Student understanding of calorimetry in introductory calculus-based physics

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Abstract

We report on students' thinking regarding calorimetry concepts in an introductory calculus-based physics course. We analyzed student responses to a variety of questions in diverse contexts and found that despite overall good performance (> 60% correct responses), only about half of all students were able to provide correct answers with satisfactory explanations. A number of persistent student difficulties were found to affect up to 40% of the student population even after instruction, including apparent confusion about the meaning of specific heat and misunderstanding of the nature of thermal energy exchange. Student response patterns varied significantly depending on the context of the question and often reasoning did not appear to be consistent among contexts, instead seeming to favor algorithmic or "rule-based" reasoning. Interviews with students suggested that difficulty with algebraic manipulations was a significant contributor to incorrect responses on calorimetry questions.

I. Introduction

Calorimetry is often one of the first topics discussed in the introductory physics course after basic concepts of heat and temperature are introduced. It is considered to be relatively easy since calorimetry problems require only straightforward application of a few fundamental concepts along with basic algebraic skills. Depending on instructor preference, students are expected to make use of relationships involving specific heat (such as $Q = mc\Delta T$) or molar specific heat (such as $Q = nc_{molar}\Delta T$). The definition of specific heat does not vary significantly from textbook to textbook, most often discussed in a fashion similar to that of Reese: "You can think of specific heat as the heat transfer to one kilogram of the material needed to raise its temperature by one Kelvin."[1] Nonetheless, the topic of calorimetry depends on concepts such as thermal equilibrium and heat transfer, both found to be challenging in previous studies of physics students' thinking as described in Sec. II below. In view of those conceptual challenges and the central role calorimetry often plays in introductory discussions of thermal physics, investigation of students' thinking in this area merits focused investigation.

II. Previous research on student learning of calorimetry concepts

The science education literature includes numerous studies reporting on the difficulties students encounter with the concepts of heat and temperature[2,3,4,5,6]; some of these investigations have been in the specific context of thermochemistry.[7,8,9,10,11,12] For example, Greenbowe and Meltzer[13] investigated student thinking regarding calorimetry concepts in the context of solution chemistry as studied in introductory chemistry courses. In addition to previously reported student learning difficulties regarding the relationship between heat and temperature, they found significant misunderstanding related to the role of chemical reactions in the heating process. Jasien and Oberem[14] investigated calorimetry-related ideas among various groups of college students who had diverse backgrounds in the physical sciences. They reported student difficulties with concepts involving thermal equilibrium, heat capacity, and specific heat, with no significant correlation between observed difficulties and the number of physical science courses that the students had taken.

The most wide-ranging investigation concerning student thinking about calorimetry in a physics context is that of Matt Cochran and co-workers at the University of Washington.[15] Consistent with findings of other investigators,[9, 13, 14, 16, 17] Cochran found that many students have substantial difficulty distinguishing between heat and temperature. He pointed out that this may impair their ability to understand other thermodynamic concepts, such as recognizing that objects in thermal equilibrium with each other are at the same temperature. Cochran also found that students occasionally focused on *rates* of heat transfer or of temperature change when it wasn't appropriate to do so,[18] and that students would often incorrectly treat the amount of heat transfer as being solely dependent on only a single property in the interaction. For example, the change in temperature of a hot copper block in water was thought to be due *only* to the specific heat or initial temperature of the block, ignoring the role of the block's mass.

III. Objectives of the Investigation

As part of our ongoing research into the learning and teaching of thermodynamics, we recognized a need for further research on students' understanding of calorimetry at the university level. In this section we enumerate a set of concepts, the understanding of which we attempted to assess during our investigation of student thinking on calorimetry. We also discuss the questions we used to assess students' understanding of these concepts.

A. Concepts targeted for assessment

i. There is no heat transfer between the inside and the outside of a perfectly insulated container.

Although it is usually assumed that students recognize the need for this assumption, we wanted to determine whether they actually made use of it in practice.

ii. When two objects are in an insulated container, the magnitude of heat transfer from one object is equal to the magnitude of heat transfer to the other object.

Solution of quantitative calorimetry problems hinges on recognizing this consequence of energy conservation in the context of an insulated container.

iii. The specific heat is the amount of energy per unit mass required to change the temperature of some object.

