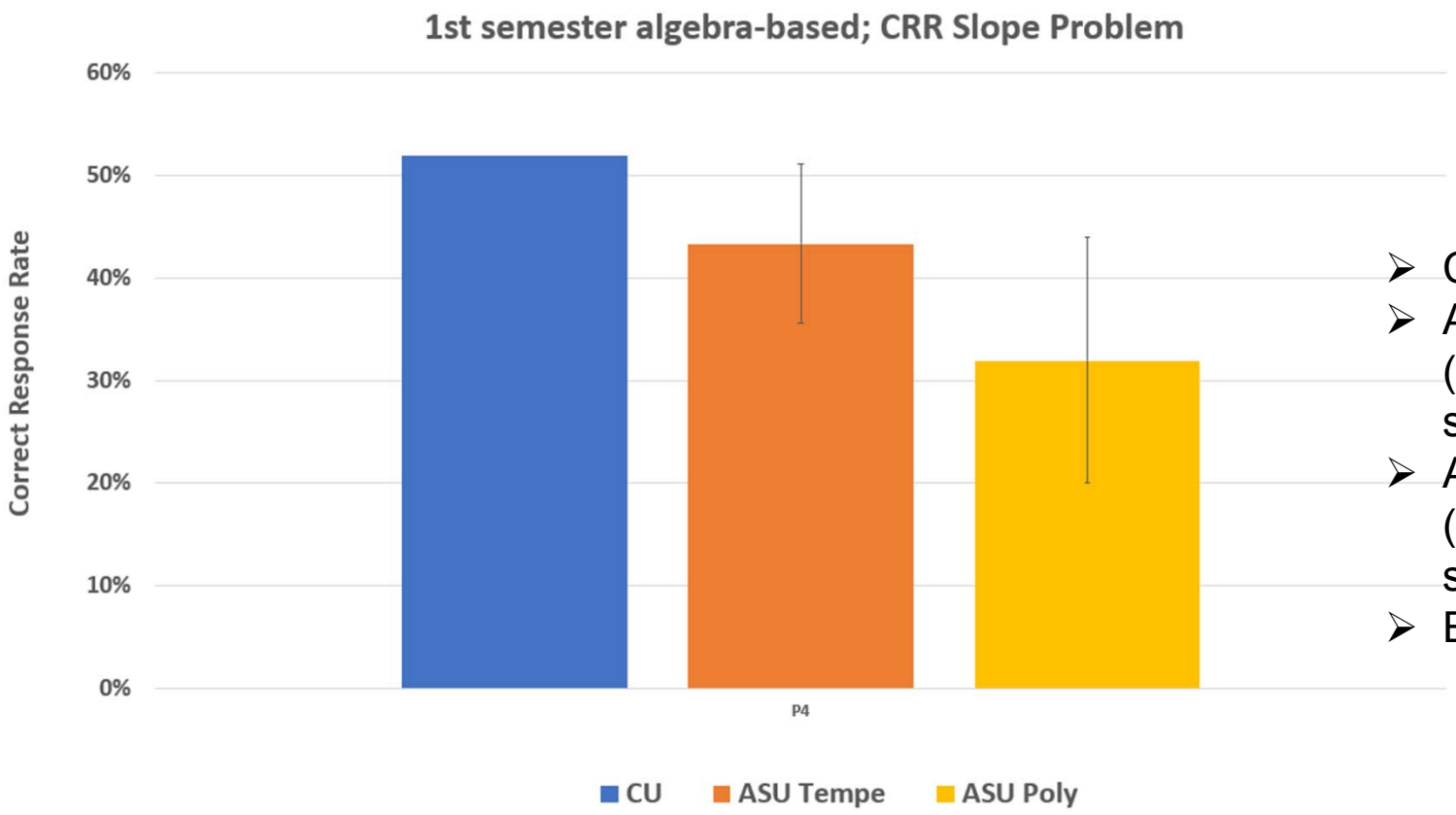


A. $\frac{1}{3}$ m/s because the object moves 1 meter in 3 seconds.

B. $\frac{1}{3}$ m/s because the line rises 1 box while it goes 3 boxes in the horizontal direction.

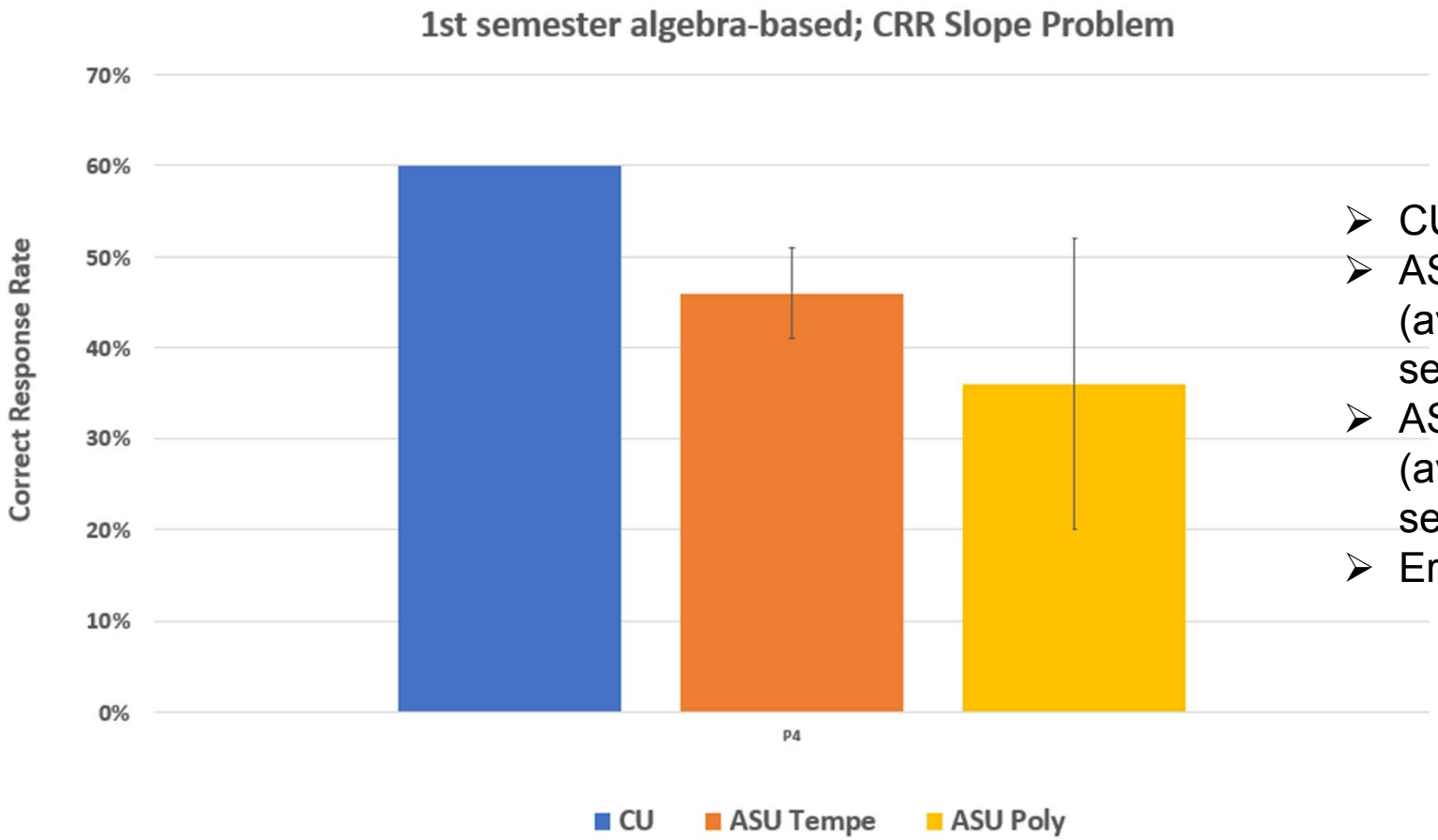
C. $\frac{2}{3}$ m/s because the object moves 2 meters in 3 seconds.

