1. Introduction and Goals

It is well known and documented that many students have significant difficulty with using mathematics in introductory physics courses, and that mastery of requisite math is a strong predictor of success in these courses (e.g., Meltzer 2002). This project is aimed at addressing this critical STEM education issue through the combination of two important ideas. First, as discussed in Sections 2a-c, it is becoming clear that in addition to difficulties with the underlying mathematical structure or skills, a major component contributing to student difficulties with using mathematics in physics is the rich and varied contexts represented in physics such as the complex notation of physics expressions themselves. Second, as detailed in Section 2d, there are strong theoretical, empirical, and practical reasons to consider that well-designed online mastery practice units with immediate feedback and spaced practice may be a powerful tool for helping students to become more fluent with math skills in the context of introductory level physics tasks. The central hypothesis of this project is that carefully designed systematic practice with representations that use common physics notation will allow for more automated processing of these expressions, leading to improved fluency with math in physics contexts, ultimately resulting in better performance in the course. Specifically, this project will focus on improving fluency of specific skills in three math topics relevant to physics: trigonometry, algebra, and graphing. The goal is to develop, implement, and assess a set of practice assignments called *Math Practice* that is based on *Essential Skills*, an existing and empirically successful online platform and theoretical framework, developed by one of the project PIs (Heckler) and used at Ohio State University (OSU) in its introductory physics courses. The project also relies on the considerable physics education expertise of David Meltzer at Arizona State University (ASU) and his experience in studying and helping students with difficulties in math used in physics, and on the significant introductory physics instructional experience and emerging educational research interests of Beatriz Burrola Gabilondo (OSU). The ultimate goal of this project is to improve student fluency with math skills critical for introductory physics which in turn leads to student success in the course, with a special focus on topics and practice materials that help underprepared students.

The project has at its heart three research questions:

RQ1. To what extent does Math Practice improve student academic performance, motivational factors, and retention in introductory physics?

RQ2. Which practice topics and methods and are most beneficial?

RQ3. To what extent do benefits from Math Practice depend on student-level demographic factors, motivational factors, and academic performance measures?

To address these research questions, the project has five goals:

Goal 1. <u>Iteratively build a list of target math skills</u>

Critical Elements:

- Based on existing literature and/or from the data accrued by the project.
- Special focus on skills that low-achieving students find difficult.
- Skills critical for algebra or calculus-based introductory physics.

Outcomes: list of specific target math skills in trigonometry, algebra and graphing relevant to introductory physics.

Goal 2. <u>Iteratively develop effective materials for practice</u>

Critical Elements:

- Design based on effective practices from Cognitive/Educational Psychology and Discipline-based Education research, and will investigate the effectiveness of various methods.
- Empirical data from assessments (Goal 4) will be used for continual improvement.

Outcomes: Effective materials for Math Practice for both algebra and calculus-based courses.

Goal 3. Implement Essential Skills platform using controlled-study designs

Critical Element:

- Seamless logistical integration of Math Practice into OSU and ASU courses.

Outcomes: Successful implementation of experimental studies to > 2500 students/semester.

Goal 4. Assess student performance and motivational factors

Critical Element:

- Iteratively build relevant math skills assessment based on PI Meltzer existing work.

Outcomes: pre, post data on math skills, physics assessments, surveys on motivational factors, grade and retention data, and in-task performance data from ES application.

Goal 5. Analyze results, evaluate progress, disseminate findings

Critical Elements

- Timely analysis of data for use in iterative improvement.
- Systematic engagement of External Advisory board.

Outcomes: Data-informed iterative design of materials and experimental studies. Full analysis and reporting of results. Begin engagement of 2-3 potential future pilot institutions.

2. Background and Rationale

This project is a natural continuation and synthesis of several other projects carried out in recent years by the PIs, including *Identifying and Addressing Mathematical Difficulties in Introductory Physics Courses (DUE #1504986; PI: D. Meltzer).* In that project, PI Meltzer and his students administered written diagnostic instruments to several thousand introductory physics students and interviewed more than 70 students. This allowed documentation with high confidence in the extent and nature of difficulties with basic mathematics skills that constitute obstacles to physics students' performance. Below we discuss how these results, together with work by others, provide context and perspective for the current project.

a. The issue of mathematics skills in introductory physics

Introductory physics courses, both algebra- and calculus-based, pose a substantial challenge to students' mathematical skills, and these challenges are to some degree widely recognized among physics instructors. For example, it is common to hear instructors complain that their students have trouble with seemingly "simple" skills such as algebraic and trigonometric manipulation. What is perhaps less well appreciated is (1) how much of an impact these difficulties with basic skills can have on students' course performance, and (2) the extent to which students' difficulties are rooted in discomfort with symbolic representation itself, and with the very particular ways symbols are used in physics courses, in contrast to mathematics courses.

Much as physics instructors might try to minimize the burden of sheer calculation in their classes, their students are inevitably required to make sense of and manipulate many mathematical expressions in order to be successful. If students get bogged down with such tasks, needing much time and effort to carry out basic mathematical procedures, their ability to focus on, understand, and apply physics principles can be greatly reduced. In this way, difficulties with basic mathematics skills can have a disproportionate negative impact on students' course performance. Research by ourselves and others has revealed a previously underappreciated degree of such difficulties, one that is traceable in part to challenges posed by the use of symbols to represent constant and variable quantities, and by the "messiness" of mathematical expressions as used in physics.

Students whose primary exposure to and practice with algebra is in mathematics courses are often surprised and unsettled by the complicated appearance of simple algebraic expressions when used in a physics context. Most numbers are attached to physical units involving a diverse array of symbols such as Ω, eV, kg-m/s^2, N-s, and J/C; variables represent physical quantities that also have units, and graphs—and slopes of graphs—nearly always involve manipulation of units. Trigonometric functions typically appear in complicated expressions involving both known and unknown quantities, with units present or implied, and require substantial manipulation before they can be evaluated numerically. Symbols such as those representing vectors are used in ways that are rarely or never seen in a typical mathematics class. Research by Torigoe and Gladding (2011) and more recently by ourselves (Meltzer and King, 2018) has shown that the mere use of symbols to replace numerical quantities (such as velocity, acceleration, and force) can lead to immediate degradation in students' problem-solving success. Research indicates that a large majority of physics students use highly inefficient "arithmetical" procedures to solve algebra problems, in which premature numerical substitution leads both to algebraic errors and to a complete inability to check and balance the physical units involved in an expression.