The concept of specific heat (or, "specific heat capacity") is fundamental to an understanding of calorimetry processes. In the context of introductory physics, the complication of heating due to chemical reactions is not present and, as long as no phase transitions occur, solution of calorimetry problems can be accomplished through straightforward algebra.

B. Assessment questions

Our investigation focused on student responses to two related questions that were each administered in multiple formats. The "Object in Liquid" question (Fig. 1) and the "Two-Liquid" question (Fig. 2) were used to probe student thinking regarding substances of equal masses but different specific heats. The questions both require students to apply the idea that energy is conserved during the process, and that energy transfer to a substance changes its temperature by an amount that depends on its specific heat. There are some key differences between these two questions: (1) the Two-Liquid question involves common initial temperatures and equal rates of heating, while the Object in Liquid question involves two substances at different initial temperatures within an insulated container;[19] (2) the Two-Liquid question includes a graphical component which is absent from the Object in Liquid question.[20] Both questions require application of simple algebra or proportional reasoning. Questions were administered “pre-instruction” (before all instruction), “after lecture instruction” (after lecture instruction but before recitation instruction), and “post-instruction” (after all instruction was completed). Both questions were administered simultaneously on a single sheet of paper unless otherwise noted. Below we present student response data obtained at various times during the second semester of an introductory calculus-based physics course.

The bulk of this study was conducted with students in the second semester of a year-long calculus-based introductory physics course at Iowa State University. This sequence usually enrolls 700–800 students per calendar year. Most of the students are engineering majors, but a few physics majors and computer science majors are included. The course content varies slightly; the first semester usually covers mechanics and electric fields, and the second semester covers magnetism, ac circuits, waves, fluids, and thermal physics.

IV. Results of Assessments

A. Before all instruction

We administered the Object in Liquid question to all students attending the first week of recitation in Fall 2005; this was prior to all instruction on thermodynamics. (In other semesters this question was almost always used after lecture instruction but before recitation instruction, and frequently was coupled with a separate calorimetry question; see IV.B below.) We found that even before any instruction on thermodynamics, students' previous exposure to this material was evident. Half of all students (exactly 50%) answered correctly that the substance with the lower specific heat would have greater temperature change than the substance with the higher specific heat, and 80% of those who gave a correct answer also provided acceptable explanations. (See below for sample explanations and Appendix I for details.)

B. After lecture instruction

i. Object in Liquid question

After lecture instruction, 63% of students overall correctly answered the Object in Liquid question by saying that the substance with the lower specific heat would have a greater temperature change than the substance with the higher specific heat ($N = 1036$ over three semesters; see Tables I and II). [Note that in Fall 2005 this question was administered both before all instruction and after lecture instruction.]. Those students giving a correct answer with a correct explanation (53% of all students) relied on the equation $Q = mc\Delta T$ or the definition of specific heat to explain their answer. Some representative explanations follow:

> *“Object* A *will change less than liquid* B *because the specific heat of object* A *is greater so it takes more heat to change its temperature by one degree.”*

> *“Less than, the object has a higher specific heat so it takes more energy to change its temperature.”*

Nearly one quarter of all students (22%) stated that the temperature change of the object and the liquid would be the same. Explanations for this response include the idea that equal energy transfer is assumed to imply equal temperature change. For example, here is one student’s argument:

> *"Same. The system will reach an equilibrium since the copper will gain the heat that the water gives up, they will both change the same amount of ℃."*

A different justification was offered by this student:

> *“The temperature change of the copper and the water will be the same. Any heat lost by the copper will be gained by the water, or any heat gained by the copper will lost from the water. So* ΔT *of both are the same.”*

The remaining 18% of students answered that the substance with the lower specific heat would have a smaller temperature change than the substance with the greater specific heat. Most students offering this response stated that the temperature change was proportional to the specific heat, for instance:

> "*The temperature change of copper will be less than that of the ΔT of the water, because the specific heat of water is greater, and the masses are the same."*

A note on the term "equilibrium"