D. $\frac{2}{3}$ m/s because the line rises 2 boxes while it goes 3 boxes in the horizontal direction.

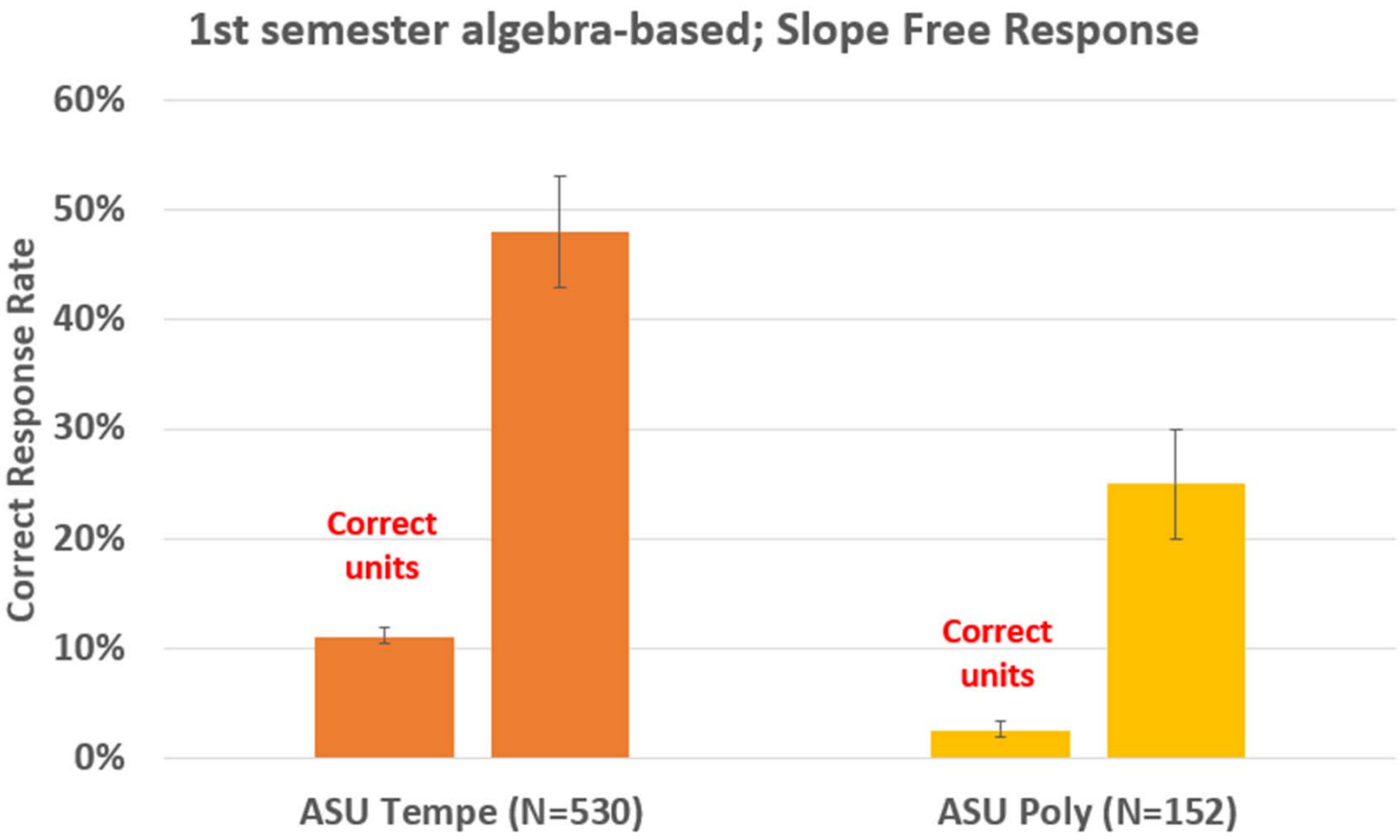


- CU: *N*=388
- ASU Tempe: *N≈750* (average from 3 semesters)
- ASU Poly: *N≈200* (average from 3 semesters)
- Error bars are ±σ

Numbers based on including all multiple-choice responses of "2/3" as correct



- CU: *N*=388
- ASU Tempe: *N≈750* (average from 3 semesters)
- ASU Poly: *N≈200* (average from 3 semesters)
- Error bars are ±σ


1st semester algebra-based; Slope Free Response
Correct Response Rate
60%
50%
40%
30%
20%
10%
0%
Correct units
Correct units
ASU Tempe (N=530)
ASU Poly (N=152)

Symbolic Single Equation Algebra Problems

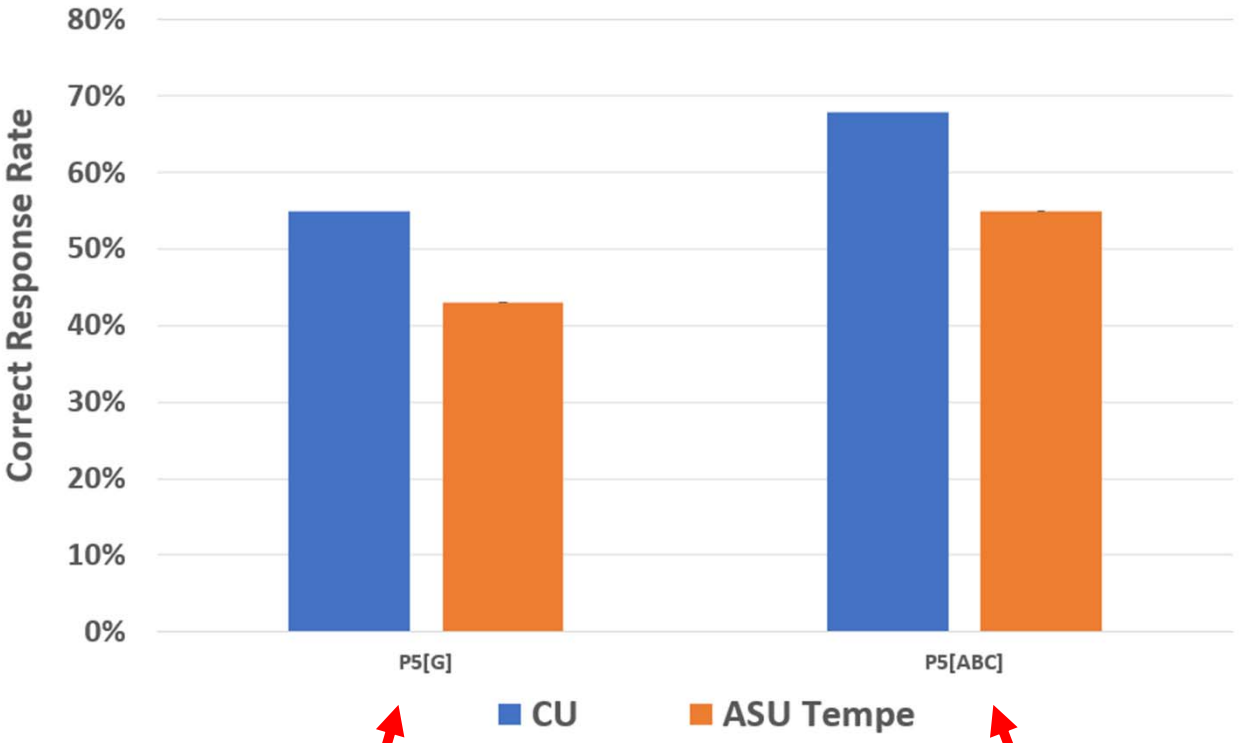
Solve for θ.

$$\gamma\theta + \eta = \lambda\theta + \omega$$

Solve for x.

$$ax + b = cx + d$$

1st semester algebra-based; CRR Single Equation Algebra Problems



Solve for θ.

$\gamma\theta + \eta = \lambda\theta + \omega$

Solve for x.

