Student understanding of entropy and the second law of thermodynamics

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Overview

- Introduction
- State-function property of entropy
  - Cyclic process question
  - First entropy tutorial
- Entropy in Spontaneous Processes
  - General context questions
    - Free-response
    - Multiple-choice
  - Concrete context question
  - Second entropy tutorial
- Conclusions

Thermodynamics Project

- Objectives: (a) To investigate students' qualitative understanding of entropy, the second law of thermodynamics, and related topics in a second-semester calculus-based physics course*; (b) To develop research-based curricular materials
- In collaboration with John Thompson at the University of Maine and David Meltzer at the University of Washington on investigations in an upper-level undergraduate thermal physics course

**Previous work on related topics: M. Cochran (2002)*

Context of Investigation

Second semester calculus-based introductory physics course

≈ 90% of students have taken high school physics

≈ 90% have completed college chemistry course where entropy is discussed

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Cyclic process question

Consider a heat engine that uses a fixed quantity of ideal gas. This gas undergoes a *cyclic process* which consists of a series of changes in pressure and temperature. The process is called "cyclic" because the gas system repeatedly returns to its original state (that is, same value of temperature, pressure, and volume) once per cycle.

Consider one complete cycle (that is, the system begins in a certain state and returns to that *same* state).

a) Is the *change* in temperature (ΔT) of the gas during one complete cycle *always equal to zero for any cyclic process* or ***not*** *always equal to zero for any cyclic process*? Explain.

b) Is the *change* in internal energy (ΔU) of the gas during one complete cycle *always equal to zero for any cyclic process* or ***not*** *always equal to zero for any cyclic process*? Explain.

c) Is the *change* in entropy (ΔS) of the gas during one complete cycle *always equal to zero for any cyclic process* or ***not*** *always equal to zero for any cyclic process*? Explain.

d) Is the net heat transfer to the gas during one complete cycle *always equal to zero for any cyclic process* or ***not*** *always equal to zero for any cyclic process*? Explain.

Cyclic Process Question Data

Cyclic Process Pre-Instruction (*N* = 190)							
a. Temperature		**b. Internal Energy**		**c. Entropy**		**d. Heat transfer**	
=0	≠0	**=0**	≠0	**=0**	≠0	=0	**≠0**
84%	16%	**84%**	16%	**55%**	45%	46%	**54%**

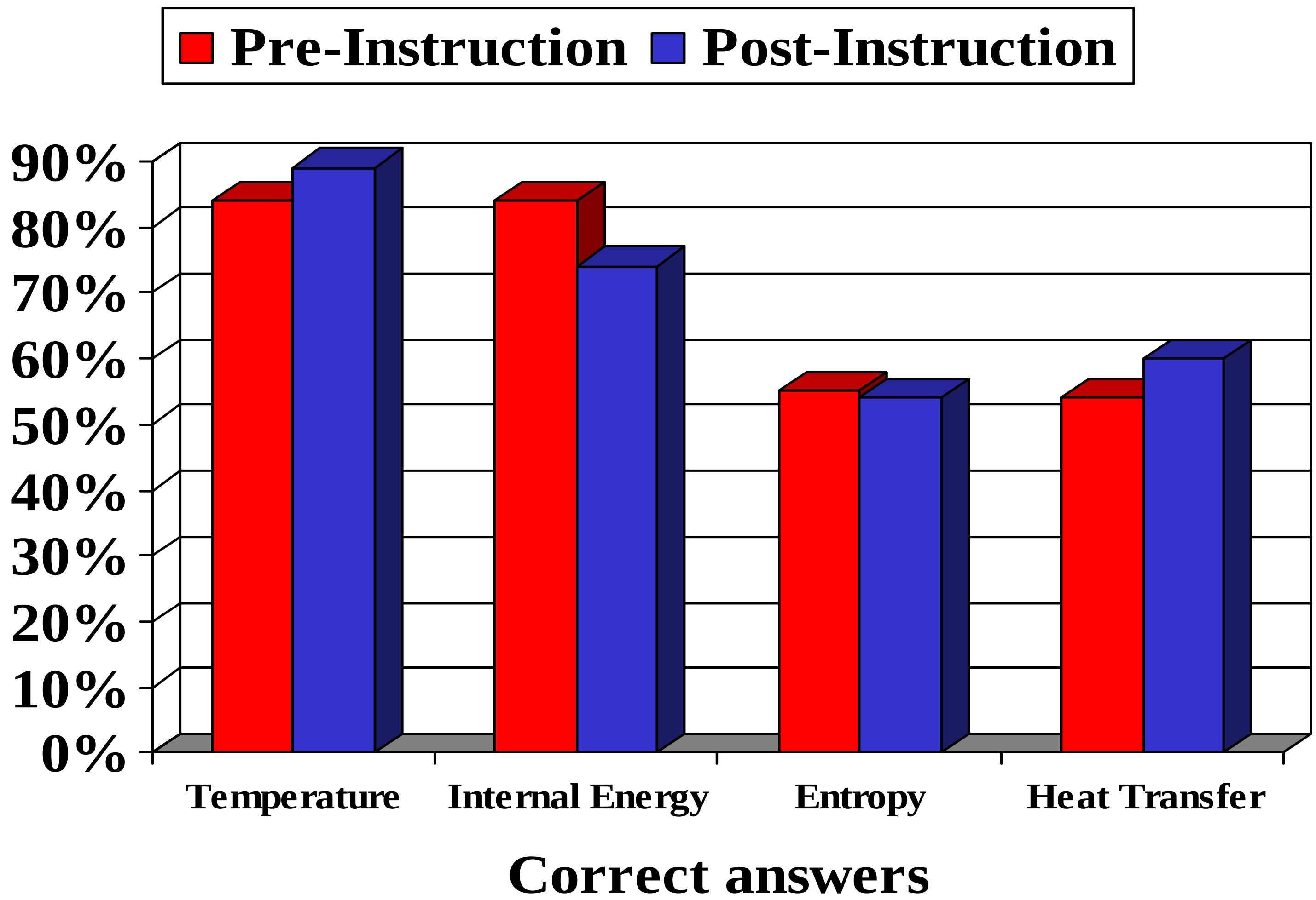
- 16% said the change in temperature would not be equal to zero
- 55% stated the change in entropy for the cycle would equal zero

Correct answers in red boxes

Entropy Tutorial Spring 2005

- Focused on the state-function property of entropy
- Built off first law worksheet that students had done the previous week
- Developed from U Maine question about three different processes
- Stripped down version for algebra-based course using only two of three processes

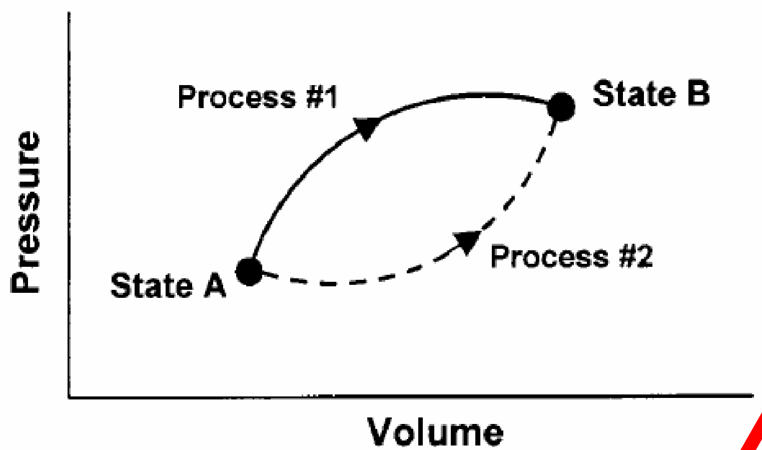
Pre-/Post-Instruction Comparison



Consistent with previous research

Meltzer (*2004*)

This *P-V* diagram represents a system consisting of a fixed amount of ideal gas that undergoes two ***different*** processes in going from state A to state B:



Which would produce the largest change in the total energy of all the atoms in the system: ***Process #1, Process #2, or both processes produce the same change?***

2001: **73%** correct answer ($N = 279$)