Approximately one third of students who said that the temperature changes for the object and the liquid would be equal justified their answer by stating that the object and the liquid go to "equilibrium." Although the technical definition of this term was explicitly discussed in the course texts and, presumably, in the lectures, we were uncertain as to the actual meaning attributed to it by the students who used it in their explanations. Reports in the literature had suggested significant difficulties with this concept (see, e.g., Ref. 14). To address this issue and to minimize potential confusion we administered the Object in Liquid question with a change in the wording for the Fall 2005 and Spring 2006 courses; the change was as follows: the phrase "*During the time it takes for the system to reach equilibrium…"* was changed to, *"During the time it takes for the object and the liquid to reach a common final temperature…"* However, this re-wording seemed to affect only the manner in which students expressed their explanations; it did not lead to any significant changes in responses rates for the different answer options. Students didn't use the specific term *equilibrium* as frequently as they had done with the original form of the question. Some students continued to use it, while others instead used the term "common final temperature" to justify their responses that temperature changes of the object and liquid would be equal.

ii. Two-Liquids question

This question asked students to graph the temperature as a function of time for two liquids with different heat capacities, each placed in a container on a heating plate. The plate delivered equal heating at a constant rate to both containers.

Since the answer depends to some extent on one's ability to properly graph two lines, we decided to accept as correct any response that showed the slope of line *B* (representing the substance with lower specific heat) as being greater than that of line *A*. Interview data supported this criterion since, although many students initially failed to draw graphs that reflected a quantitatively accurate ratio of the slopes, they were able to recognize this defect and almost always were able to correct it when pressed to do so.

Using this "B-slope > A-slope" criterion for correctness, we found that two-thirds of students (72%) correctly identified the slope of the liquid with the lower specific heat as being greater than that of the liquid with the higher specific heat ($N = 788$ over two semesters; see Table III). Over half of all students (58%) gave a correct explanation along with the correct response. Roughly a third of all students (27%) stated that the slope of *B* would be less than the slope of *A*, and there were essentially zero students who answered that the slope of the two liquids would be the same, despite the fact that 22% of all students had given an analogous answer on the Object in Liquid question (see Table I).

We have no convincing explanation for the sharp disparity in proportions of students who chose the "temperature changes are equal" response on the two questions;

however, we will address students' related use of "rule-based reasoning" in the next section.

iii. Student performance inconsistency across questions

As is described above, we observed inconsistent results on the two assessment questions regarding the "equal temperature change" response, with 22% offering such a response on the Object in Liquid question and none at all on the Two-Liquids question. We tracked correlations among student responses between these two questions to determine the consistency of student thinking (see Appendix II). We found that a high proportion (82%) of those students who answered the Object in Liquid question correctly *also* answered the Two-Liquids question correctly, while only a relatively small number of them (15%) selected an incorrect answer of "Greater c, Greater ΔT" on the Two-Liquids question

Students who stated that the temperature changes were equal for the Object in Liquid question split their answers between the correct answer "Greater c, Smaller ΔT" (48%) and the incorrect answer "Greater c, Greater ΔT" (45%) on the Two-Liquids question. *None* of these students offered an answer on the second question that was consistent with their (incorrect) answer on the first question.

Similarly, students who gave an incorrect answer consistent with "Greater c, Greater ΔT" on the Object in Liquid question split their answers almost evenly between the correct answer "Greater c, Smaller ΔT" (51%) and the incorrect answer "Greater c, Greater ΔT" (47%) on the Two-Liquids question; none of them gave an "Equal ΔT" response

These findings suggest that students are employing reasoning that is strongly context-dependent in answering these two questions. This could be called “rule-based” reasoning since students typically justify their answers by citing one or another presumed “rules” which they tend to employ instead of trying to arrive at an answer by reasoning from more elementary principles; see discussion in Section V below.

C. Post-Instruction Results

We probed student thinking after all instruction was complete during the summer of 2002. We administered the Object in Liquid and Two-Liquids questions to thirty-two students during a recitation; the responses (see Appendix III and Appendix IV) were roughly consistent with those obtained after lecture but before recitation instruction in subsequent offerings of the same course (as reflected in Table II and Table III).