$ax + b = cx + d$

- CU: *N*≈200
- ASU Tempe: *N≈100*

Fraction Problems

7. $\left(\frac{a}{3}\right)^3 = ?$

A. $\frac{a^3}{3}$ B. $\frac{a}{27}$ C. $\frac{a^3}{27}$

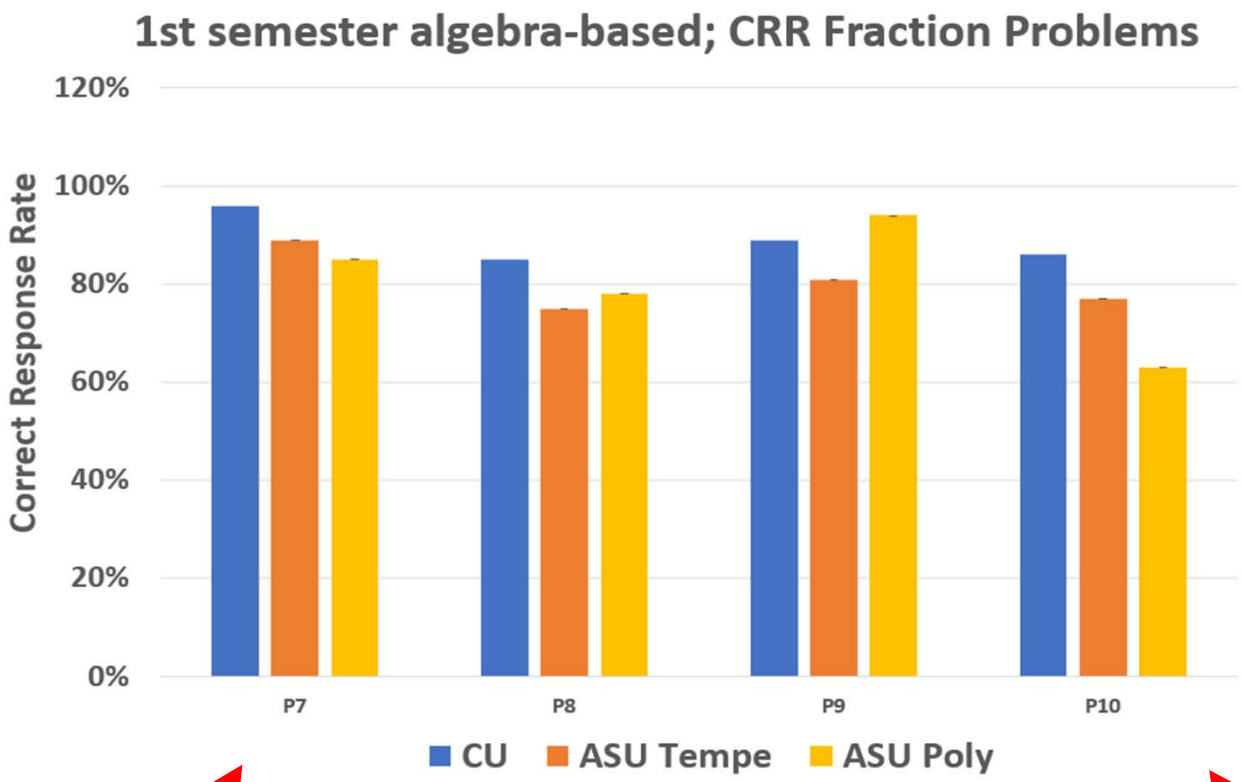
8. $2\left(\frac{a}{b}\right) = ?$

A. $\frac{2a}{b}$ B. $\frac{2a}{2b}$ C. $\frac{a}{2b}$

9. $2\left(\frac{3}{4}\right) = ?$

10. $\frac{a/b}{c^2/d} = ?$

A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$



- CU: *N*=388
- ASU Tempe: *N≈200*