Below we present some examples from the pilot research of PI Meltzer that demonstrates the additional difficulty faced by physics students when numerical coefficients are replaced by symbols as commonly used in physics courses. Table 1 shows a dramatic reduction in performance for students in an algebra-based course when simple numbers are replaced with more complex notation or symbols. There is a similar, though not as pronounced, reduction for students in the calculus-based course.

Table 1: Student performance on solving two equations with two unknowns

Algebra Based Physics		**Calculus-based Physics**	
Equations to solve (for x and y)	% correct	Equations to solve (for x)	% correct
$3x = 2y$ $5x + y = 26$	70% (*N*=123)	$78.4 - y = 8x$ $0.5y = 2x$	87% (*N*=733)
$x \cdot \cos(20°) = y \cdot \cos(70°)$ $x \cdot \cos(70°) + y \cdot \cos(20°) = 10$	25% (*N*=150)	$a - y = bx$ $cy = dx$	63% (N=733)
$ax = by$ $bx + ay = c$	15% (*N*=150)		

Even in a simple, one-variable equation, in which students need only solve for "*x*," these differences persist, as shown in Table 2:

Table 2: Student performance on solving one equation with one unknown

	Equation to solve $5x + 3 = 2x + 5$ $x = ?$	 $ax + b = cx + d$ $x = ?$
Algebra based physics	96% (N=222)	57% (N=222)
Calculus based physics	96% (N=899)	87% (N=899)

Within the mathematics education literature itself, Booth et al. (2014) have enumerated specific difficulties with algebraic computations and manipulations encountered by introductory students *outside* the context of physics. Earlier, Kieran (1992) provided an extensive review of this topic. What is notable is that our investigations have clearly identified identical and persistent difficulties among college physics students, showing that issues thought to have been resolved within the context of high-school mathematics courses often linger into students' college studies. The implications of these research findings and empirical observation is that most introductory physics students require some improvements in their mathematical skills to have the best chance of success in their courses. The evidence is strong that the experience students have in their mathematics courses is simply inadequate to fully resolve the difficulties they encounter in applying mathematics to physics. In previous work, we have addressed these challenges by developing a research-based online "Essential Skills" tool that students are required to use each week. (Mikula & Heckler) We have documented significant positive impacts on students' course performance resulting from the use of this tool. The present project is to construct Math Practice content using the Essentials Skills platform and framework which will greatly expand the scope and utility of this tool, and to make it readily available to the broader community of physics instructors.

b. Physics Students Need Practice with Mathematics as Used in Physics

The equations that students encounter in introductory physics courses contain a welter of features and symbols not ordinarily present in typical mathematics classes. For example, virtually any introductory physics textbook will include extensive algebraic calculations that incorporate a bewildering variety of different physical units, both simple and composite, that students may never have previously encountered, such as: N, m/s^2, J, W, kg, Ω, Hz, s^{-1}, J/C, T. Students are expected to preserve these dimensional units in their equations and in each algebraic step, in order to check the consistency of units in each step and in their final answer. Moreover, even in the most basic introductory course, more than a dozen Greek letters are *commonly* used in algebraic equations, and provide an additional challenge for students who may not have seen them before. All of these symbols are commonly used in association with multiple subscripts and superscripts, adding to the symbolic complexity. Figure 1 to the right, for example, presents just a small sampling of equations that are found in virtually any introductory physics textbook. Note that the mathematical form of these equations is simple; most of them are in fact structurally identical. Yet, even though algebraic equations with familiar coefficients such as *a*, *b*, and *c* and variables *x* and *y* are formally identical to those that simply substitute Greek letters, letters with subscripts, or append "Ω," "s^{-1}," or "m/s^2," this formal equivalence is of little comfort to the introductory physics students who are bewildered by the differences.

$$f_s = \mu_s N$$
$$c = f\lambda$$
$$v_i = r\omega_i$$
$$a_c = r\omega^2$$
$$\gamma = \frac{C_p}{C_v}$$
$$E = h\nu$$
$$\rho = \frac{m}{V}$$
$$\tau = rf sin\varphi$$
$$\Phi_B = \mathrm{BA}\cos\theta$$
$$\alpha = \frac{\Delta\omega}{\Delta t}$$
$$v = 2\pi rf$$

Figure 1: Common equations

c. Review of the Literature, and Relevance to Current Project

The literature on the relation between mathematics knowledge and performance in physics courses is extensive. Here we review key research themes and identify their relationship to the current proposal.

To begin, we note that PI Meltzer (2002) reported a significant correlation between students' algebra skills and learning of physics as measured by performance on qualitative, conceptual physics questions which themselves required virtually no algebraic manipulation. However, the reasons for this relationship were not explored at that time and remained (and still remain) somewhat obscure.

Sherin (2001) has pointed out that "successful (physics) students learn to understand what equations say in a fundamental sense; they have a feel for expressions, and this guides their work." Sherin notes that students locate mathematical or physical meaning in certain equation patterns he called "symbolic forms." Recently, Rodriguez, Bain et al. (2018) and Rodriguez et al. (2018) applied this to graphing in a chemistry context, and Dorko and Speer (2015), discussed calculus students' difficulties in the context of symbolic forms. We appreciate this caution against focusing too narrowly on mathematical techniques and skills *in themselves* as presumed keys to success for introductory physics students. However, work by ourselves and others has shown that difficulties with those mathematical techniques and symbols can form a serious stumbling block to success of introductory physics students. For example, Redish and Kuo (2015) make this point; further, Dray and Manogue (1999; 2003; 2004) identified difficulties posed to physics students by the sharply divergent symbols and techniques used by the physics and mathematics communities for common procedures such as line and surface integrals. Still, the mathematical difficulties encountered by students are by no means all traceable to language mismatches in themselves. In their analysis of students' ideas and confusions regarding the "arrow" representation of electric field vectors, Gire and Price (2013) show that students' difficulties with symbolic representations used in physics may not have *direct* sources or analogues in material encountered in their mathematics studies. Therefore, we can't assume that difficulties that are apparently "mathematical" in origin are necessarily caused by *or* addressable through actions taken in the students' mathematics courses. In fact, Ivanjek et al. (2016), found that students' strategies of graph interpretation were largely context dependent and domain specific.

