# Semi-intuitive thinking and reasoning inconsistencies in calorimetry

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# Physics Students' Reasoning in Calorimetry

- Investigation of reasoning regarding calorimetric concepts among students in a calculus-based general physics course

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- A free-response quiz was administered after lecture instruction to 311 students in an attempt to assess their understanding of calorimetry

# Free-Response Question

*Written pretest given after lecture instruction completed*

The specific heat of water is ***greater*** than that of copper.

A piece of copper metal is put into an insulated calorimeter which is nearly filled with water. The mass of the copper is the ***same*** as the mass of the water, but the initial temperature of the copper is *lower* than the initial temperature of the water. The calorimeter is left alone for several hours.

During the time it takes for the system to reach equilibrium, will the temperature change (number of degrees Celsius) of the copper be *more than, less than,* or *equal to* the temperature change of the water? Please explain your answer.

# Free-Response Question Solution

$$Q = mc\,\Delta T$$

$$|Q_{Cu}| = |Q_W| \quad \text{and} \quad m_{Cu} = m_W$$

$$\Rightarrow c_{Cu}\,\Delta T_{Cu} = c_W\,\Delta T_W$$

***Notation: ΔT ≡ absolute value*** *of temperature change*

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$$\Rightarrow c_{Cu}\,\Delta T_{Cu} = c_W\,\Delta T_W$$

$$\Delta T_{Cu} = \frac{c_W}{c_{Cu}}\,\Delta T_W$$

$$c_W > c_{Cu} \Rightarrow \Delta T_{Cu} > \Delta T_W$$

***Notation: ΔT ≡ absolute value*** *of temperature change*

# Free-Response Question Results

*Second-semester calculus-based course (PHYS 222)*

| | $N$=311 |
|---|---|
| Correct | |
| $\Delta T_{LSH} > \Delta T_{GSH}$ | 62% |
| ***With correct explanation*** | ***55%*** |

LSH = lower specific heat
GSH = greater specific heat

*(five different versions of question were administered)*

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| **Incorrect** | |
| $\Delta T_{LSH} = \Delta T_{GSH}$ | 22% |
| $\Delta T_{LSH} < \Delta T_{GSH}$ | 16% |

LSH = lower specific heat
GSH = greater specific heat

*(five different versions of question were administered)*

# Free-Response Explanations

*N=311*

| | |
|---|---|
| **Incorrect ($\Delta T_{LSH} = \Delta T_{GSH}$)** | **22%** |
| Temperature changes are equal since energy transfers are equal | 9% |
| Temperature changes are equal since system goes to equilibrium | 6% |
| Other | 6% |

# Example of Incorrect Student Explanation

*"Equal, to reach thermal equilibrium the change in heat must be the same, heat can't be lost, they reach a sort of 'middle ground' so copper decreases the same amount of temp that water increases."*

**"Equal energy transfer" is assumed to imply "equal temperature change"**

# Free-Response Explanations

*N=311*

| | |
|---|---|
| **Incorrect ($\Delta T_{LSH} < \Delta T_{GSH}$)** | **16%** |
| Specific heat directly proportional to temperature change | 7% |
| Other | 8% |

# Example of Incorrect Student Explanation

*"The temperature change of copper will be less than that of the $\Delta T$ of the water, because the specific heat of water is greater, and the masses are the same."*

**"Greater specific heat" is assumed to imply "Greater temperature change"**

# Verbal Multiple Choice Question

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  - *An attempted intervention using modified instruction was unsuccessful on this question, although possibly effective on related questions.*

# Verbal Multiple Choice Question

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An object is immersed in a liquid within a sealed and insulated container. The mass of the object is the same as the mass of the liquid. The initial temperature of the object is lower than the initial temperature of the liquid, but the specific heat of the object is ***greater*** than that of the liquid.