- ASU Poly: *N=54*

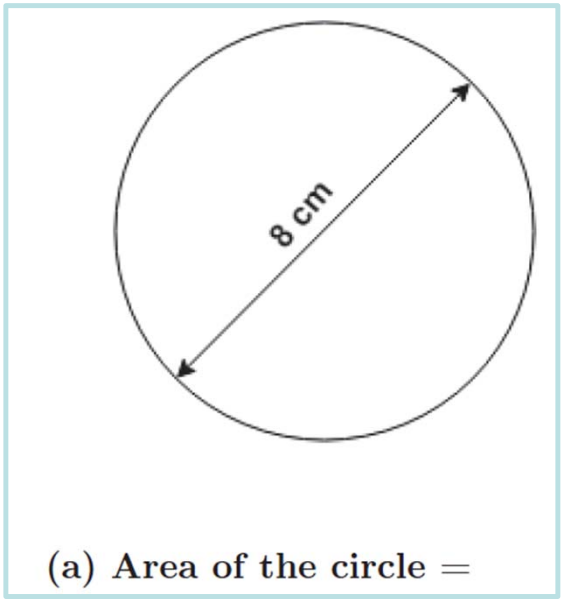
7. $\left(\frac{a}{3}\right)^3 = ?$

8. $2\left(\frac{a}{b}\right) = ?$

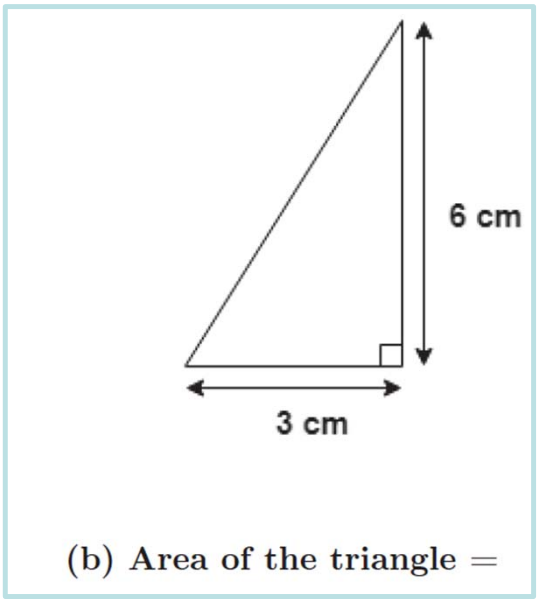
9. $2\left(\frac{3}{4}\right) = ?$

10. $\frac{a/b}{c^2/d} = ?$

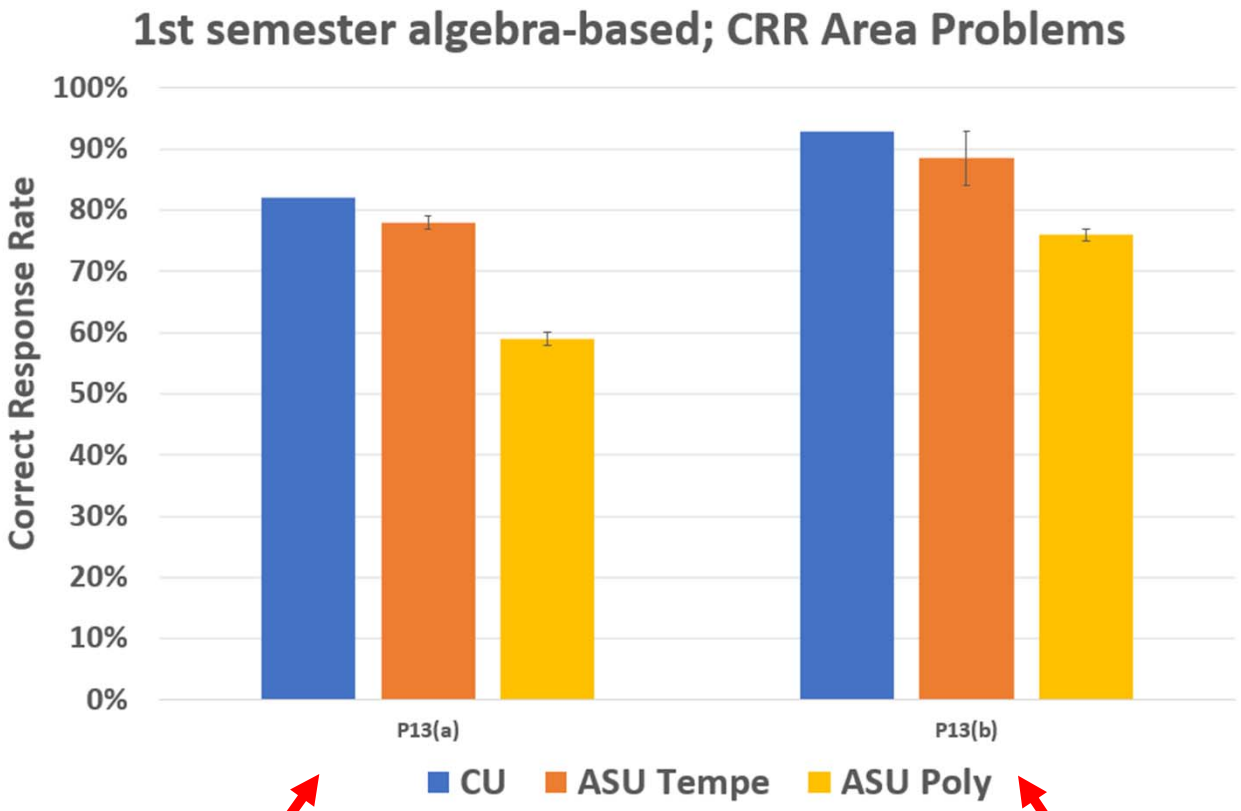
Finding The Area Problems



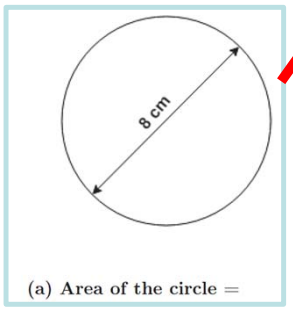
(a) Area of the circle =

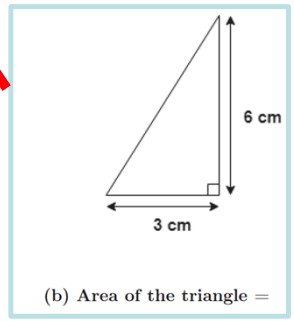


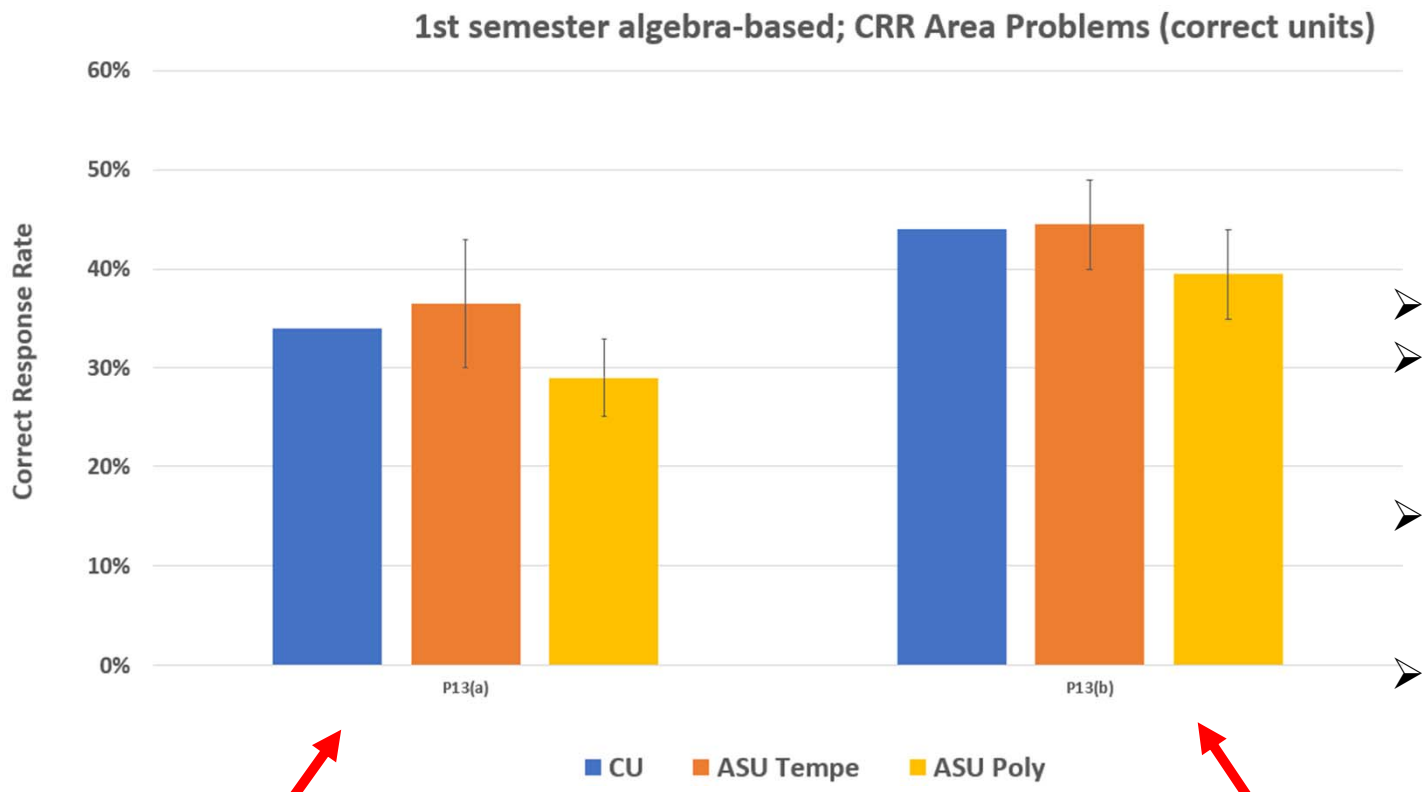
(b) Area of the triangle =



- CU: *N*=383, 382
- ASU Tempe: *N≈500* (average from 3 semesters)
- ASU Poly: *N ≈100* (average from 2 semesters)
- Error bars are ±σ

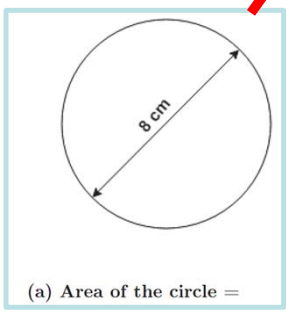


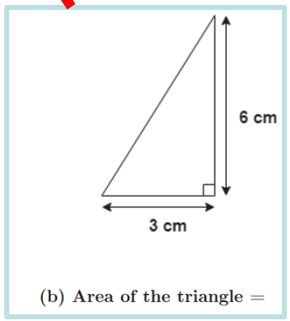




Numbers based on requiring units to be correct

- CU: *N*=383, 382
- ASU Tempe: *N≈500* (average from 3 semesters)
- ASU Poly: *N ≈100* (average from 2 semesters)
- Error bars are ±σ