PI Meltzer and Nguyen built upon work by Knight (1995) and probed students' reasoning on graphical representations of vectors (Nguyen & Meltzer, 2003). They developed a diagnostic test and used it to explore students' thinking, and others such as Barniol and Zavala (2014) have amplified and extended these findings. PI Heckler has further extended (Heckler & Scaife, 2015) and applied this work to develop an online tool that has been shown effective in aiding students' vector manipulation abilities (Heckler & Mikula, 2016; Mikula & Heckler, 2017). This work, taken as a whole, demonstrates that (1) there are significant obstacles to physics students' performance attributable to specific difficulties with mathematical symbols and skills, and (2) these obstacles may be addressed with some effectiveness by specifically targeted instruction that is carried out with regularity and at length, within the context of the physics courses themselves.

An example of work that is particularly relevant to our project is that of Torigoe and Gladding (2007a,b; 2011). In a series of investigations, they demonstrated convincingly that "confusions of symbolic meaning," and not mere manipulation errors of the symbolic equations, are what in fact underlie many of the difficulties students have in understanding physics equations. In fact, they suggest that "an inability to interpret physics equations may be a major contributor to student failure in introductory physics." They showed that even when faced with physics problems that were *precisely* identical, differing only by using common symbols (such as "M") for numerical quantities (such as "1.5 kg"), students displayed sharply decreased levels of correct responses in the symbolic versions. PI Meltzer's work, (Meltzer & King 2018) has extended and amplified these findings. Thus it is clear that we must be sensitive to the role of symbol-meaning confusion in mathematical difficulties manifested by our students.

In work that is related but has a distinctly different approach, the University of Maryland group has focused attention on "student's perception or judgment of the kind of knowledge that is appropriate to bring to bear in a particular situation." They emphasize that "students often 'get stuck' using a limited group of (mathematical) skills or reasoning and fail to notice that a different set of tools (which they possess and know how to use effectively) could quickly and easily solve their problem" (Bing and Redish, 2009; Gupta and Elby, 2011). Our own investigation will explore the possibility of analogous "utilization failures" among our own students. However, our preliminary studies suggest that our students often do not possess alternative sets of tools that they "know how to use effectively"; instead, it seems that they have never actually mastered certain key tools in the first place. Still, our investigation will address possible utilization failures by *varying the context* in which students are asked to apply specific mathematical concepts, thus providing multiple opportunities for students to access their knowledge and—one hopes—avoiding perceptual traps that might manifest in one or another *specific* context.

d. Rationale for the Math Practice format

Math Practice will employ the Essential Skills platform, an existing, free, research-based online application used by thousands of students every week in introductory physics courses at OSU developed by one of the PI's (Heckler). More details about the structure of practice assignments will be described in section 3, but in brief, students complete short weekly online assignments for credit in which they practice simple skills, such as vector addition, that are necessary for success in introductory physics. The assignments are mastery-based in that students must answer 3-5 questions correctly in a row in order to complete a skills category, and there are typically several categories within the assignment. The median completion time is about 15-20 minutes.

There are strong theoretical, empirical, and practical reasons for Math Practice to employ the Essential Skills practice platform and framework. From the theoretical perspective, Math Practice aims to not only increase student accuracy with a skill but also to reduce the amount of time needed to complete the task, thus achieving procedural automation or *fluency* through repeated practice. Novice students often have not automated the basic procedural skills needed to solve a problem, resulting in increased time on task as well as increased cognitive load (Bransford et al., 1999; Ericksson et al., 1993). Even if novices correctly carry out a procedural step, they tend to do so slowly and with effort, leaving them with fewer cognitive resources available to attend to the rest of the problem and its conceptual underpinnings (Sweller et al., 1998; van Merrienboer & Sweller, 2005).

Researchers in cognitive psychology and the learning sciences have emphasized that improving fluency in these skills is critical to expert-like performance in problem solving (e.g, Kellman & Massey, 2013; Goldstone et al., 2017; Koedinger et al., 2012). In fact, Kellman, Massey & Son (2010) have pointed out that difficulties with math may arise precisely from the difficulties of low-level perceptual processing of mathematical expressions, and that perceptual learning training of these expressions (that is the automation of differentiating and encoding mathematical expression) results in improved performance. Further, Goldstone et al. (2010) have argued that methods that train the perceptual system to adapt (i.e. automate) to the complex representations, for example in mathematics, can result in significant benefits for higher level processing and "offer a promising approach to educational reform".

The structural design of the Math Practice assignments is based on several empirically validated methods and principles:

- Computer-based training with feedback: Training is online, outside of class. Shown to be effective in a wide range of topics (Hattie & Timperly, 2007; Van der Kleij, 2015). The instant feedback given to students has been shown to be effective in our context (Heckler & Mikula, 2016), and includes optional general explanations.
- Mastery-based training: Completion of assignment requires achieving a standard of performance (here getting 4-5 questions correct in a row). This is more efficient and flexible for the variety of student backgrounds compared to conventional summative course assignments (Block and Burns, 1976; Gutmann et al., 2018).
- Distributed and interleaved practice: Practice of a given skill is spaced throughout the term to improve learning and retention (Roher & Pashler, 2010). Interleaving problem types (as opposed to blocking all problems types together) within a practice session improves learning, as well as student ability to distinguish between and appropriately apply the different types of similar skills (Dunlosky et al. 2013)
- Multiple representations*:* Practice will include multiple representations, including diagrams and equations, and variation of mathematical symbols. Such variation can aid in fluency and the transfer to other aspects of course work (Rau et al. 2012; Bransford & Schwartz, 1999)

It is important to note that the Essentials Skills platform has been empirically shown to dramatically improve student accuracy and fluency for vector math for introductory physics (Mikula & Heckler, 2017), and reading logarithmic graphs for engineering students (Heckler, et al., 2013).

Further, there are strong practical and logistical reasons for Math Practice to employ the Essential Skills platform for this project:

- The Essential Skills platform is a successful, existing system used at a large university and can be relatively easily modified for adoption at other institutions. Currently, Essential Skills can only be used at OSU, but we propose to modify the application (a straightforward process) such that it could be used by any university using the Canvas Learning Management System (LMS) which OSU uses and ASU will use starting Autumn 2019. (Essential Skills may be adopted for other systems such as Blackboard).
- Math Practice assignments easily integrate into existing courses. We have found that making Essential Skills a graded weekly assignment (full credit for completion) requires only about 15-25 minutes per week, and student feedback reveals that these added assignments are not a strain on their time, and that the vast majority recommend that it be used for future classes. Further our experience is that instructors are happy to include it as part of the course once they learn about it.
- Math Practice is free of charge to students. This is an important feature of this added course component and avoids issues of financial stress for students.