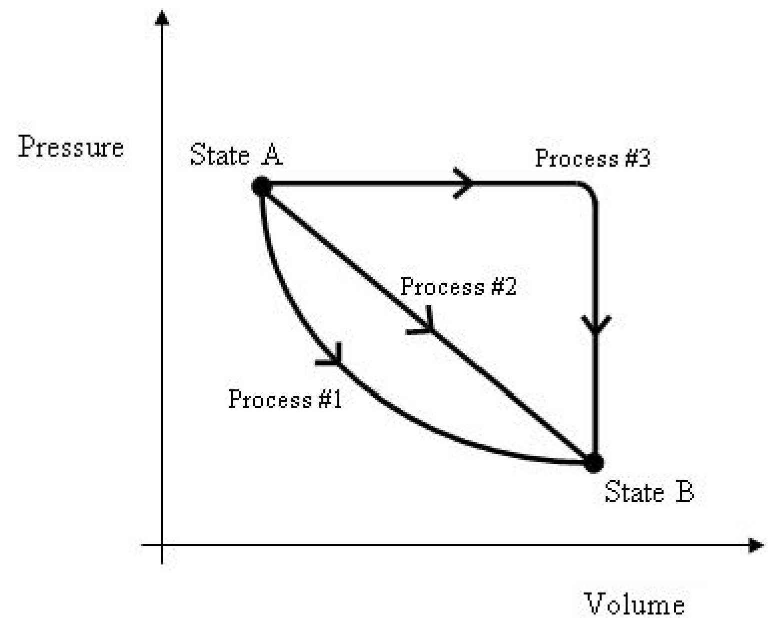
Is Q for Process #1 ***greater than***, ***less than***, or ***equal to*** that for Process #2?

	1999	2000	2001
Incorrect	$N = 186$	$N = 188$	$N = 279$
$Q_1 = Q_2$	31%	43%	41%

Cyclic Process Post-Instruction ($N = 190$)							
Temperature		Internal Energy		Entropy		Heat transfer	
=0	≠0	**=0**	≠0	**=0**	≠0	=0	**≠0**
89%	11%	**74%**	26%	**54%**	46%	40%	**60%**

PV-diagram question

This *P*-*V* diagram represents a system consisting of a fixed amount of ideal gas that undergoes three different processes in going from state A to state B:



Rank the change in entropy of the system for each process.

NOTE: ΔS_1 represents the change in entropy of the system for Process #1, etc.

A. $\Delta S_3 < \Delta S_2 < \Delta S_1$

B. $\Delta S_1 < \Delta S_2 < \Delta S_3$

C. $\Delta S_1 = \Delta S_2 < \Delta S_3$

D. $\Delta S_1 = \Delta S_2 = \Delta S_3$

E. Not enough information

PV-diagram post-test results

Algebra-based course	Sample	% correct
Control Group	($N = 109$)	61%
Intervention Group	($N = 60$)	78%

$p < 0.03$ (Binomial Proportions Test)

Calculus-based course	Sample	% correct
All students	($N = 341$)	67%

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Spontaneous Process Question

For each of the following questions consider a system undergoing a naturally occurring ("spontaneous") process. The system can exchange energy with its surroundings.

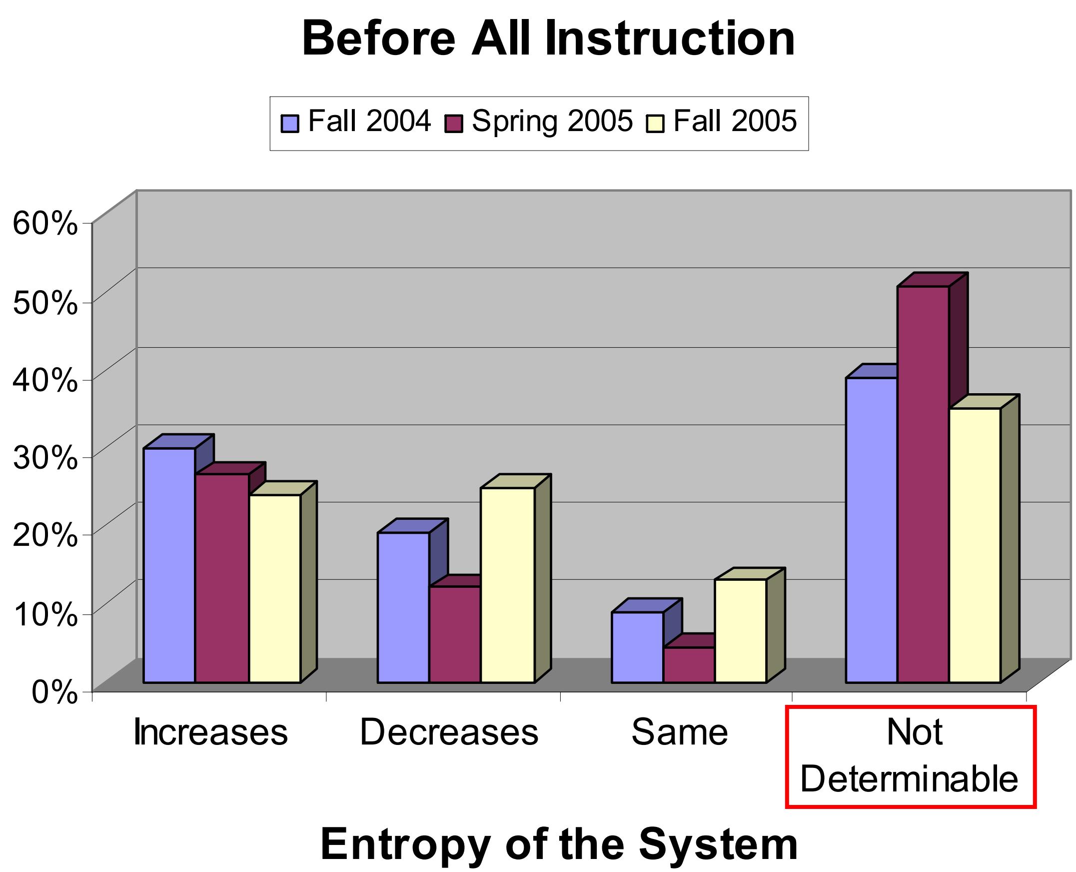
A. During this process, does the entropy of the **system** [S_{system}] *increase*, *decrease*, or *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

B. During this process, does the entropy of the **surroundings** [$S_{surroundings}$] *increase*, *decrease*, or *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

C. During this process, does the entropy of the system *plus* the entropy of the surroundings [$S_{system} + S_{surroundings}$] *increase*, *decrease*, or *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

Responses to Entropy Question

Fall 2004 (N = 406), Spring 2005 (N = 132), & Fall 2005 (N = 360)



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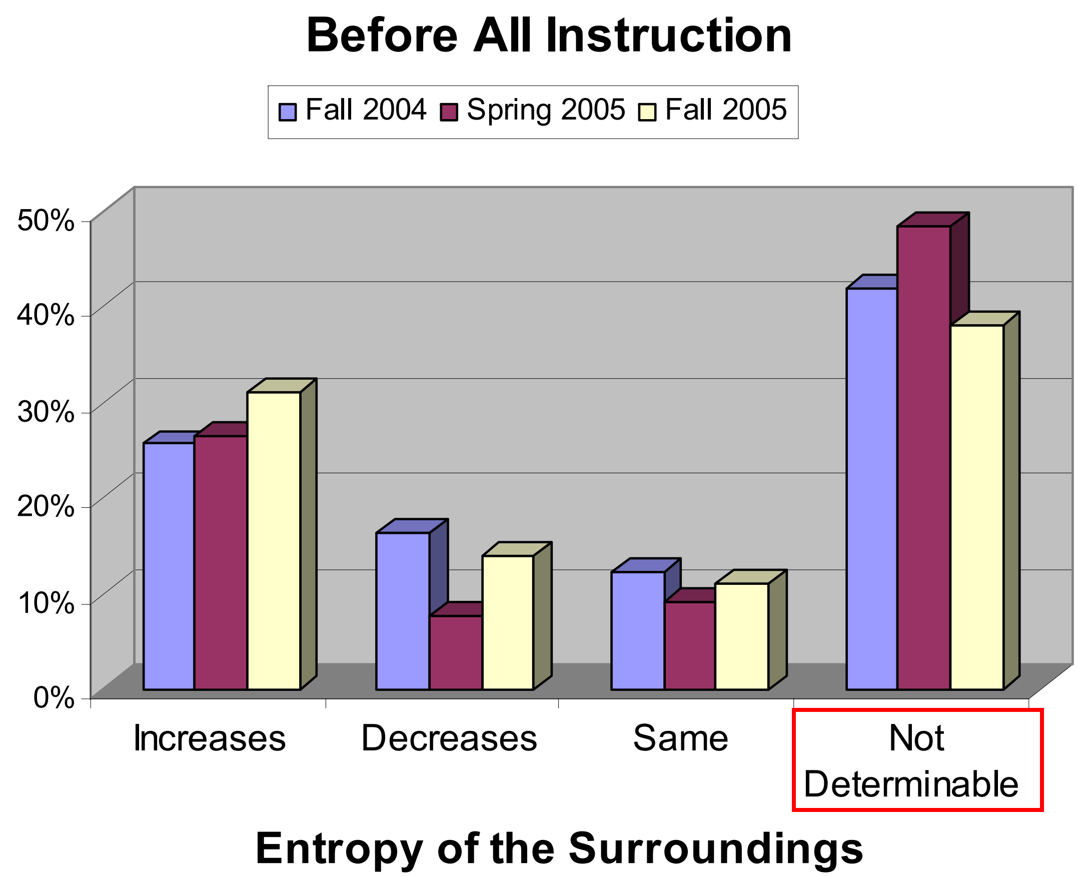