We created a multiple-choice equivalent of the Two-Liquids question (see Fig. 3), and administered it on a midterm exam after all instruction was complete in Spring 2004. Due to the somewhat subtle difference between choices *A* and *B* on this question we group responses by combining together all those students who gave an answer that was consistent with greater specific heat corresponding to a smaller change in temperature; this is the sum of those who answered either *A* or *B* (Appendix IV, Spring 2004). Similarly, we categorized both *C* and *D* responses as being incorrect under the common heading of “greater specific heat implies greater temperature change.” (Response E corresponds to “equal temperature change.”)[21]

This grouping of answers on the multiple-choice version of the Two-Liquids question yields a response pattern that is almost identical to that obtained on the free-response version of the same question given after lecture instruction, across all three answer categories (compare Appendix IV to Table III), as well as being very similar to the response pattern obtained on the free-response version after all instruction during Summer 2002 (Appendix IV). As before, responses that are consistent with the liquids having equal changes in temperature are non-existent, although such answers were often given in the different context of the Object in Liquid question.

A further follow-up using the Object in Liquid question in a multiple-choice format ("Text Multiple Choice," Fig. 4a) was carried out in spring of both 2003 and 2004; this version had wording similar to that of the free-response version of this same question. Another version of this multiple-choice question which uses common mathematical symbols instead of text, hereafter called the "Symbol Multiple Choice" version (Fig. 4b) was administered in spring of 2004 and 2006.[22,23] (The text version was administered on the midterm exam during 2004, and the symbol version on the final exam during the same semester; only minor differences in responses were found; see Appendix VI.) Despite all instruction having been completed (including lecture, recitation, homework, and exam preparation), student performance on both multiple-choice versions of this question is very similar to that on the free-response version that had been given as a quiz after lecture instruction only, suggesting that little to no effective learning occurred (on these concepts) as a result of the additional instructional activities following the lecture. The proportion of correct responses on all multiple-choice versions of the Object in Liquid question after instruction (68%, $N = 1491$ over four samples; see Appendix VI) is highly consistent with that on the corresponding free-response version of that question (63%, $N = 1036$ over three samples) which was given after lecture instruction but before recitation (see Table IV). However, on the multiple-choice questions, the “Equal ΔT” response frequency (11%) was lower than that seen on the free-response question (22%). By contrast, the “Greater c, Greater ΔT” response was slightly more popular than it was on the free-response question (22% vs. 15%).

D. Interview data regarding students’ mathematical errors

We conducted twenty-six one-on-one student interviews during the course of three different semesters. Interviews consisted of the questions described above, as well as related questions on energy, temperature, and specific heat capacity. Students succeeded with the interview tasks at a high level (~80% correct), with only two distinct identifiable tendencies regarding incorrect reasoning.

Four of the twenty-six students stated that the initial temperature would affect the magnitude of the temperature change. This is a much higher frequency for this response than our free-response data would suggest, and it remains unclear to what extent this is representative of student thinking. The other, and perhaps more interesting, student difficulty was related to the surprising number of mathematical errors made by the students during the interviews. Nearly one quarter of all interviews (23%) involved students making algebraic errors while working problems. For instance, while answering the Object in Liquid (free-response) question, some students would set up a correct expression comparing heat transfers from the object to the liquid, and vice-versa. After obtaining a correct expression that related the magnitudes of temperature changes to the specific heat capacities, students would incorrectly interpret the proportional relationship as implying that higher specific heat was associated with a larger change in temperature.

This error was not directly observed among the written responses to the free-response question; however it seems plausible that this difficulty may have been reflected in the incorrect "greater *c*, greater ΔT" answers on the written diagnostic. Such an outcome would be consistent with other research on the relationship between mathematics skill and physics performance[24] in that it seems to suggest that weak algebra skills might be a significant source of student difficulties with questions requiring simple proportional reasoning, such as those in calorimetry.

The mathematical errors that arose during the interviews consistently interfered with students' ability to solve the problems correctly even when other, intuitive reasoning approaches eventually allowed them to arrive at a correct answer. Meltzer's previous work had examined apparently analogous correlations between students' algebraic skills and their performance on qualitative physics questions. These and other results had appeared to suggest that mathematical errors on simple algebraic operations could threaten the physics performance of a segment of the student population, even on a relatively simple topic such as calorimetry.[25]

V. Discussion

A. Student reasoning about calorimetry

In Section III.A we enumerated three specific concepts targeted in our investigation of student thinking about calorimetry. These concepts and our related findings are as follows:

i. There is no heat transfer between the inside and the outside of a perfectly insulated container.