Simultaneous Equations

15. $cy = dx$

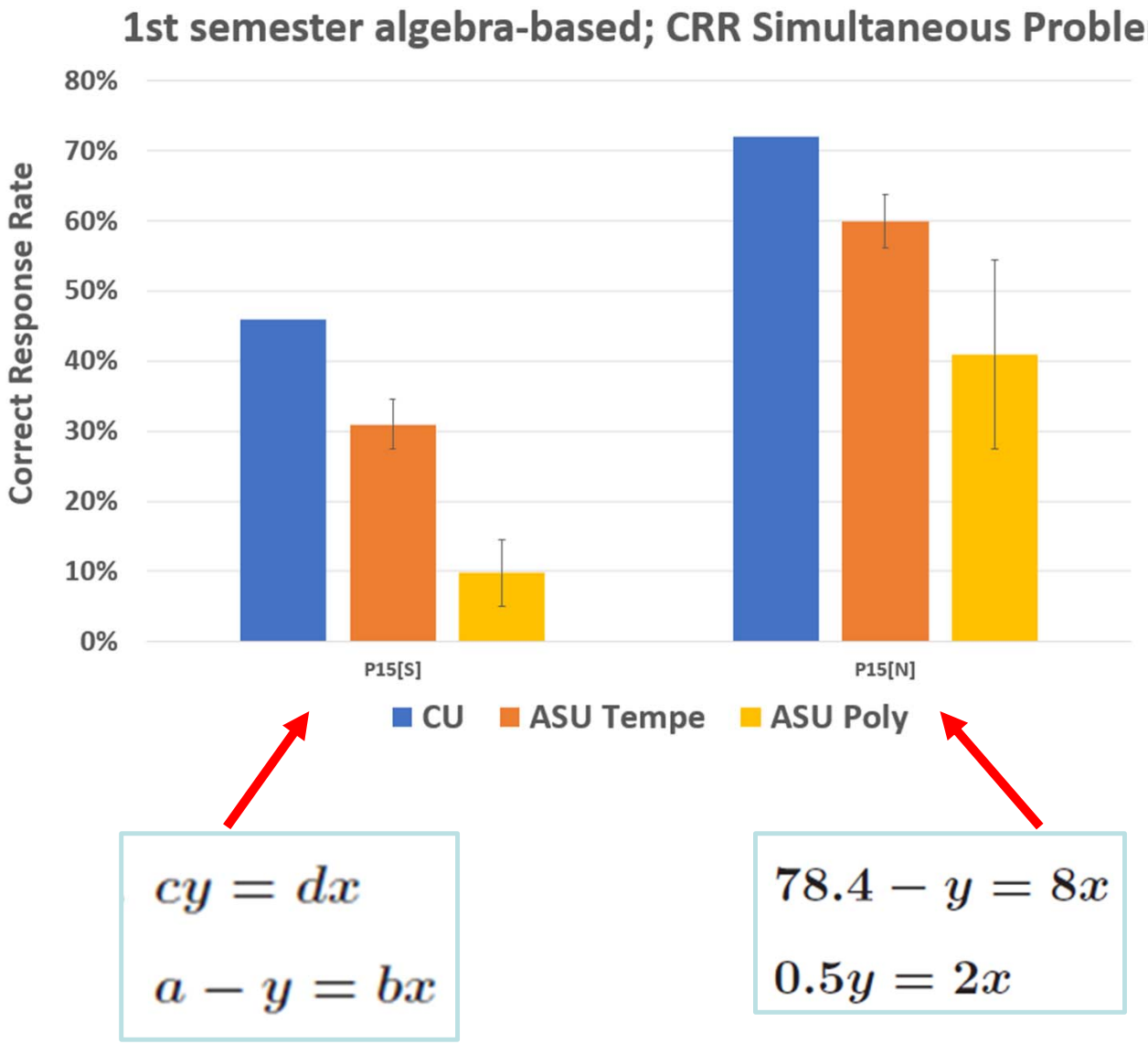
$a - y = bx$

$x = ?$

15. What is the numerical value of x?

$78.4 - y = 8x$

$0.5y = 2x$



- CU: *N*=167, 180
- ASU Tempe: *N=326, 423* (average from 3 semesters)
- ASU Poly: *N =169, 166* (average from 4 semesters)
- Error bars are ±σ

Kinematic Equations

12. What is the numerical value of d?

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \dfrac{\Delta v}{\Delta t}$

$\Delta v = 60$

$\Delta t = 8$

$v = 30$

$d = ?$

(Please clearly *circle* your answer and show all work.)