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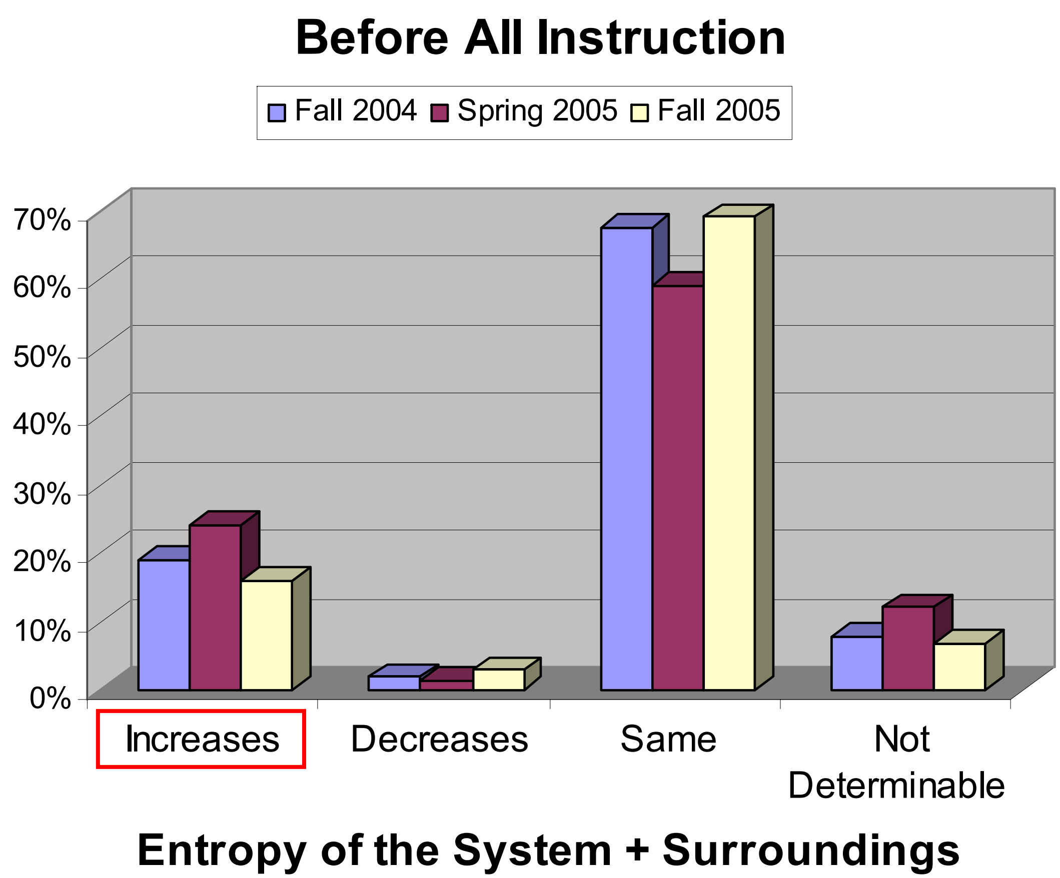


Pre-Instruction Results

Fall 2004 & Spring 2005 (*N* = 538)

- 48% of student responses were consistent with some sort of "conservation" principle, for example:
  - A. increases [*decreases*], B. decreases [*increases*], and so C. stays the same
  - A. not determinable, B. not determinable, but C. stays the same because entropy [*energy*, *matter*, *etc*.] is conserved
- Only 4% gave a correct response for all three parts

Post-Instruction Question

Final Exam, Fall 2004 (N = 539)

A subsystem *A* is in thermal contact with its environment *B*, which together comprise an isolated system. Consider the following situations:

I. Entropy of system increases by 5 J/K; entropy of the environment decreases by 5 J/K.
II. Entropy of system increases by 5 J/K; entropy of the environment decreases by 3 J/K.
III. Entropy of system increases by 3 J/K; entropy of the environment decreases by 5 J/K.
IV. Entropy of system decreases by 3 J/K; entropy of the environment increases by 5 J/K.

Which of the above four situations can actually occur in the real world?

A. I only
B. II only
C. III only
D. II and III only
E. II and IV only

S_{TOT} remains the same

S_{TOT} increases **(Correct)**

A. 54%
B. 5%
C. 7%
D. 4%
E. 30%

Pre- and Post-Instruction Comparison

The results of the final-exam question are most directly comparable to the responses on part C of the pretest:

C. During this process, does the entropy of the system *plus* the entropy of the surroundings [$S_{system} + S_{surroundings}$] *increase*, *decrease*, or *remain the same*, or is this not determinable with the given information? ***Explain your answer.***

S_{TOT} stays the same	
Pretest	Final Exam
67%	**54%**

S_{TOT} increases	
Pretest	Final Exam
19%	**30%**

Correct answer

Interview Data

Fall 2004 & Spring 2005 ($N = 16$)

- Hour-long interviews with student volunteers
  - conducted after instruction on all relevant material was completed

- Students asked to respond to several questions regarding entropy and the second law

Interview Results

- Nearly half asserted that total entropy could either increase ***or*** remain the same during spontaneous process

➡ Multiple-choice options altered for Spring 2005 to allow for "increase or remain the same" response

Post-Instruction Question

Spring 2005 ($N = 386$)

A subsystem *A* is in thermal contact with its environment *B* and they together comprise an isolated system that is undergoing an irreversible process. Consider the following situations:

I. Entropy of system increases by 5 J/K; entropy of the environment decreases by 5 J/K.
II. Entropy of system increases by 5 J/K; entropy of the environment decreases by 3 J/K.
III. Entropy of system increases by 3 J/K; entropy of the environment decreases by 5 J/K.
IV. Entropy of system decreases by 3 J/K; entropy of the environment increases by 5 J/K.