We found no evidence to indicate that students found this idea difficult or challenging. Although we did not probe student thinking about this idea explicitly, neither students' written responses nor the interview data provided any hint of related conceptual confusion.

ii. When two objects are in an insulated container, the magnitude of heat transfer from one object is equal to the magnitude of heat transfer to the other object.

The two multiple-choice versions of the Object in Liquid question explicitly probed student thinking about this idea. Responses from over 1000 students across three semesters indicate that between 12% and 25% of students believed that the magnitude of heat transfer to the colder substance was *not* equal to the magnitude of heat transfer away from the hotter substance (see Table IV for a summary of the data and Appendix VI for a semester-by-semester breakdown). Explanations of student reasoning were not required on these questions and the issue was not explicitly probed during the interviews, so no further information is available.

iii. The specific heat is the amount of energy per unit mass required to change the temperature of some object.

We probed students' thinking about this concept by asking them to compare temperature changes for two substances that had identical masses but different specific heats, and which are in thermal contact with each other or with a common heating source. Most students answered correctly that the substance with higher specific heat would have a smaller temperature change. About half of all students who gave correct answers and who were also asked to provide explanations of their reasoning did give adequate explanations for their responses. However, substantial numbers of students either failed to provide adequate explanations, or responded incorrectly regarding the temperature changes.

Some students answered that the temperature changes of the two substances would be equal. On all versions of the Object in Liquid question, a significant fraction of students gave this response, implying substantial confusion about the specific heat concept. Between 20-25% of all students gave this response on the free-response version of the question both after lecture instruction, and after all instruction. Between 7-12% of students gave that response on the multiple-choice versions of the question after all instruction. (On the multiple-choice versions, the response option that corresponded to equal temperature changes also stated that heat transfers would be equal, potentially excluding students who might have believed that equal temperature changes were linked to *unequal* heat transfers.) The most common justification for this response was either that the substances ended up with the same final temperature or that they reached "equilibrium," with the specific response depending on how the question was worded. Other popular explanations hinged on the equality of the substances' masses or that of the heat transfers.

Despite the fact that all versions of the Object in Liquid question yielded substantial numbers of "equal temperature change" answers, responses to the alternative-context Two-Liquids question, by contrast, yielded *no* answers that were consistent with an "equal temperature change" idea. This is a striking example of the well-known fact that questions based on identical physical principles but posed in different contexts can lead to very diverse student response patterns.[26,27]

The other common incorrect student response in regard to specific heat was to claim that the substance with higher specific heat would have the *larger* temperature change. The proportion of all students who gave this response after instruction (out of nearly 1500 students overall) was within the range of 14% to 28%, with only one low-enrollment summer class falling below this range. Responses fell within this range regardless of whether the questions were administered after lecture instruction only or after all instruction, and on both the Two-Liquids question and the Object in Liquid question (including both free-response and multiple-choice versions of both questions). Student explanations tended to be very straightforward assertions, with no supporting arguments, that larger specific heat implied a larger temperature change. The implication seemed to be that students had made this assumption either without critical examination or on the basis of persuasive, albeit faulty reasoning.

B. Relation to previous work

Our findings are quite consistent with the only other detailed study on related concepts, that is, the one reported by Cochran in his Ph.D. dissertation.[15] Using a variety of questions involving different substances undergoing heat exchange with each other and approaching a mutual equilibrium temperature, Cochran probed student difficulties with calorimetry concepts as reflected in rankings of temperature changes and magnitudes of heat transfers. Several of the questions used by Cochran are very similar to those used in our investigation. For example, Cochran administered the following question to students in an algebra-based physics course after all instruction on thermal physics was completed (Ref 15, p. 62):

> A block of lead at 100°C is put into an equal mass of cold water at 0°C in a perfectly insulated container. The specific heat of water is much greater than that of lead. The lead and the water are allowed to come to thermal equilibrium.
>
> 1. Is the magnitude of heat transfer from the lead *greater than*, *less than*, or *equal to* the heat transfer to the water. Explain.
>
> 2. Is the temperature change of the lead *greater than*, *less than*, or *equal to* the temperature change of the water? Explain.

