College Physics Students' Mathematical Difficulties and Their Implications for High School Teachers

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ABSTRACT

Over the past two years, we have administered a diagnostic test of basic mathematical skills to over 2000 students enrolled in introductory physics courses at Arizona State University, including both the algebra-based and calculus-based courses, We find widespread difficulties with basic trigonometric and algebraic operations, with evidence that symbolic operations pose particular problems for many students. Interviews show that a large proportion of errors are due to lack of fluency or simple carelessness. We speculate that putting additional focus on certain trigonometric facts, combined with stronger emphasis on careful execution and checking of algebraic work, may yield significant dividends in physics at both the high school and college level.

STUDENT PERFORMANCE ON TRIGONOMETRY PROBLEMS

- We administered and analyzed a written diagnostic, given to 1400 students in 18 algebra- and calculus-based physics classes over four semesters at Arizona State University during 2016-2017.
- We carried out individual interviews with 65 students enrolled in those or similar courses during same period.

Correct Response Rate, #1-3 combined

ASU Polytechnic campus, Spring + Fall

Algebra-based course, 1st semester, (*N* = 116): 37%

Algebra-based course, 2nd semester, (*N* =79): 48%

ASU Polytechnic campus, Spring

Calculus-based course, 1st semester, (*N* = 146): 66%

Errors observed:

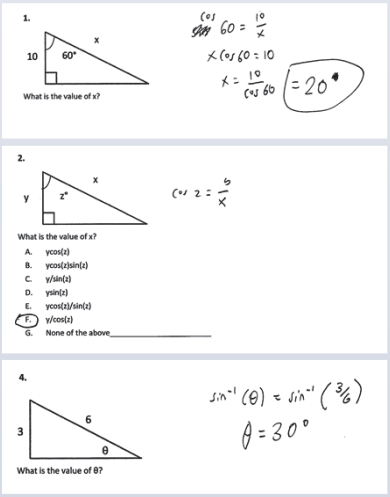
(i) use of incorrect trigonometric function (e.g., cosine instead of sine), or misunderstanding of definition;

(ii) unaware (or forgot) about inverse trigonometric functions, e.g., arctan, arcsin, arccos [$\tan^{-1}$, $\sin^{-1}$, $\cos^{-1}$]

35-70% of students confused on basic trigonometry relations

Trigonometry Questions

with samples of correct student responses



Trigonometry Questions: Summary

Regardless of course, semester, campus, or question type, between 35% and 70% of introductory physics students at ASU have significant difficulties with basic trigonometric relationships.

Students frequently tended to self-correct errors during interviews, suggesting that many of the errors were "careless" or due to insufficient review or practice.

STUDENT PERFORMANCE ON ALGEBRA PROBLEMS

Our Findings: Significant Differences in Performance between Numerical and Symbolic Versions of Simultaneous-Equations Problem

Summary of the Data

- *Algebra-based course:* 70% correct on numerical version, 20% on symbolic version;
- *Calculus-based course:* 70% correct on numerical version, 43% correct on symbolic version.

Algebra: Simultaneous Equations

$3x = 2y$
$5x + y = 26$ — Numerical Version

What are the values of x and y? Show all your steps. For example, $x = 2$, $y = 5$ (These are NOT the correct answers).

Correct Response Rate, ASU (% correct responses)
Algebra-based course, second semester (*N* =123): 70%

$x \cos(20°) = y \cos(70°)$
$x \cos(70°) + y \cos(20°) = 10$ — "Semi-Symbolic" Version

What are the values of x and y? Show all your steps. Note: The value for x should NOT include y, and the value for y should NOT include x.

Correct Response Rate, ASU (% correct responses)
Algebra-based course, second semester (*N* =150): 20-30%
(different campuses, slightly different versions)

$ax = by$
$bx + ay = c$ — Symbolic Version

a, b, and c are constants.
What are the values of x and y in terms of a, b, and c? Show all your steps. Note: The value for x should NOT include y, and the value for y should NOT include x.

Correct Response Rate, ASU (% correct responses)
Algebra-based course, second semester (*N* =150): 10-20%
(different campuses, slightly different versions)

Only 10-20% correct responses!

Simultaneous Equations Questions in Calculus-based course, first semester (Polytechnic campus) [*N* = 91]

$3x = 2y$
$5x + y = 26$
What is the numerical value of x?

70% correct

In the two equations below, a, b, c, and d represent (unknown) numbers, e.g., 3, 8, 9, 14.

$ax = by$
$cx + y = d$
$x = ?$

(Your answer for x should have a, b, c, and d in it, but *not* y.)

43% correct

Difficulties with Mathematical Operations (Booth et al., 2014)

Negative Sign, e.g., moving a term without changing its sign; deleting or adding a negative sign;

Equality, e.g., performing operations without maintaining balance on both sides of an equals sign;

Mathematical Property, e.g., inappropriately applying the distributive property;

Fraction, e.g., moving a term from the numerator to the denominator or vice versa.

➡ All of these observed in our investigation

Why the Difficulties with Symbols? Some Hints From the Interviews

- In elementary math courses, "simplified forms" of equations are emphasized (i.e., few messy symbols and functions)
- Students get "overloaded" by seeing all the variables, and are unable to carry out procedures (e.g., multiplying each term in an expression by a constant [symbol]) that they do successfully with numbers (e.g., multiply through by a number)
- Other procedural failures that occur more often with symbols: cancellation, factoring out a constant, retaining coefficients from one line to the next

Sources of Difficulties

- Carelessness
  - Students *very frequently* self-correct errors during interviews
  - Evidence of carelessness on written diagnostic
- Skill practice deficit: Insufficient repetitive practice with learned skills
- Inability to efficiently access previous learning

SUMMARY AND IMPLICATIONS FOR PHYSICS TEACHERS

- Some mathematical difficulties relate to relatively narrow, well-defined procedures and may be subject to significant improvement by short-term, targeted instruction (e.g., trigonometry).
- Other difficulties (e.g., algebraic operations) are likely more long-standing, resistant to quick improvement, and not easily addressable within college physics courses themselves.
- Guidance to strengthen physics students' self-checking, "care-taking" skills offers hope for short-term improvements.
- It may be possible for high school physics teachers to address these problems *before* students arrive in college, if they are made aware of them in advance.