Finally, it is worth explicitly addressing the question: Why should one consider developing Math Practice when there are already products out there, such as ALEKS (Canfield, 2001), which is also research-based, and online products from publishers such as Pearson? The primary answer is that the existing products are not designed to address the critical and widespread problem targeted by this project, namely that students often have significant difficulty with math in physics contexts. For example, while ALEKS does employ some research-based principles such as spaced practice and research on math learning, it is not targeted for mastering math in physics contexts. Further, it is our experience (which is in concurrence with our discussions with colleagues at various institutions) that the online resources provided by textbook publishers also do not address these student difficulties with math (as evidenced by the persistence of poor student performance in math). Specifically, these resources do not provide substantial practice, and the practice tasks provided simply randomize various parameters in the problems, rather than employ more careful attention to which dimensions to vary in a question and how to vary them, as discussed more in Section 3a below. Further, Math Practice uses a variety of question types, such as self-explanations, as discussed below. Finally, another important advantage of Math Practice is that it is free of charge to students, and we envision Math Practice (and the Essential Skills platform) to follow a non-profit model so that it may remain free (or very inexpensive) for institutions to use, similar to the model of the highly successful online PhET simulations (Wieman, Adams & Perkins, 2008).

3. Research Plan: Methods, materials and experiment design

The research plan consists of the construction, implementation, and assessment of Math Practice assignments, including the controlled investigation of the effectiveness of various practice question methods. The sections below provide more details on each of these aspects of the plan.

a. Construction of Math Practice assignments

The project will focus on specific topics and categories directly relevant to introductory physics (Table 3).

Table 3. Proposed content topic categories and examples of potential categories within the topics.

Topic	Category
Trigonometry	• Determining sides of triangles using sine, cosine, tangent in various orientations and contexts • Identifying the correct right triangle in a physics context
Algebra	• Solving 1 eq. 1 unknown • Solving 2 eqs. 2 unknowns • Solving quadratic equations
Reading graphs	• Determining slope using units and axis scales • Interpreting simple kinematics graphs

Students will see significant variations in a given kind of physics task throughout the course, thus it will be critical for the practice questions to span the space of relevant variations commonly seen in the course. Therefore, we will construct practice tasks that vary along a number of dimensions relevant to the specific content category, such as varying geometrical orientations, scalings, symbolic notation, question verbiage, question task, question storyline, special cases (e.g. angle $\theta = 0$, perfect algebraic squares), units, and algebraic representations. For example in comparing Figures 2a and 2b we see several kinds of variation: question posed, orientation of the triangle, amount of symbols used, angle of interest, answer choice representation (product or quotient) and answer choice space. All of these variations provide a rich set of differences in which the student may practice seeing relevant information.

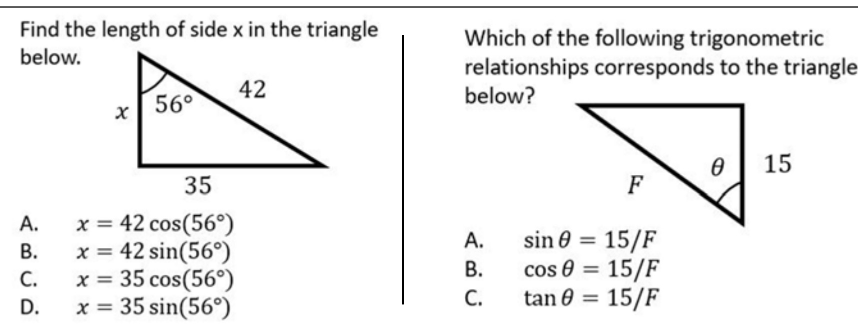


Figure 2: Examples of variations in practice questions in trigonometry

It is important to keep in mind that most of the students are likely to answer these practice questions correctly. The point is, however, that we want to improve their *fluency*, not just accuracy. We want students to be able to answer these questions quickly and effortlessly to reduce the cognitive load so they may attend to deeper physics aspects of more complex problems. Certainly some students will also be challenged by these practice questions, and Math Practice is designed to improve accuracy first, then fluency comes with repeated practice. To further illustrate the potential for variation, Figures 3a and 3b are practice questions for finding the slope of a graph. Here we vary the question wording and context, scale (a common oversight of some populations of students) units, the presence of data points, and the answer-choice space.

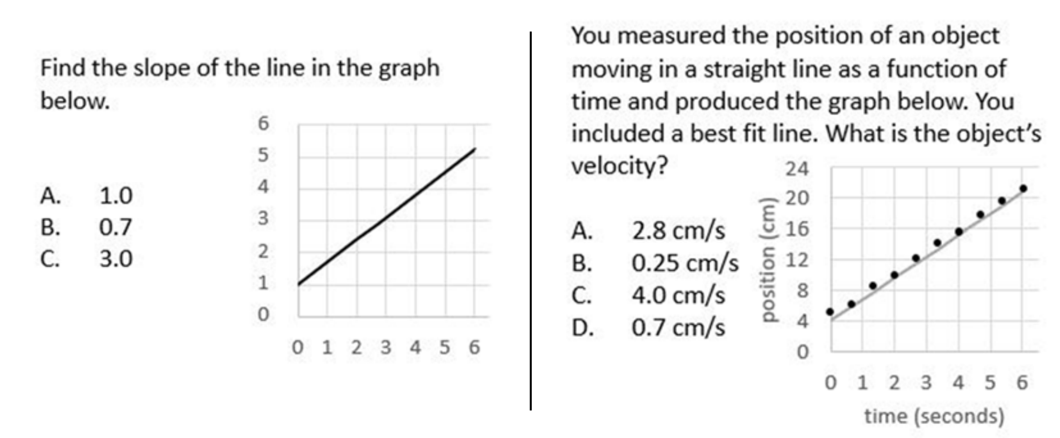


Figure 3. Examples of variations in practice questions in graphing

Similarly, Figures 4a and 4b show examples which are rich in common physics symbols.