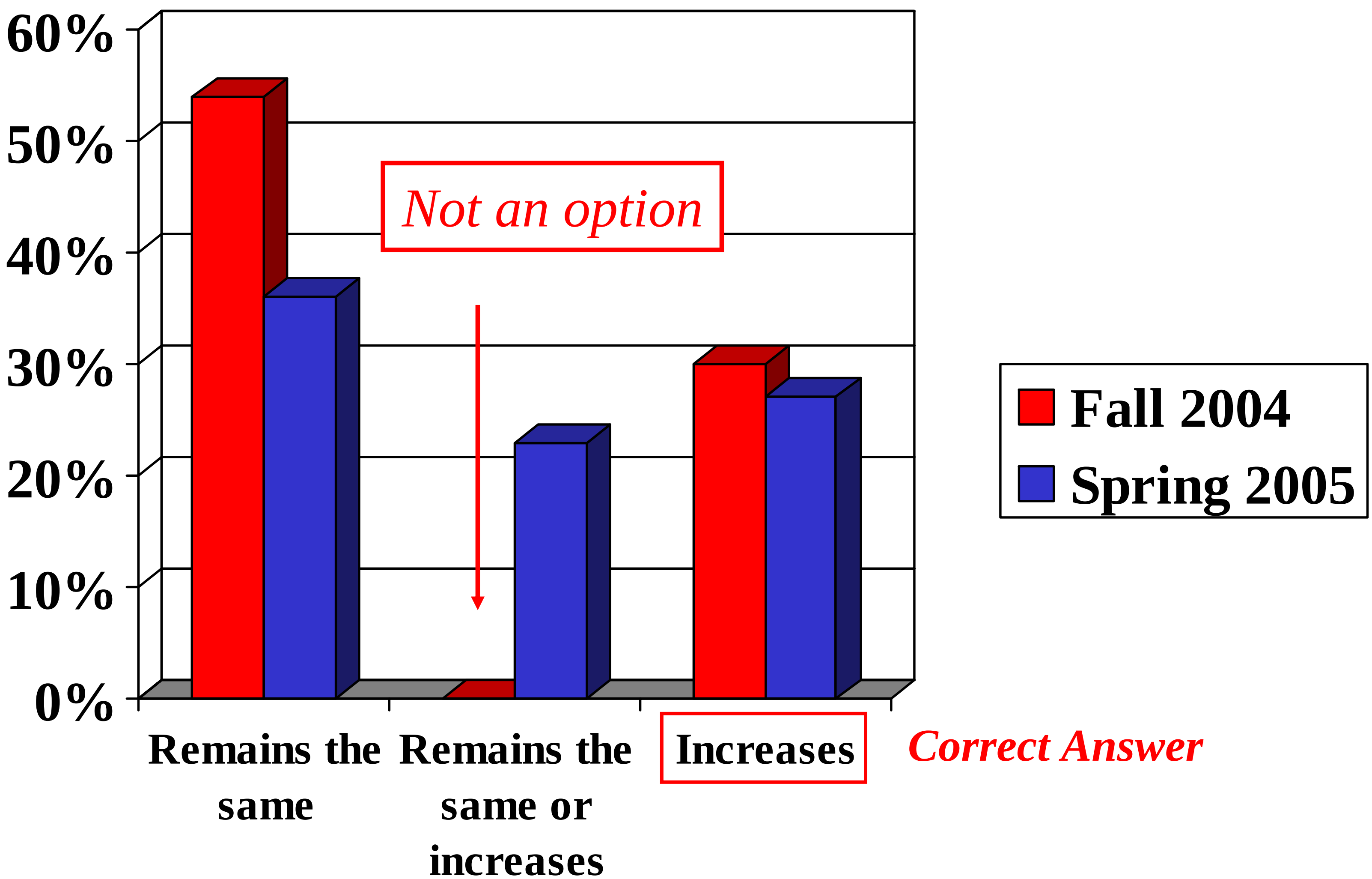
Which of the above four situations can actually occur ?

A. I only	A. 36%
B. II only	B. 12%
C. III only	C. 2%
D. II and IV only	D. 27%
E. I, II, and IV only	E. 23%

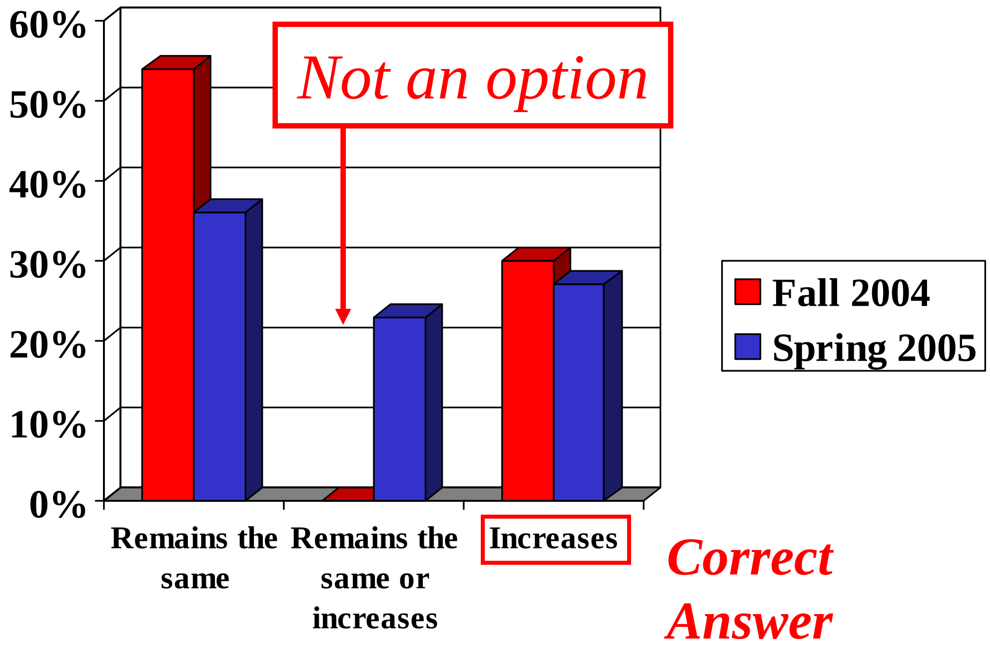
***S_{TOT} increases* (Correct)**

S_{TOT} remains the same or increases

Post-Instruction responses for S_{TOT}



Post-Instruction responses for S_{TOT}



Allowing for entropy to either remain the same or increase appears to more accurately reflect student thinking

Spontaneous Process Question

For each of the following questions consider a system undergoing a naturally occurring ("spontaneous") process. The system can exchange energy with its surroundings.

A. During this process, does the entropy of the **system** [S_{system}] *increase*, *decrease*, or *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

B. During this process, does the entropy of the **surroundings** [$S_{surroundings}$] *increase*, *decrease*, or *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

C. During this process, does the entropy of the system *plus* the entropy of the surroundings [$S_{system} + S_{surroundings}$] *increase*, *decrease*, or *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

Is the Question too General?

Spontaneous Process Question

For each of the following questions consider a system undergoing a naturally occurring ("spontaneous") process. The system can exchange energy with its surroundings.

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C. During this process, does the entropy of the system *plus* the entropy of the surroundings [$S_{system} + S_{surroundings}$] *increase*, *decrease*, or *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

Entropy Question in Context

Spring 2005

An object is placed in a thermally insulated room that contains air. The object and the air in the room are initially at different temperatures. The object and the air in the room are allowed to exchange energy with each other, but the air in the room does not exchange energy with the rest of the world or with the insulating walls.

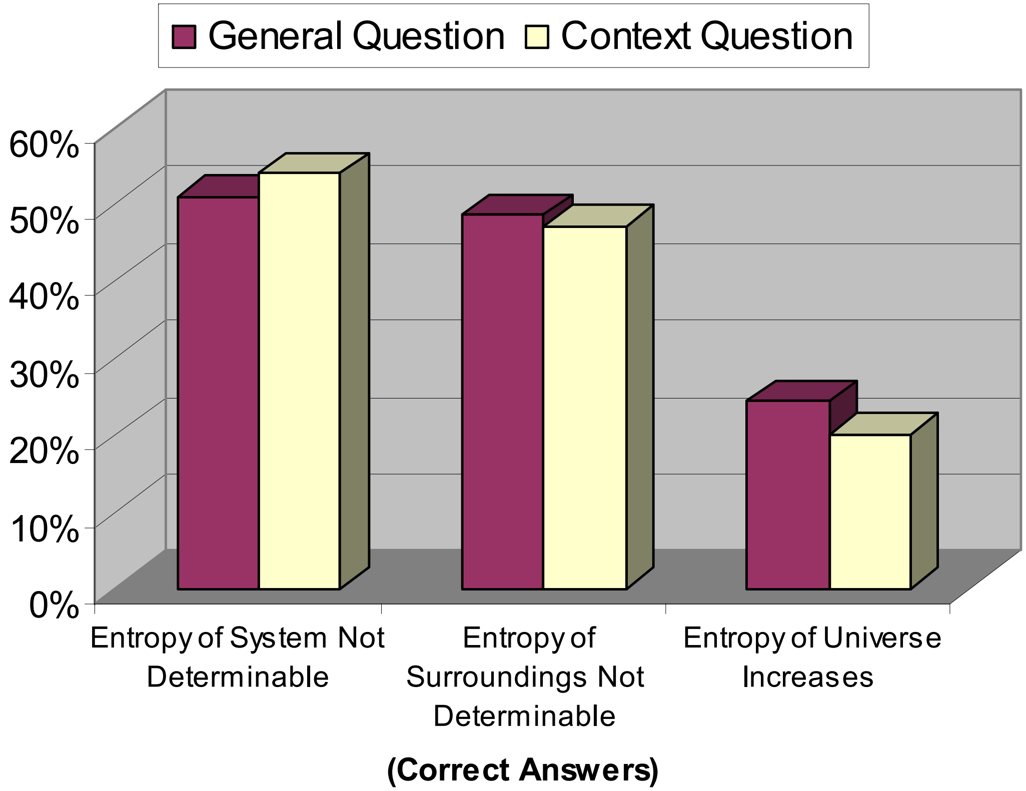
A. During this process, does the entropy of the **object** [S_{object}] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