A. $d = 30$ B. $d = 60$ C. $d = 120$ D. $d = 240$ E. $d = 480$

12. $v^2 = v_0^2 + 2ad$

$v_0 = 0$

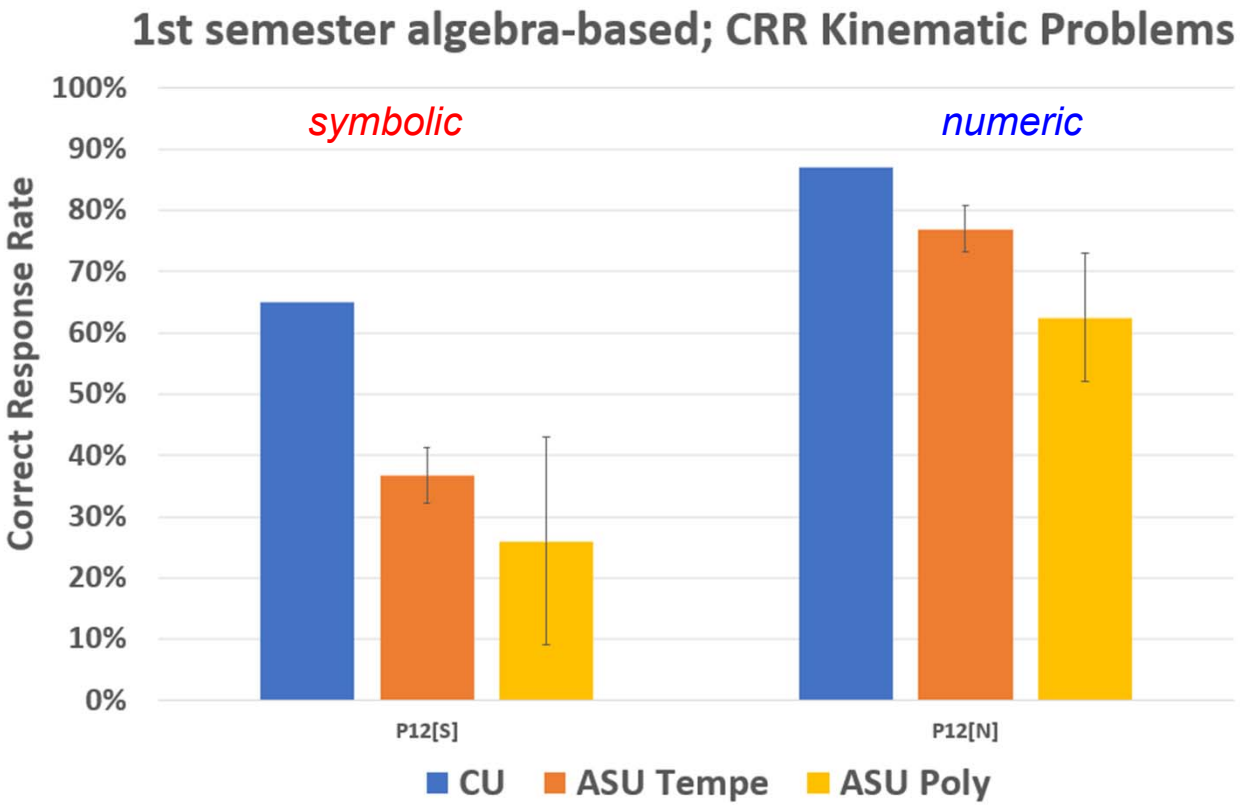
$a = \dfrac{v_1}{t_1}$

$v = \dfrac{v_1}{2}$

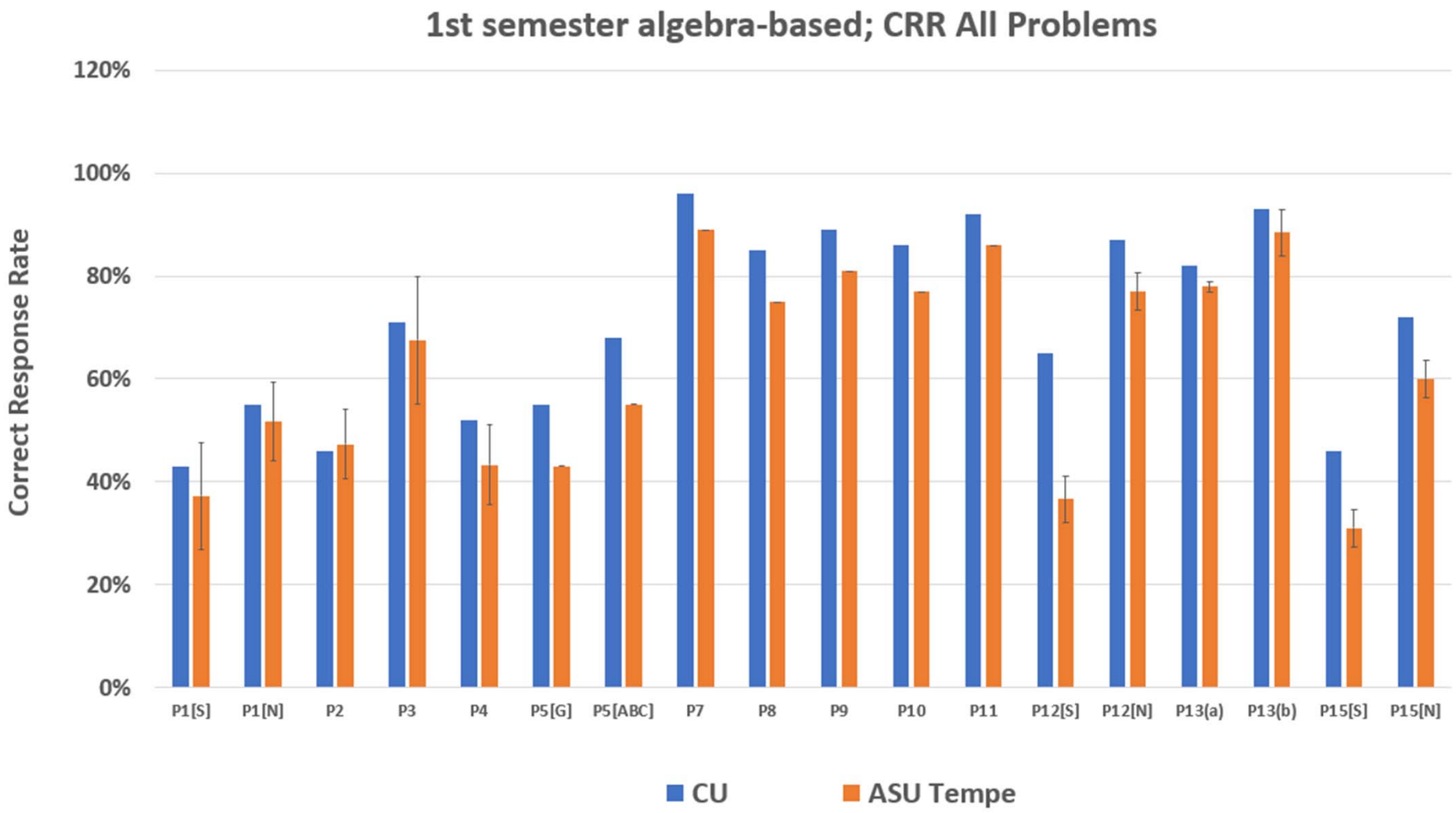
$d = ?$

(Please clearly *circle* your answer and show all work.)

A. $d = v_1 t_1$ B. $d = \dfrac{v_1 t_1}{2}$ C. $d = \dfrac{v_1 t_1}{4}$ D. $d = \dfrac{v_1 t_1}{8}$ E. $d = \dfrac{v_1 t_1}{16}$



- CU: *N*=191, 196
- ASU Tempe: *N=461, 471* (average from 3 semesters)
- ASU Poly: *N =136, 147* (average from 3 semesters)
- Error bars are ±σ


1st semester algebra-based; CRR All Problems
Correct Response Rate
120%
100%
80%
60%
40%
20%
0%
P1[S]
P1[N]
P2
P3
P4
P5[G]
P5[ABC]
P7
P8
P9
P10
P11
P12[S]
P12[N]
P13(a)
P13(b)
P15[S]
P15[N]
CU
ASU Tempe

Summary: Implications for Instruction

- Difficulties due to **skill-practice deficits** might be addressed by short-term, in- and out-of-class tutorials and assignments, designed to refresh students' previously learned knowledge and skills (e.g., Mikula and Heckler, 2017)
  - Current project, OSU + ASU, NSF DUE #1914709/1914712
- Difficulties due to **"carelessness"** might be addressed by guiding students to (1) carefully check and re-check key steps in their calculation; (2) slow down, review problem statements, and re-solve when possible