In solving a physics problem, you set up the following equations:

$$E_i = \frac{1}{2}mv_i^2 + \frac{1}{2}I_0\omega_i^2$$
$$I_0 = mr^2$$
$$\omega_i = v_i/r$$

Assume that E_i, m and r are known. What is the initial velocity v_i?

a. $v_i = \sqrt{\frac{E_f}{m}}$ b. $v_i = \sqrt{\frac{2E_f}{m+1}}$ c. $v_i = \sqrt{\frac{2E_f}{m(1+r^2)}}$

Consider the equations:

$$F_N - mg\cos\theta = 0$$
$$mg\sin\theta + \mu_k F_N = ma_x$$

If the coefficient of friction μ_k and the angle θ are given, what is the acceleration a_x in the x direction?

a. $a_x = mg(\sin\theta - \mu_k\cos\theta)$
b. $a_x = g(\sin\theta - \mu_k\cos\theta)$
c. $a_x = mg(\sin\theta + \mu_k\cos\theta)$
d. $a_x = g(\sin\theta + \mu_k\cos\theta)$

Figure 4. Examples of variations in practice questions in algebra

Of course students do see a few questions such as those in Figures 2-4 during a course, but Math Practice will offer the students the opportunity to practice these kinds of questions repeatedly with feedback, so that they can become fluent in these basic skills. This method of variation of dimensions relevant to the category was successfully used in PI Heckler's studies using Essential Skills in the domains of vector math and reading logarithmic plots (Mikula and Heckler, 2017; Heckler, Mikula & Rosenblatt, 2013), and math education researchers continue to investigate the issue of variation in examples (see, e.g, Kullberg, Kemp & Marton. 2017). Note that in the sections below we describe Studies 3 and 4 which will investigate additional kinds of variation of question types.

b. Implementation of Math Practice Assignments

Once the questions are created, they will be uploaded to the Essential Skills platform and weekly graded Math Practice assignments will be constructed for the students. Each week, the students will be assigned to "master" 3-5 categories. Here, mastery means that the student must correctly answer 4-5 questions in a row (this mastery condition level can be set by the instructor). The difficulty level of the questions will be such that highly prepared students will likely complete the assignment quickly without missing any questions, and students with average-level preparation will likely need to practice with about 10 questions (per category) to reach mastery. Students receive full credit for mastering the categories assigned for the week, and, within the given due dates, they can practice until reaching mastery. For a given category the Essential Skills platform randomly chooses the questions from a pool of (we propose) at least 30 questions (per category) so that it is unlikely that any two students will see the same questions or in the same order.

The assignments will be constructed such that students will see each category several times spaced throughout the term. Within a given assignment, there is the (instructor) option of interleaving (rather than blocking) the practice of categories. Furthermore, we will provide general explanations for each category that a student may click on while thinking about a particular problem. This kind of static explanation feedback has been shown to be effective in this context of Essential Skills practice (Heckler & Mikula, 2016).

c. Experimental methods and design

In addition to studying the effect of Math Practice on math skills, course performance, and motivational factors, this project also proposes to investigate the relative effectiveness of several theoretically and empirically motivated question types (i.e. interventions). The investigation will consist of 4 Studies employing a few kinds of interventions, described further below in this section, but first we will describe the design. The general experimental design for a given semester at OSU and ASU is shown in Table 4.

Table 4. Outline of experimental design for each semester

Pretest (1st week of class)	**Intervention** (weekly)	**Posttest & academic performance** (last week of class)
• Math content assessment • Physics content assessment • Motivational factors survey	<u>OSU</u> (between semester/section design): • Study 1, 2, 3, or 4 <u>ASU</u> (within lecture section design): • Study 2,3, or 4 (within-section controls)	• Math content assessment • Physics content assessment • Motivational factors survey • Course performance (exams etc.) • Academic and demographic data

As is inevitable in education research, the experimental design across and within semesters must include the logistical considerations of the different course contexts of OSU and ASU.

<u>OSU: between-semester and/or between-section designs</u>. At OSU the project will encompass entire courses (first semester Algebra and Calculus-based physics), each having 4-7 lecture sections per semester, with ~220 students per section (for a total of ~22 sections/year ≈ 5000 students/year). Currently, OSU implements the same instructional structures and policies for any given section for a given semester.

Therefore, at OSU comparisons between interventions will be made between lecture sections within and/or across semesters. Threats to validity include the fact that different lecture sections may be taught by different lecturers, and that Autumn and Spring semesters commonly have slightly different populations of students, as measured by ACT scores, rank of student etc. To account for this, we will collect data for all interventions in both Autumn and Spring semesters. At OSU there will be 4-7 lecturers per semester (depending on the course and semester) and we will carefully assign sections to interventions in order to counterbalance and control for varying lecturers to the extent possible.

Each semester, two of the Studies will be conducted at OSU in a between-section design. (See Table 5). Overall, a comparison of Studies 1 and 2 will allow for inferences on the effectiveness of Math Practice for specific math topics (RQ1 and RQ2), and Studies 3 and 4 will allow for inferences on the effectiveness of several kinds of Math Practice question types (RQ2). A statistical analysis of results will include (i.e. control for) student level factors such as physics pretest scores, ACT scores, and GPA (RQ3). We will also consider using multi-level modeling in order to help account for this hierarchically clustered data (students within lecture sections), but the numbers of lecture sections is small (even pooling semesters together), so caution will be used in employing the appropriate analysis methods and interpreting results (Gelman & Hill, 2006).

ASU: within section design. At ASU, the implementation will be with one lecture section per semester with 40-70 students per semester. This relatively small number of students makes it difficult to reliably compare results between semesters (a significant semester-to-semester variation in pre and post scores has been observed by PI Meltzer). Therefore, this project will adopt a within-section design at ASU, that is, the sample population will be partitioned and serve as a control on itself.

At ASU, the confound of different lecturers will be removed (compared to OSU) because the design is within section, though the drawback is the smaller number of participants within a section. Nonetheless, this design allows for within-section controls. Specifically, students will be randomly assigned intervention conditions within a section. For example: Condition 1 could include an intervention on math topic A and not on Topic B, while Condition 2 could include an intervention on topic B but not on topic A. In this case the experimental design can employ a 2-way repeated measures ANOVA analysis (math topic x intervention), where the results of interest would be a main effect of intervention (RQ1) and an interaction of intervention with math topic (RQ2).

Note that the within-section design has the potential of producing systematic differences in course performance between students in different conditions. These differences will be mitigated in 2 ways. First, the instructor will analyze average student performance of each condition and adjust grades for each condition if needed to help ensure equity. Second, all students in the section will be allowed to voluntarily complete additional math practice assignments for categories not originally assigned to them in their condition. We will have a record of any students who do so and can then account for this in our analysis.