B. During this process, does the entropy of the **air in the room** [S_{air}] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

C. During this process, does the entropy of the object *plus* the entropy of the air in the room [$S_{object} + S_{air}$] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

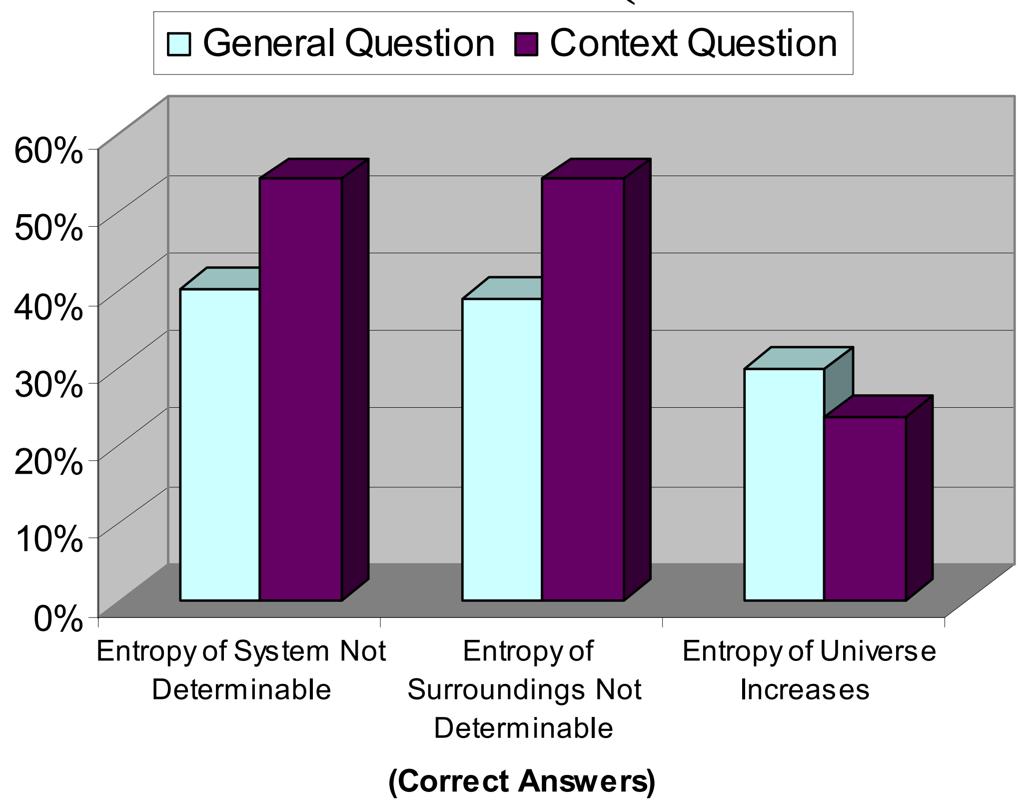
D. During this process, does the entropy of the **universe** [$S_{universe}$] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

General vs. Context (Pre-Instruction)



- Students' correct responses initially show consistency in and out of context

General vs. Context (Post-Instruction)



- Student responses initially show consistency in and out of context
- After instruction students seem willing to apply different rules for a problem in context

General and Context Comparison

Placing the question in context:

- does not yield a higher proportion of correct answers concerning entropy of the universe, pre- or post-instruction

- *does* yield a higher proportion of correct answers concerning entropy of the system and surroundings, post-instruction only

More on Concrete Context Question

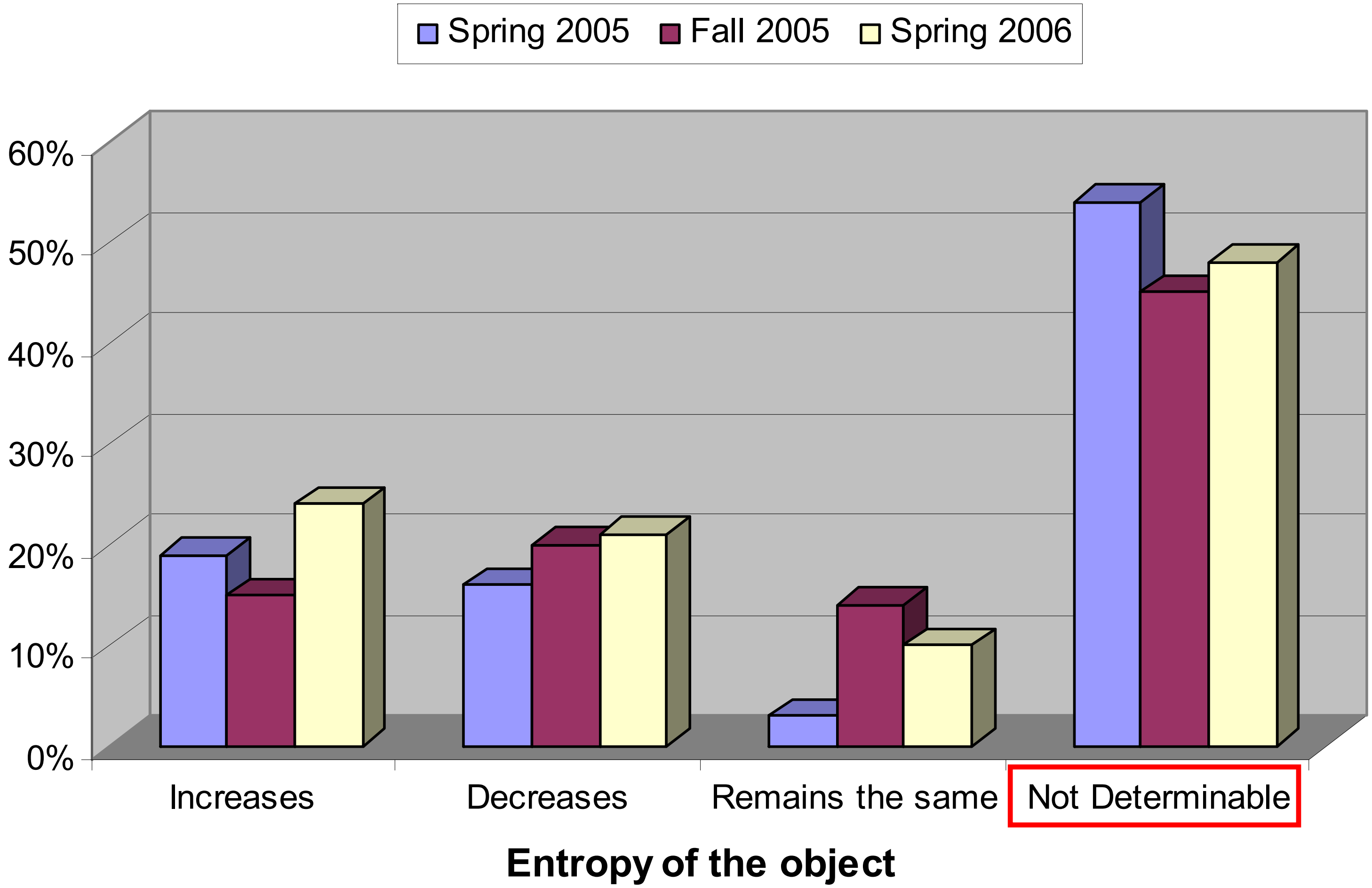
An object is placed in a thermally insulated room that contains air. The object and the air in the room are initially at different temperatures. The object and the air in the room are allowed to exchange energy with each other, but the air in the room does not exchange energy with the rest of the world or with the insulating walls.