# Verbal Multiple Choice Question

An object is immersed in a liquid within a sealed and insulated container. The mass of the object is the same as the mass of the liquid. The initial temperature of the object is lower than the initial temperature of the liquid, but the specific heat of the object is ***greater*** than that of the liquid. The calorimeter is left alone for several hours until it reaches equilibrium. Which of the following is true? *Note: Here, "temperature change" means "number of degrees Kelvin increased* ***or*** *decreased."*

# Verbal Multiple Choice Question

A. The energy transfer to the object is *not* equal to the energy transfer away from the liquid, and the temperature change of the object is greater than the temperature change of the liquid.

B. The energy transfer to the object is *not* equal to the energy transfer away from the liquid, and the temperature change of the object is less than the temperature change of the liquid.

C. The energy transfer to the object is equal to the energy transfer away from the liquid, but the temperature change of the object is greater than the temperature change of the liquid.

D. The energy transfer to the object is equal to the energy transfer away from the liquid, and the temperature change of the object is equal to the temperature change of the liquid.

E. The energy transfer to the object is equal to the energy transfer away from the liquid, but the temperature change of the object is less than the temperature change of the liquid.

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D. The energy transfer to the object is equal to the energy transfer away from the liquid, and the temperature change of the object is equal to the temperature change of the liquid.

**E. The energy transfer to the object is equal to the energy transfer away from the liquid, but the temperature change of the object is less than the temperature change of the liquid.**

# Verbal Question Results

| Spring 2003 (*N* = 311) | | | | |
|---|---|---|---|---|
| A | B | C | D | **E** |
| 4% | 13% | 13% | 12% | **57%** |

# Verbal Question Results

| Spring 2003 ($N$ = 311) | | | | |
|---|---|---|---|---|
| A | B | C | D | **E** |
| 4% | 13% | 13% | 12% | **57%** |

| $\Delta T_{LSH} < \Delta T_{GSH}$ | $\Delta T_{LSH} = \Delta T_{GSH}$ | Q's unequal | $\Delta T_{LSH} > \Delta T_{GSH}$ |
|---|---|---|---|
| 17% | 12% | 17% | 71% |

# Free-Response and Verbal

| | F-Resp | Verb MC |
|---|---|---|
| $\Delta T_{LSH} > \Delta T_{GSH}$ | 62% | 71% |
| $\Delta T_{LSH} = \Delta T_{GSH}$ | 22% | 12% |
| $\Delta T_{LSH} < \Delta T_{GSH}$ | 16% | 17% |
| $Q_{AWAY} \neq Q_{TO}$ | -- | 17% |

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*The free-response question doesn't ask the students to make any claims about the relation of the two heat transfers.*

# A Second Test Run

- We administered a similar verbal multiple-choice question to 461 students in the same course one year later [Spring 2004] to test the reliability of our results.
- The question was given as an extra credit question on the first exam covering calorimetry.

# Comparative Results

| | Spring 2003 | | Spring 2004 |
|---|---|---|---|
| | F-Resp | Verbal | Verbal |
| $\Delta T_{LSH} > \Delta T_{GSH}$ | 62% | 71% | 60% |
| $\Delta T_{LSH} = \Delta T_{GSH}$ | 22% | 12% | 13% |
| $\Delta T_{LSH} < \Delta T_{GSH}$ | 16% | 17% | 27% |
| $Q_{AWAY} \neq Q_{TO}$ | -- | 17% | 25% |

# Equation-based Problem

- Student comments suggested that the timed-exam environment in which the verbal question was administered in combination with the extensive "*legalese*" of the verbal question may have caused confusion

# Equation-based Problem

- Student comments suggested that the timed-exam environment in which the verbal question was administered in combination with the extensive “*legalese*” of the verbal question may have caused confusion
- An equation-based version was created in an attempt to by-pass this problem and was administered on the Spring 2004 final exam

# Equation-based Problem

Object $A$ has mass $m_A$, specific heat $c_A$, and initial temperature $T_{initial\ A}$. Liquid $B$ has mass $m_B$, specific heat $c_B$, and initial temperature $T_{initial\ B}$.