Cochran found that 70% of the students gave a correct answer to Question #2, while 20% of students responded that the temperature changes would be equal. Also, 10% stated that the temperature change of the higher specific heat substance (the water) would be greater (Ref. 15, Table 6.1, p. 63). These results are remarkably similar to those on the *non-graphical* questions in our investigation, such as those reported in Table I (correct response: 63%; equal temperature change: 22%; greater change for higher specific heat: 15%) and Appendix III (correct response: 72%; equal temperature change: 22%; greater change for higher specific heat: 6%). Similarly, Cochran found that 10% of the students responded to Question 1 by claiming that the heat transfer magnitudes would *not* be equal; this is similar to our findings on a similar question as reported in Appendix VI, in which 12-25% of students made a similar incorrect assertion.

Cochran reported results on several other similar but more complicated questions. The results are consistent with those given above, but can not be compared directly because additional conceptual issues were involved in these questions.

Cochran notes that while most students are able to give correct answers to fundamental questions on calorimetry, a significant minority of introductory physics students manifest persistent problems. He cites specific student difficulties with calorimetry as including (1) students tending to equate magnitudes of heat transfer with temperature or with changes of temperature (previously reported by Kautz, Ref. 16, and by many others) and (2) students' tendency to focus on one particular variable (while ignoring the others) in the equation $Q = mc\Delta T$, thereby attributing a determining influence either to mass, or to specific heat, or to initial temperature, in order to justify answers regarding heat transfers or temperature changes.

For instance, on a question involving the immersion of two identical blocks into different volumes of water at the same temperature (the blocks had equal masses, specific heats, and initial temperatures), Cochran reports that 20% of the students identified the changes in temperature of the two blocks as being equal.

An excerpt from his dissertation reflects findings nearly identical to our own:

> About 20% of the students incorrectly answered that the final temperatures would be equal in each experiment. The responses of these students often suggested that they viewed temperature as a measure of amount of heat.

> *"They both start at the same temperature and heat is neither gained or lost in the process so both of the temperatures reached in the process should be the same."*

> A few students claimed that the final temperatures were equal because thermal equilibrium was reached and, "*…thermal equilibrium is when heat lost equals heat gained.*" [Ref. 15, pp. 72-73].

One key feature uncovered by our study and not reported by Cochran was the striking disappearance of "equal temperature change" answers on the graphical (Two-Liquids) questions. It appears that the graphical nature of this task might have altered student responses, since similar questions in a different context (that is, the Object in Liquid question) failed to show this same pattern of responses.

C. Rule-Based Reasoning

Various reports in the physics-education and science-education literature suggest that the behavior we have called "rule-based reasoning" may originate in part from perceptions that students have regarding their role in the classroom. A "rule-learner" has been described by Herron and Greenbowe as a student who views their primary task as memorizing rules and algorithms which are then practiced until they can be applied flawlessly.[28] Herron and Greenbowe point out that successful problem solvers may utilize a similar pattern, but more often include a step where they check the validity of their answer or evaluation method before reporting a final answer.

The Maryland Physics Expectations Survey[29] and the Colorado Learning Attitudes about Science Survey[30] both probed student expectations and attitudes about science and science learning. Both studies found that a substantial number of students both before and after instruction feel that they must *memorize* all the information, and then simply *find* the right equation to solve a problem. It would seem that this notion of needing to "determine the rule" often leads students to try to learn the material without bothering to search for any underlying conceptual framework or unifying ideas. Stavy and Tirosh have described analogous behaviors in a variety of contexts.[31]

We have interpreted our findings within the framework of students' specific learning difficulties tied to a particular physical context, a model that has proven highly effective in improving physics instruction.[32] Alternative interpretations (for instance, involving student "resources" as described by Hammer) could be employed as well.[33] Some of the behaviors we observed could be interpreted as students making use of "phenomenological primitives" as identified by diSessa,[34] such as "More means more" or, in the terms of Stavy and Tirosh, "More A-More B."[31] However, as plausible as such an interpretation might be, we have no direct evidence either from the written-response data or the interview data that students were employing a context-independent mode of reasoning that led to their incorrect responses.