A. During this process, does the entropy of the **object** [S_{object}] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

B. During this process, does the entropy of the **air in the room** [S_{air}] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

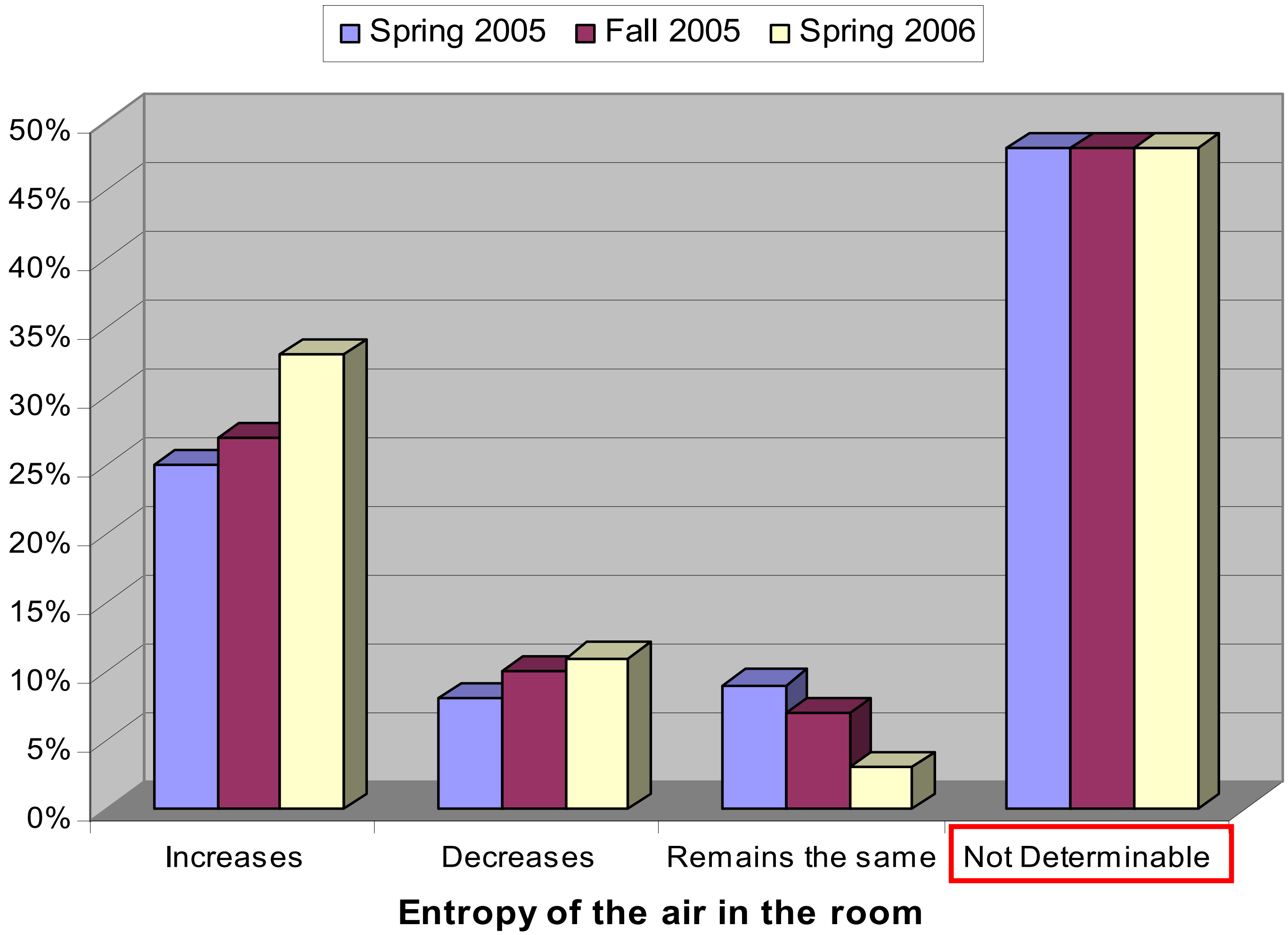
Pre-Instruction Results - Entropy of object

Spring 2005 (*N* = 155), Fall 2005 (*N* = 207), Spring 2006 (*N* = 75)



Pre-Instruction Results – Entropy of air in room

Spring 2005 (*N* = 155), Fall 2005 (*N* = 207), Spring 2006 (*N* = 75)



Student explanations

Total Sample $N = 437$

≈ 50% of students gave a correct response ("not determinable")

≈ 30% gave a correct response with acceptable explanation

Example of acceptable student response:

"[not determinable because] depends on which is the higher temp. to determine increase or decrease"

Student explanations

Total Sample $N = 437$

Tendency to *assume* direction of heat flow for "system"

- Cited as justification for claiming object (or air) entropy increases (or decreases)
- About 60% of all increase/decrease responses were based on this assumption

Concrete Context Question

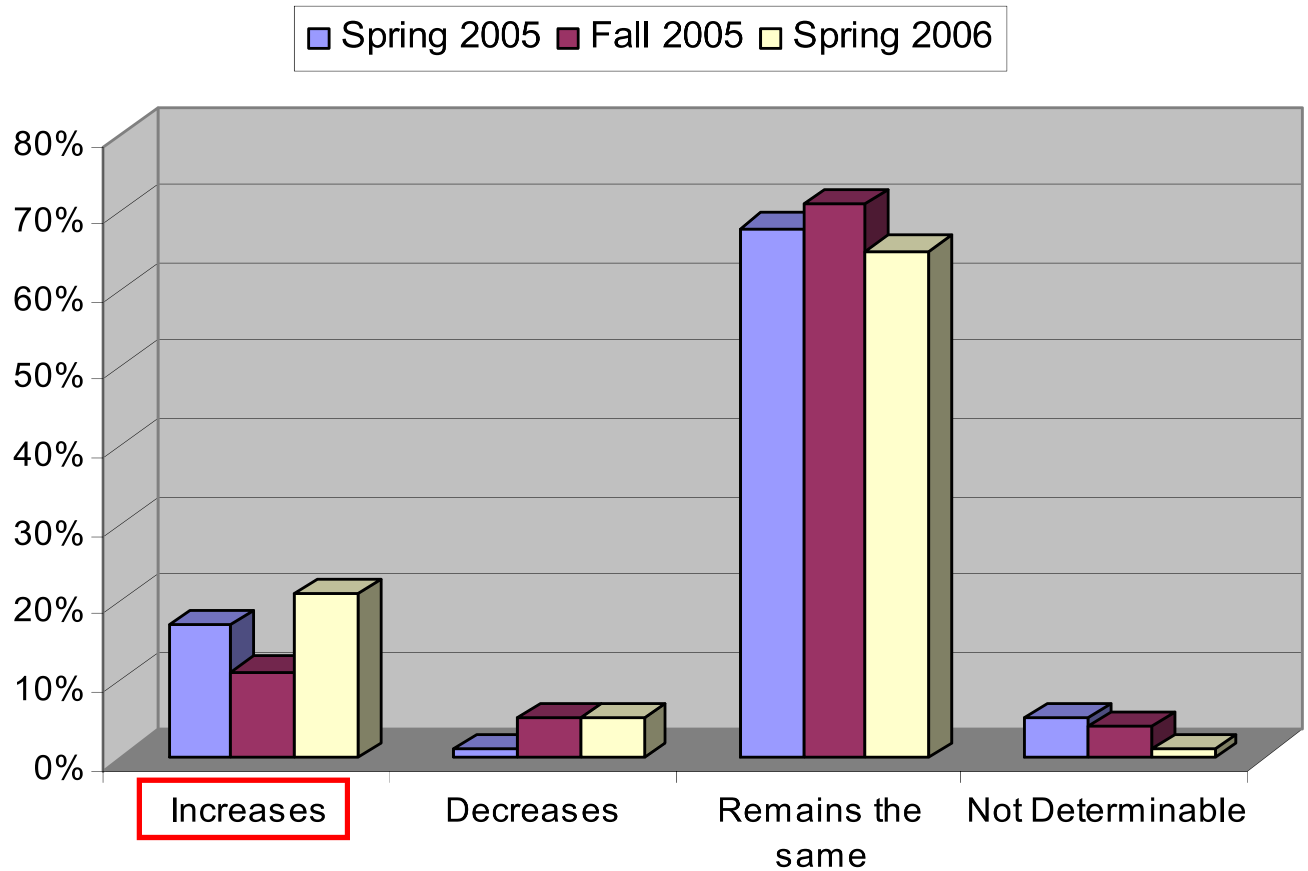
An object is placed in a thermally insulated room that contains air. The object and the air in the room are initially at different temperatures. The object and the air in the room are allowed to exchange energy with each other, but the air in the room does not exchange energy with the rest of the world or with the insulating walls.