# Equation-based Problem

Object $A$ has mass $m_A$, specific heat $c_A$, and initial temperature $T_{initial\ A}$. Liquid $B$ has mass $m_B$, specific heat $c_B$, and initial temperature $T_{initial\ B}$. Object $A$ is immersed in Liquid $B$ within a sealed and insulated container (i.e., a calorimeter).

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$$m_A = m_B$$

$$c_A > c_B$$

$T_{initial\ A} < T_{initial\ B}$ but after a long time, $T_{final\ A} = T_{final\ B}$

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$$m_A = m_B$$

$$c_A > c_B$$

$T_{initial\ A} < T_{initial\ B}$ but after a long time, $T_{final\ A} = T_{final\ B}$

Which of the following is true?

[$Q$ is heat transfer; $\Delta T \equiv T_{final} - T_{initial}$]

# Equation-based Solution

A. $Q_{\text{to } A} \neq Q_{\text{away from } B}; |\Delta T_A| > |\Delta T_B|$

B. $Q_{\text{to } A} \neq Q_{\text{away from } B}; |\Delta T_A| < |\Delta T_B|$

C. $Q_{\text{to } A} = Q_{\text{away from } B}; |\Delta T_A| > |\Delta T_B|$

D. $Q_{\text{to } A} = Q_{\text{away from } B}; |\Delta T_A| = |\Delta T_B|$

E. $Q_{\text{to } A} = Q_{\text{away from } B}; |\Delta T_A| < |\Delta T_B|$

# Equation-based Solution

*A.* $Q_{\text{to } A} \neq Q_{\text{away from } B}$; $|\,\Delta T_A\,| > |\,\Delta T_B\,|$

*B.* $Q_{\text{to } A} \neq Q_{\text{away from } B}$; $|\,\Delta T_A\,| < |\,\Delta T_B\,|$

*C.* $Q_{\text{to } A} = Q_{\text{away from } B}$; $|\,\Delta T_A\,| > |\,\Delta T_B\,|$

*D.* $Q_{\text{to } A} = Q_{\text{away from } B}$; $|\,\Delta T_A\,| = |\,\Delta T_B\,|$

***E.*** $\boldsymbol{Q_{\text{to } A} = Q_{\text{away from } B}}$**;** $\boldsymbol{|\,\Delta T_A\,| < |\,\Delta T_B\,|}$

# Results are Consistent

| | Spring 2003 | | Spring 2004 | |
|---|---|---|---|---|
| | F-Resp | Verbal | Verbal | Eqn |
| $\Delta T_{LSH} > \Delta T_{GSH}$ | 62% | 71% | 60% | 66% |
| $\Delta T_{LSH} = \Delta T_{GSH}$ | 22% | 12% | 13% | 8% |
| $\Delta T_{LSH} < \Delta T_{GSH}$ | 16% | 17% | 27% | 26% |
| $Q_{AWAY} \neq Q_{TO}$ | -- | 17% | 25% | 20% |

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**Results appear to be consistent across question format, instructor, and semester.**

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*“During the time it takes for the system to reach equilibrium…”*

was changed to:

*“During the time it takes for the object and the liquid to reach a common final temperature…”*

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*“During the time it takes for the system to reach equilibrium…”*

was changed to:

*“During the time it takes for the object and the liquid to reach a common final temperature…”*

► No significant change in the number of “*temperature changes are equal*” responses.

# Follow-up Interviews

Summer and Fall 2003, Spring and Summer 2004
(Different instructors and class formats)
($N = 34$)

- Math errors appeared more frequently than on the free response quizzes (~30%)
- Few conceptual errors observed

# Mathematical Errors

- Errors resulting from manipulations of equations (such as $Q = mc\Delta T$ as well as proportional reasoning difficulties)
- Not necessarily indicative of poor conceptual understanding (based on evidence of interview responses)
- Not often seen in answers to free response quizzes
  - Interviews allow us to probe student responses in depth
- Apparently a significant source of student confusion

# Conclusion

- Students' reasoning in calorimetry appears to be reproducible across semesters, instructors, and class format
- Weak mathematical skills often appear to function as a roadblock to qualitative understanding