Table 5. Project Timeline

	Autumn 2019	Spring 2020	Summer 2020	Autumn 2020	Spring 2021	Summer 2021	Autumn 2021	Spring 2022	Summer 2022
Goals	1,2,3,4	1,2,3,4	1,2,5	3,4	2,3,4,5	2,5	3,4,5	3,4,5	5
OSU	Stdy 1,2	Stdy 1,2		Stdy 1,2	Stdy 1,2		Stdy 3,4	Stdy 3,4	
ASU	Stdy 2	Stdy 2		Stdy 2	Stdy 2		Stdy 3	Stdy 4	
Evaluation	Init.mtg		Y1 eval			Y2 eval			Fnl eval

The interventions described in the studies below will be constructed in the context of variation along relevant question dimensions discussed in Section 3a.

Study 1: Control. This study will collect student pre- and post-instruction data to be used as a control for comparison with the various interventions used in Studies 2, 3 and 4. The Study 1 "intervention" will consist of assignments that are not relevant to math skills but are equivalent in time length to the interventions of Studies 2-4. More specifically, the assignments will consist of weekly sets of simple non-quantitative questions based on the assigned textbook chapters. Students will receive full credit for answering. Note that ASU does not need to employ Study 1 since it is a within section design with the treatment and control of individual topics within the same lecture section.

Study 2: Practicing with context-rich physics notation

In a sense, Study 2 is the core intervention of the project. As described in Sections 2a-c, it is clear that students particularly struggle with the often significantly more complex notation used in physics courses, even for "simple" equations. The practice questions for Study 2 are focused on student practice with the common symbols used in physics. For examples see Figure 2-4, in which symbols like θ and ω are used instead of numbers, and units are included.

The hypothesis of Study 2, and a central hypothesis of this proposal, is that the repeated and spaced practice with a variety of expressions and representations that use common physics symbols and notation will allow for more automated processing of these expressions, leading to improved accuracy and fluency with math in physics contexts (Kellman & Massey, 2013; Goldstone Landy, & Son, 2010; Koedinger et al., 2012), and ultimately resulting in better performance in the course.

Study 3: Concreteness fading

It is well documented that learning, transfer, and performance can be significantly affected by the relative concreteness of the representations used in the learning domain (e.g., Fyfe et al., 2014). By *concreteness*, we mean the perceptual and/or conceptual richness of a representation (Kaminsky, Sloutsky & Heckler, 2013). In short, there are advantages and disadvantages to concreteness. Relatively concrete representations (in contrast to abstract ones) can be easy to learn if familiar and relevant to the learning domain, difficult if not, and performance typically does not transfer well (Kaminsky et al., 2008). Concreteness can enhance performance if it cues prior relevant knowledge, but can also hinder performance by cuing spurious, interfering concepts or procedures (e.g., Heckler, 2010). One way of mitigating these differences between concrete and abstract is to employ a practice regime called "concreteness fading", in which the learners begin on representations with relevant (and familiar) concreteness, gradually "fading" to representations that are more abstract (Fyfe et al., 2014).

The issue of concreteness is likely relevant to this project. For example, it has been shown that numbers such as "5" tend to be more concrete (and familiar) than simple symbols such as "*a*" (Kaminsky et al., 2013). Therefore, in Study 3 we will investigate the extent to which Math Practice examples using concreteness fading are beneficial for achieving accuracy and fluency compared to Study 2 which focuses more on using symbolic notation only, without any regimen of fading. Figure 5 shows an example of a concrete problem (using numbers) and a relatively less concrete algebra problem, which has been shown to be significantly more difficult (Torigoe & Gladding, 2007a).

In solving a kinematics problem to find the distance a car travels while accelerating, you set up the following equations:
$v^2 = v_0^2 + 2ad$ and $a = \frac{\Delta v}{\Delta t}$
where $v_0 = 0$, $\Delta v = 60\frac{m}{s}$, $\Delta t = 8\ s$, and $v = 30\frac{m}{s}$.
Find the value of the distance d.

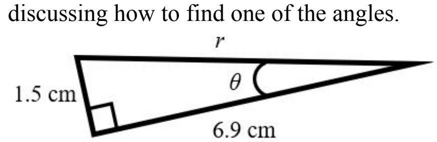
In solving a physics problem, you set up the following equations:
$v^2 = v_0^2 + 2ad$ and $a = \frac{\Delta v}{\Delta t}$
where $v_0 = 0$, $\Delta v = v_1$, $\Delta t = t_1$, and $v = \frac{v_1}{2}$.
Find the value of d.

Figure 5. Example of a concrete question with numbers, and a similar, less concrete question.

Study 4: Self-explanation and Comparison of alternative solutions procedures.

Study 4 will investigate the effectiveness of two research-based methods in the context of Math Practice, compared to the interventions in Studies 2 and 3. The first is self-explanation. Prompting students to consider explanations for particular solution methods can be helpful in a variety of tasks, though it is clear that more research is needed (Dunlosky et al., 2013). In a recent review on the effect of the self-explanation method for math learning, Rittle-Johnson et al. found that the self-explanation may lend itself well to the structured-response format (e.g., multiple choice) of Math Practice, especially for gaining conceptual understanding, though gains in procedural knowledge are mixed (Rittle-Johnson, Loehr & Durkin, 2017). Since this method shows promise, we will prompt students to choose among explanations for worked examples, including among (explicitly) correct and incorrect solutions (Booth et al., 2013). See Figure 6 for a trigonometry example.

Two students are working on a physics homework problem. They set up the triangle below and they are discussing how to find one of the angles.



Student 1: We can use 'sine equals opposite over hypotenuse'. In this case, sine of the angle equals 1.5 over 6.9, and the centimeter units cancel.

Student 2: I think sine of the angle equals 1.5 over *r*, because *r* is the hypotenuse, not 6.9. We can use Pythagoras to find r first, then use sine equals 1.5 over *r*.

Which student do you think is correct? (multiple choice: Student 1, Student 2, Both, Neither.)

Figure 6. Example of self-explanation question.

The second method is comparison of alternative solutions procedures. One way to help students gain procedural knowledge in math is to prompt comparison of pairs of worked solutions illustrating two different correct procedures, especially for students with appropriate prior knowledge. (Rittle-Johnson & Star, 2007; Rittle-Johnson, Star & Durkin, 2009). In our project we will prompt students to compare solutions, for example asking which solution has a given strategy, or asking them to match one of the two solution methods with a solution method to a separate similar problem, in the sense of an analogical comparison task (Gentner, Lowenstein and Thompson, 2003; for an example in a physics context see also Lin and Singh, 2007; Badeau et al, 2017).

d. Assessments

Math assessments

Since we have found no directly relevant instruments that test student math skills in a physics notation context. We propose to adapt the considerable initial work on a valid assessment already begun by PI Meltzer (for a separate project) to construct research validated scales to assess student skills with trigonometry, graphs, and algebra relevant for an introductory physics context.