C. During this process, does the entropy of the object *plus* the entropy of the air in the room [$S_{object} + S_{air}$] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

D. During this process, does the entropy of the **universe** [$S_{universe}$] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

Pre-Instruction Results – Object + Air

Spring 2005 (*N* = 155), Fall 2005 (*N* = 207), Spring 2006 (*N* = 75)



Entropy of the object plus the air in the room

Object + Air Explanations

Entropy remains the same because…

- energy or entropy is "conserved"
- system is isolated by walls (or it's a "closed system")
- total entropy of object <u>and</u> air in room doesn't change

Entropy of Object + Air Conserved

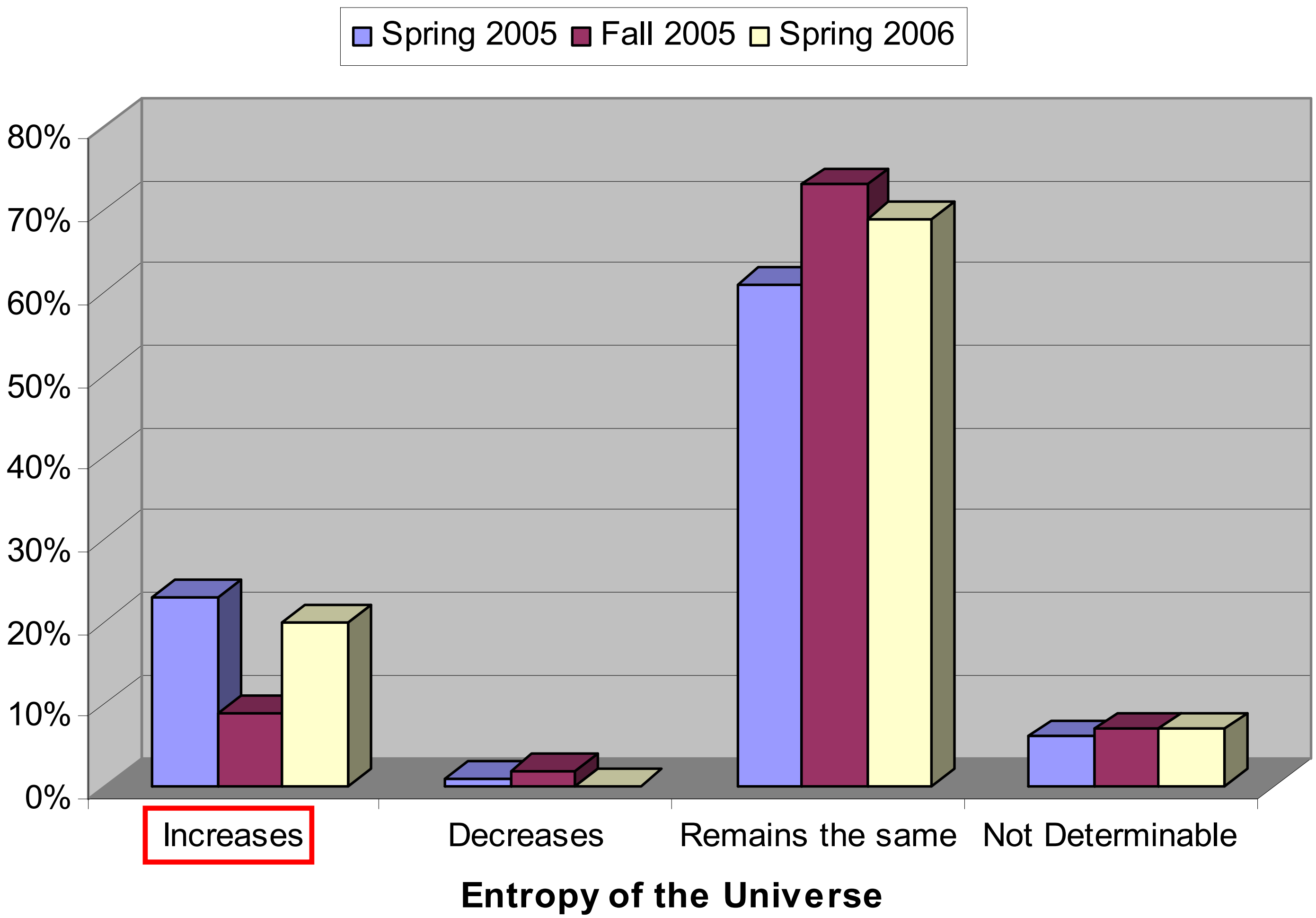
~50% of all student responses were consistent with some sort of "conservation" principle, for example:

- A. increases [*decreases*], B. decreases [*increases*], and so C. stays the same
- A. not determinable, B. not determinable, but C. stays the same because entropy [*energy*, *matter*, *etc*.] is conserved

Nearly identical to results of general context question

Pre-Instruction Results – Universe

Spring 2005 (*N* = 155), Fall 2005 (*N* = 207), Spring 2006 (*N* = 75)



Entropy of the Universe Explanations

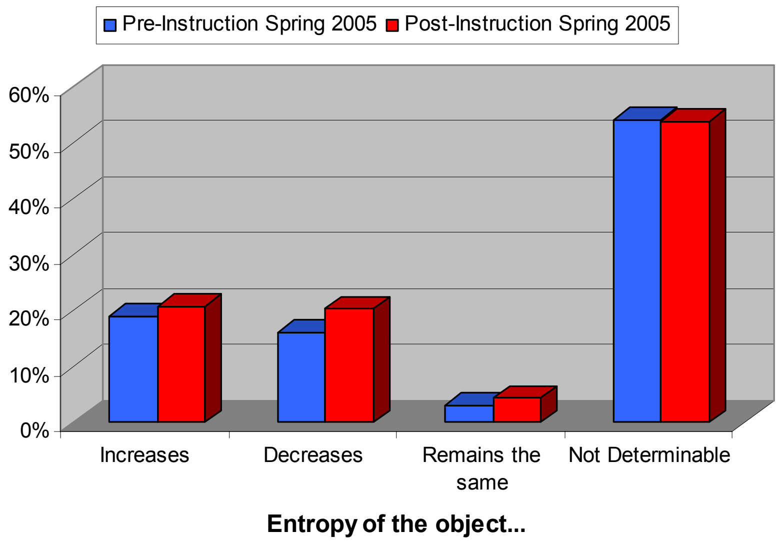
Entropy remains the same because…

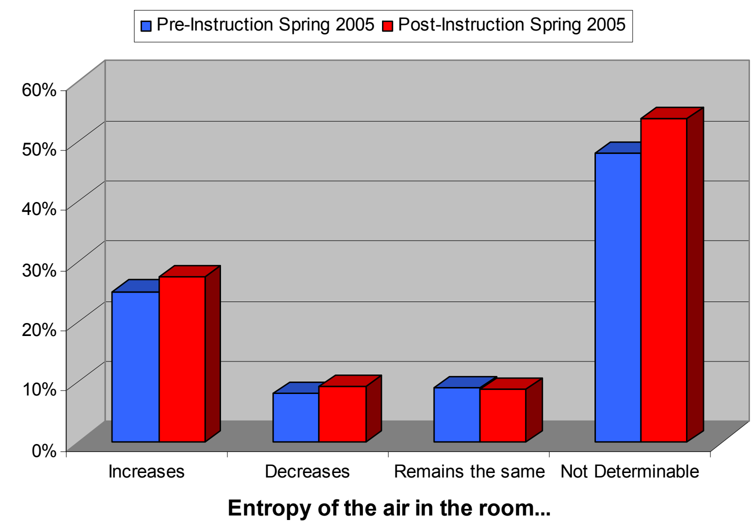
- process doesn't affect the universe due to insulation
  - consistent with "universe" being defined as only that which is *outside* the room
- entropy is constant
- universe is too large to change in entropy