VI. Conclusion

We found that students' explanations for solutions to calorimetry problems, whether correct or incorrect, generally involved very basic, stripped-down "rules-of-thumb" rather than detailed elaborate arguments. An example of such a rule might be, "greater specific heat implies that temperature change is greater" [or, "is smaller]." Many students employed algebraic calculations to justify their answers, although students didn't seem to show a strong preference for such algebraic approaches. Instead, many simply employed a straightforward qualitative argument involving rather perfunctory rule-based reasoning.

It seemed clear that some students had been misled by faulty rules-of-thumb such as "equilibrium means equal temperature change" or "rate of temperature change is directly proportional to specific heat." Students' ideas about what *should* happen appeared to lead them to form conclusions to fit their expectations. We do not have data that would allow us to account for the origins of such student ideas.

It is possible that refining this rule-based reasoning so that students are guided to *test* their conclusions by reference to basic principles might be an efficient way of promoting improved problem solving in calorimetry. We attempted to develop curricular materials to do this, simultaneous guiding students to explicitly address failures and inconsistencies of incorrect ideas or rules. However, we were unable to obtain consistent improvements in student performance.[35]

VII. Implications for Instruction

The model of student knowledge employed by Redish (which he describes as analogous to an archery target) identifies three "areas" of student thinking.[36,37] The innermost area (corresponding to the black "bull's eye" region of an archery target) consists of ideas that students know well, and that they are able to use to solve problems correctly regardless of specific context or representation. In our investigation, more than half of all students provided correct answers with correct explanations on calorimetry questions in diverse contexts, even after receiving only lecture instruction. Using Redish's analogy, we could identify calorimetry as falling within these students' bull's-eye region. From this we conclude that for many students, the concepts of calorimetry in a physics context appear to be relatively easily grasped. For some students, however, rule-based reasoning (in contrast to reasoning from fundamental principles) seems to dominate thinking in this physical context. For example, some students seem to become attached to a "greater c, greater ΔT" formulation without critically examining the implications of such a formulation[38]

Following a model of instruction that has been laid out in previous work, we sought to develop curricular materials that would explicitly addresses student difficulties in this area.[39] It would seem that exercises which guide students to recognize the interrelationships among mass, specific heat, temperature change, and heat transfer would be essential. We sought to guide students to resolve inconsistencies in their answers, especially when using representations that often tend to elicit inconsistent responses (such as the Object in Liquid and Two-Liquids questions); our objective was to prevent reliance on intuitive but faulty "rule-based" reasoning. It is possible that efforts to directly improve students' facility with algebraic manipulations might result in improved performance on the calorimetry assessments, but this is not something that we are able to investigate directly. In any event, we are unable to report any consistent successes in improving student performance on the calorimetry questions discussed here.[35] Apparent improvements observed among some of the experimental groups were not reproduced consistently among others, and the question of how best to improve student learning in calorimetry still awaits resolution.

VIII. Acknowledgments

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Table I. Object in Liquid, free response; after lecture instruction

"Greater c, Smaller ΔT" corresponds to a correct response that the substance with the greater specific heat would have a smaller change in temperature, while "Greater c, Greater ΔT" associates that same substance with the greater change in temperature. The statistics reported in the All Semesters column represent the 95% confidence interval of student performance for each answer category, based on score variances among the three semesters.

	Spring 2003	Fall 2005	Spring 2006*	All Semesters
	$N = 359$	$N = 427$	$N = 250$	$N = 1036$
Greater c, Smaller ΔT	64%	61%	64%	63 ± 4%
Correct with Correct Explanation	*55%*	*51%*	*53%*	*53 ± 5%*
Equal ΔT	21%	25%	20%	22 ± 7%
Greater c, Greater ΔT	15%	14%	16%	15 ± 2%

*In Spring 2006 the Object in Liquid question was administered without the Two-Liquids question.

Table II. Object in Liquid, free response, after lecture instruction. This is a breakdown of the responses outlined in Table I. Some of the students justified an "Equal ΔT" response by claiming either that the system goes to "equilibrium" or to a "common final temperature."