The mathematics diagnostic instrument used in the research by Meltzer and his students (e.g., Meltzer & King, 2018) has undergone revisions in every semester of its use, extending back to spring of 2016. Through careful analysis of students' written responses to the free-response questions, as well as analysis of more than 70 individual problem-solving interviews with students, we worked to ensure that the wording of the questions was clear and elicited the desired responses. The diagnostic consisted of sets of two or three items each on trigonometry, graphing, vectors, "numerical" algebra (preponderance of numerical coefficients), and "symbolic" algebra (preponderance of symbolic coefficients). We looked for, and found, consistent patterns of responses indicating that results of each "subscale" were consistent. For example, correct-response rates on "find the unknown side" items were similar to those on "find the unknown angle" items, as were those on slightly revised alternate versions of each item. Rates on the different items tracked each other—that is, rose and/or fell together—when we compared results from different courses, such as algebra-based physics vs. calculus-based physics. The pattern of lower correct response rates on more "symbolic" item compared to "numerical" ones was extremely consistent across item versions, course types, and semesters.

Course performance assessments

We will also collect data on course performance, including scores on all assignments and tests, and grades. We will also collect scans of tests in order to code and analyze student performance on specific questions relevant to the math topics practiced in this project. This data can help to determine the extent to which learning and mastery of Math Practice topics is related to course performance. Finally, we will also collect registrar data on enrollment and grades (including withdrawals) in physics courses in order to measure retention and success in physics classes.

Math Practice performance measures

Student metrics on the weekly Math Practice assignments can provide extremely useful information and will be collected and analyzed. This will include accuracy measures for each content topic and category such as the number of questions attempted to achieve mastery (typically to get 4 or 5 correct in a row), answer choice patterns, and the percentage of questions answered correctly. We will also collect timing data, which can provide critical information on fluency with each topic and student engagement with the assignment. Taken together, this data can provide more insight into the evolution of student skill development and feedback on which topics, categories, and questions are effective. For example if we find that, week after week, the accuracy and time to mastery for a specific category (and/or specific questions in the category) are improving, this is good evidence of the effectiveness of the assignments that complements the data we will collect from pre and posttests. If the students are not improving, then this provides us with feedback to make improvements to the practice questions in this category.

Assessing Motivational factors
In additions to studying academic performance, this project proposes to investigate the extent to which three motivational factors may interact and evolve with engagement in Math Practice.

Specifically, using validated scales, we will measure self-efficacy (cf. expectancy for success (Wigfield & Eccles, 2000), see also Sawtelle et al., 2012) for both math and physics (Midgley et al., 2000), mindset (Dweck & Leggett, 1988), and the possible mediating influence of persistence (Vollmeyer & Rheinberg, 2000) (both self-reported (Wolters, 2004) and measured through level of mastery performance in Math Practice), all of which may play important roles in how and why students learn skills using Math Practice, in student performance, and respond to intervention (Sawtelle et al., 2012; Harackiewicz et al., 2016; Blackwell et al., 2007)).

The primary goal for measuring and analyzing motivational factors in conjunction with Math Practice and academic performance is to achieve deeper insight into mechanisms underlying student improvement in basic math skills, identify potential subpopulations that may particularly benefit or be at risk for not improving, and provide insights into further interventions that might be productive.

4. Broader Impact

This project has the potential to measurably improve physics students' performance in introductory courses nationwide with an intervention that is both economical and logistically simple. We are addressing a need that is commonly recognized—that is, improved mathematical proficiency for students in introductory physics—but one that is, for the most part, not specifically targeted within the context of physics courses themselves. Although physics instructors tend to neglect this skill deficiency due to lack of time, interest, and appropriate learning materials, it persists nonetheless, and it impacts student performance. Our learning tool will be made practical to use and easily available to physics instructors everywhere, where we envision a no-cost model or an at-cost model that many institutions will find acceptable. Based on our extensive previous work, we expect that our tool will lead to improved student skills on mathematics procedures widely used in introductory physics. We anticipate that this will also lead to improved performance on physics problem-solving as well. The tool will be well adapted for use in the standard algebra- and calculus-based introductory courses that enroll nearly half a million students nationwide each year. (NRC, 2013)], and we plan on recruiting 2-3 institutions to begin the process of piloting broader adoption. Furthermore, the Math Practice Intervention will likely especially help underprepared students to succeed and continue in physics courses and in this way can help to increase participation in STEM and the STEM workforce.

5. Intellectual Merit

This project is potentially transformative in that it assembles a unique and powerful synthesis of cognitive psychological learning theories and methods to address a critical physics education issue. This project will determine the extent to which online weekly practice assignments based on a coherent network of established cognitive principles can *automate* critical math skills in a physics context, and the extent to which this automation can in turn help students succeed in physics courses. Thus this project will advance our knowledge of student difficulties with essential math skills in a physics context—and the mechanisms underlying these difficulties — and will provide a theoretical and practical framework for constructing effective practice assignments to help students become fluent in critical math skills in a physics context. Further, this project will advance our knowledge of the effectiveness of several *kinds* of empirically and theoretically promising math practice tasks in this framework. This project will also determine the extent to which student level factors and motivations interact with this intervention, thus potentially providing information on mechanisms of the difficulties and additional avenues for interventions. Finally, the intellectual merit reaches beyond physics education to all of STEM education: It is virtually certain that the difficulties students have with symbolic notation are found in all areas of STEM.

6. Evaluation

The Advisory Board consists of three nationally recognized researchers in physics education, mathematics education, and cognitive science, respectively: Prof. Steven Pollock (physics) of the

University of Colorado, Boulder; Prof. Michelle Zandieh (mathematics) of Arizona State University, and Prof. John Dunlosky (cognitive psychology) of Kent State University. Prof. Pollock and his collaborators have published pioneering research into mathematical challenges experienced by students in upper-level physics courses on mechanics, electromagnetism, and quantum mechanics. He has also led the design of nationally disseminated diagnostic tests of conceptual understanding in upper-level electricity and magnetism, and in quantum mechanics. Prof. Zandieh has published many papers dealing with the learning and teaching of mathematics at the university level, with an emphasis on topics that have particular significance for physics students such as linear algebra. Prof. Dunlosky is an established leader in Cognitive Psychology, specializing in self-regulated learning and metacognition, and is an expert in empirically validated learning methods.