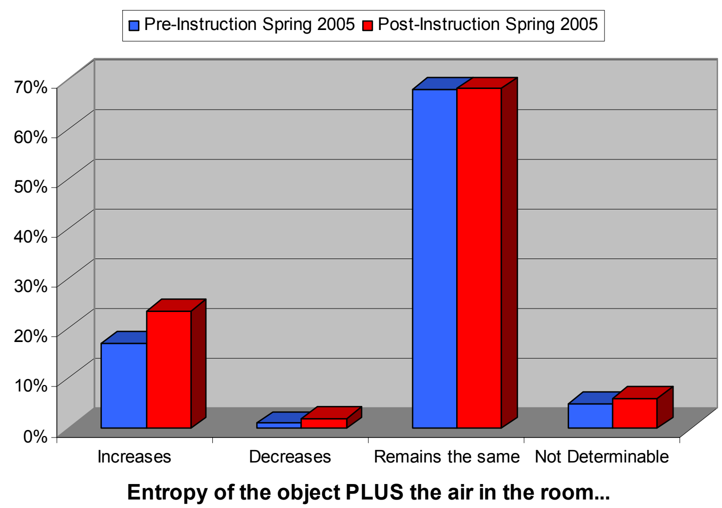
Pre- and Post-Instruction Assessment

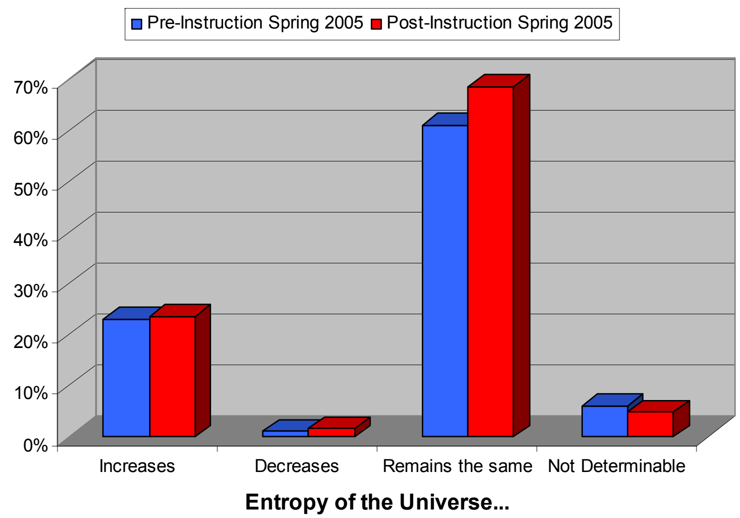
Spring 2005, attempted modified instruction using our first worksheet focusing on the state-function property of entropy

Pre- v. Post-Instruction Data







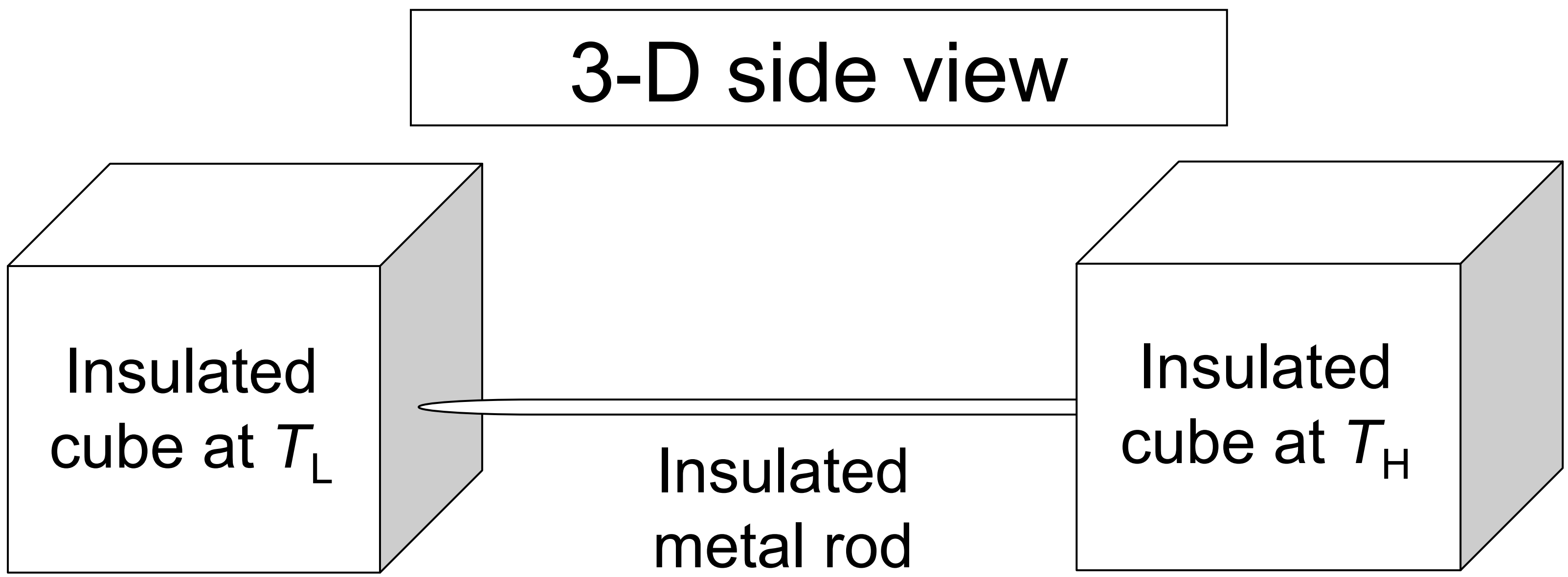


Second-tutorial Strategy and Goals

Build off of correct student ideas (e.g., heat flow direction)

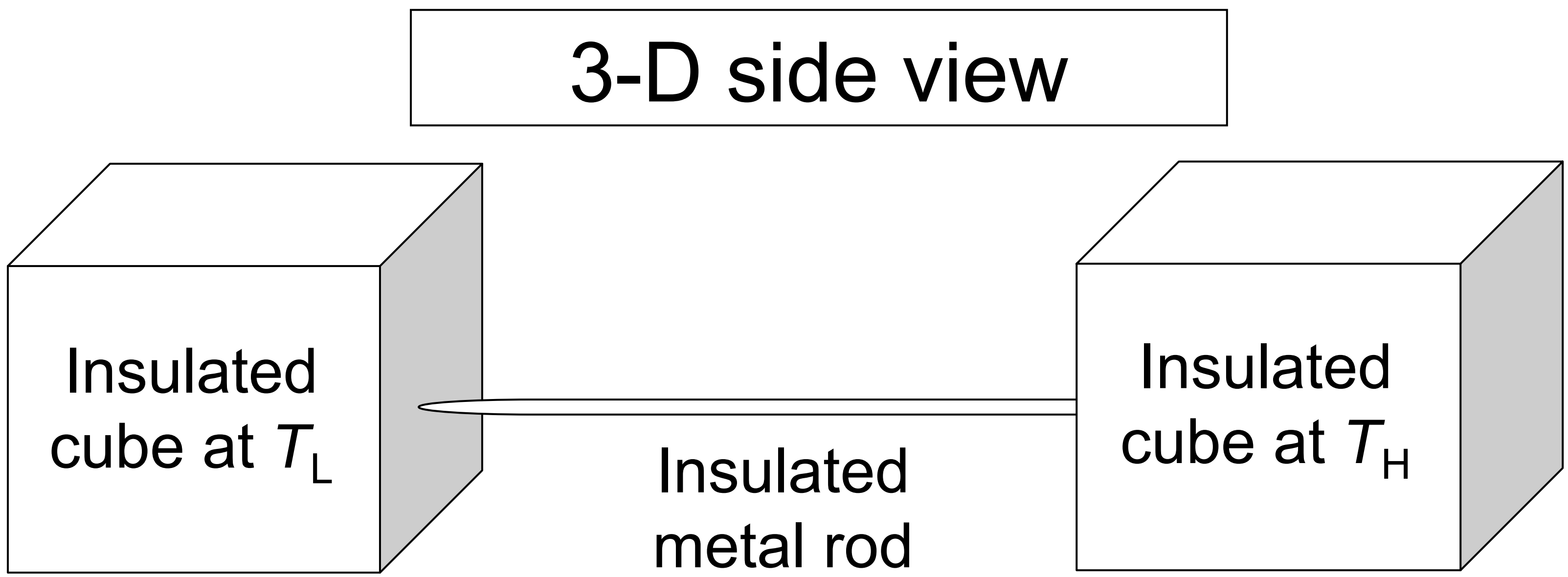
- For any real process, the entropy of the universe increases (i.e., entropy of the universe is *not* conserved).
- Entropy of a particular system can decrease, so long as the surroundings of that system have a larger increase in entropy.
- Universe = system + surroundings; that is, “surroundings” is defined as everything that isn’t the system.
- Reversible processes are idealizations, and don’t exist in the real world; however, for these ideal cases, total entropy remains the same.

Tutorial Design



- Elicit student ideas regarding entropy "conservation"
- Identify Q_H, Q_L, and discuss energy conservation
- Calculate ΔS_H, ΔS_L, compare the magnitudes, and find sign of change in total entropy

Tutorial Design



- Address ideas relating universe to system and surroundings
- Discuss arbitrary assignment of “system” and “surroundings”

Conclusions

- Observed persistent pattern of student ideas related to spontaneous processes.
- Initial attempts at tutorial worksheets were ineffective at addressing certain student difficulties.
- New worksheet created from ongoing research, currently undergoing classroom testing.