The three Board members will have an initial electronic conference with the PIs in Autumn 2019 to review project plans and goals. Formal evaluation meetings with the PIs will then occur in Summer 2020 and Summer 2021, with final evaluation in Summer 2022. Before each meeting, PIs would forward to the Board members summaries of all instructional and diagnostic materials created, descriptions of the various classroom implementations of the materials that were carried out, summaries of data collected with accompanying data analyses, and PIs' assessment of project progress. The Board members would also have access to *all* materials and all data, at their request, at any time. At the meetings with the PIs, the Board members would ask questions and provide feedback on the materials they have reviewed. After the meeting, they would each provide a short summary (one page or less) of their overall assessment of the meeting outcome and project progress.

7. Dissemination

First, we plan on disseminating Math Practice internally within OSU and ASU physics departments for adoption in all of our first and second semester introductory physics courses. On a larger scale, this project lends itself well to adoption by many institutions across the nation, and we will actively solicit 2-3 candidate institutions for pilot adoption towards the ending stages of this project. Informal conversations of PIs Heckler and Meltzer with colleagues has indicated considerable interest.

We will further promote the materials and findings 1) by presenting at national conferences (e.g. American Association of Physics Teachers winter and summer national meetings), 2) by offering workshops at professional conferences (American Association of Physics Teachers), 3) by publishing findings in relevant physics/ physics education journals 4) by further developing the Essential Skills software application so that it may be easily integrated into common Learning Management Systems such as Blackboard and Canvas. While outside the scope of this focused project, we will begin the process of planning how this application can be supported such that nationwide adoption can occur, including the possibility of submitting an IUSE Level 2 proposal to study and advance broader adoption.

8. Results from Prior NSF support

Results from closely related Prior NSF support for PI Heckler

NSF Award number: DMR-1420451; Award amount: $17,900,000; Award period: 9/1/2014 - 8/31/2020. *Note*: this is a renewal of a previous NSF award DMR-0820414 from 2008-2014, and work on this has been continuing from the previous award to this renewal.

Title of Project: Center for Emergent Materials. Note: this NSF award is for an NSF Materials Research Center; described here are results from the required education component.

Intellectual merit: There are four main projects: 1) the establishment of the OSU Master's-to-PhD Physics Bridge Program was partially supported by this grant, including the development of group-work tutorials for graduate core courses. 2) The development of a full set of group-work tutorials for an Introductory Materials Science engineering course at OSU. Results include measured dramatic gains in student learning. 3) The development and implementation of an online application to help introductory level students with STEM essential skills. We focused on skills in Materials Science engineering (e.g., dimensional analysis, reading log plots) and introductory physics (e.g. vector operations). This has resulted in measured dramatic increases in student performance on these essential STEM skills. 4) In 2016 we began a longitudinal and cross sectional survey of motivations, beliefs and the student experience of undergraduate and graduate physics majors. We are looking for factors affecting retention and for a characterization of the main "pathways" students take to obtain a graduate or undergraduate degree. In sum this program has systematically identified, characterized, and published numerous newly discovered student difficulties with core topics in graduate physics, essential skills introductory physics, and basic concepts and skills in materials science engineering.

Broader impact: This program has: 1) resulted in the matriculation of 9 underrepresented minorities (URMs) to physics PhD programs to date and with 2-3 new fellows added each year. 2) impacted over 10,000 undergraduates in introductory physics and Materials Science engineering courses with demonstrated dramatic gains in learning, and this number is increasing by ~3500/year. Further this has led to the development of novel and highly effective research-based instructional materials and methods that can be used nationwide. 3) 4 PhD thesis (including 2 women) in physics, specializing in education research. 4) Over 25 national presentations.

Publications resulting from this NSF award (since renewal in 2014): 6 journal publications (Heckler & Scaife, 2015; Heckler & Mikula, 2016; Mikula & Heckler, 2017; Amos & Heckler, 2018; Heckler and Bogdan, 2018; Young & Heckler, 2018; (and 1 more recently submitted) and 4 peer-reviewed conference proceedings (Amos & Heckler, 2015; Porter et al, 2016; Porter et al, 2017; Koenka et al. 2017)

Research products resulting from this NSF award (in addition to Publications above)

1. A full (one-semester) set of group-work tutorial materials for a university level introductory Materials Science Engineering course. Available online (MSE tutorials, 2016).
2. An online application for practicing skills essential for introductory physics. The application is used by the OSU physics department (about 5000 students/year). The online application is currently only available to OSU students, but public availability is planned.

Results from closely related Prior NSF support for Co-PI Burrola Gabiondo: No NSF support to date.

Results from closely related Prior NSF support for PI Meltzer

Over the past 20 years, David Meltzer has received NSF funding as PI or Co-PI on 12 distinct science-education-related projects (13 overall), including three during the past five years. Project titles, award numbers, and key publications are listed in the biography pages. The one most closely related to this current proposal is *"Identifying and Addressing Mathematical Difficulties in Introductory Physics Courses," NSF DUE #1504986; $250.000; 2015; 48 months*; PI: D.E. Meltzer.

Intellectual Merit: This project has focused on research into the specific mathematical difficulties encountered by students in both algebra- and calculus-based introductory physics courses. Detailed analysis was carried out on written diagnostic tests administered to over 3000 students in 24 algebra- and calculus-based physics classes over six semesters at Arizona State University during 2016-2018; individual problem-solving interviews were carried out with 75 students enrolled in these or similar courses during this period. Significant difficulties with trigonometry, graphing, and symbolic representation in algebra were identified and their prevalence determined among this population for the first time. Meltzer and his students have given three invited and eight contributed presentations related to this project in the U.S. and Mexico (see references); two undergraduate students have been intensively involved in the research and analysis work.

Broader Impacts: Our revelation of the prevalence of significant difficulties with basic mathematical skills among introductory physics students has resonated strongly when presented to other physics instructors. The common perception that this is a significant issue has been strongly bolstered by our findings, and the feeling that it should be addressed—rather than neglected as is customary—has been strengthened. We have found a desire to "do something about it," without knowing quite what might work, in view of the failure of traditional mathematics instruction to address the problems effectively. The direct impact of our results on the present project is twofold: our findings will provide the initial basis for developing effective mathematics practice materials and assessments, and further dissemination of our work is expected to increase the perceived need for, and utility of, the products to be developed. Because this project is relatively new, publications are in process, but have not yet been submitted. The project has produced 11 national/international presentations.