	All Semesters (3 Samples)
	$N = 1036$
Correct (Greater *c*, Smaller ΔT)	63%
With correct explanation	***53%***
With incorrect explanation:	
temperature change is larger because initial temperature is higher (or lower)	2%
Other explanations	7%
Incorrect (Equal ΔT)	22%
…because energy transfers are equal	5%
…because system goes to equilibrium/common final temperature	7%
…because masses are equal	4%
other explanations	6%
Incorrect (Greater *c*, Greater ΔT)	15%
… because specific heat is directly proportional to magnitude of temperature change	5%
"correct" explanation and incorrect answer	1%
other explanations	8%

Table III. Two-Liquids question, free response, after lecture instruction. "Greater c, Smaller ΔT" is a correct response that the liquid with the greater specific heat corresponds to a smaller slope on the temperature-time graph (that is, corresponding to a smaller rate of temperature change); An "Equal ΔT" response refers to graphs with equal slopes for both liquids; "Greater c, Greater ΔT" associates the liquid having the greater specific heat with the larger slope on the graph.

	Spring 2003	Fall 2005
	$N = 361$	$N = 427$
Greater c, Smaller ΔT	70%	73%
Correct with Correct Explanation	*50%*	*65%*
Equal ΔT	0%	0%
Greater c, Greater ΔT	28%	26%

Table IV. Comparison of three versions of Object in Liquid question: free response, text multiple-choice, and symbol multiple-choice. The free-response version was given after lecture instruction only, while the multiple-choice versions were given after all instruction was complete. "Greater c, Smaller ΔT" corresponds to the sum of answers *B* and *E* on the multiple-choice versions, "Equal ΔT" corresponds to answer *D,* and "Greater c, Greater ΔT" corresponds to the sum of answers *A* and *C*. "Heat transfers are not equal" corresponds to the sum of answers *A* and *B*, while "Heat transfers are equal" corresponds the sum of answers *C*, *D*, and *E*. (For detailed breakdown, see Appendix V.)

	Object in Liquid, Free Response All Semesters	Object in Liquid Text MC Average Response	Object in Liquid Symbol MC Average Response
	$N = 1036$	$N = 760$	$N = 731$
Greater c, Smaller ΔT	63 ± 4%	66%	70%
Equal ΔT	22 ± 7%	13%	8%
Greater c, Greater ΔT	15 ± 2%	22%	23%
Heat transfers are not equal	--	21%	16%
Heat transfers are equal	--	79%	85%

Appendix I. Before all instruction, Object in Liquid, free response: Fall 2005

	Fall 2005
	$N = 479$
Greater c, Smaller ΔT	50%
Correct with Correct Explanation	*40%*
Equal ΔT	38%
Greater c, Greater ΔT	12%

Appendix II. Consistency of Results between Object in Liquid and Two-Liquids questions, after lecture instruction. Each of the bottom three rows corresponds to a particular response category on the Two-Liquids question. Students who provided each of the three possible responses on the Object in Liquid question are broken down in each column according to which response they gave on the Two-Liquids question.

	Object in Liquid: Greater c, Smaller ΔT		Object in Liquid: Equal ΔT		Object in Liquid: Greater c, Greater ΔT	
	Spring 2003	Fall 2005	Spring 2003	Fall 2005	Spring 2003	Fall 2005
N	*230*	*262*	*77*	*107*	*50*	*58*
Two-Liquids: Greater c, Smaller ΔT	84%	81%	40%	54%	52%	50%
Two-Liquids: Equal ΔT	0%	0%	0%	0%	0%	0%
Two-Liquids: Greater c, Greater ΔT	15%	15%	55%	38%	46%	47%

Appendix III. After all instruction, Object in Liquid, free response.

	Summer 2002
	$N = 32$
Greater c, Smaller ΔT	72%
Correct with Correct Explanation	*63%*
Equal ΔT	22%
Greater c, Greater ΔT	6%

Appendix IV. After all instruction, Two-Liquids, free response vs. Two-Liquids, multiple choice. “Greater *c*, Smaller ΔT” corresponds to responses *A* or *B* on the multiple-choice version; “Equal ΔT” corresponds to response *E,* and “Greater *c*, Greater ΔT” corresponds to responses *C* or *D*.

	Two-Liquids, free response Summer 2002	Two-Liquids, multiple choice Spring 2004
	$N = 32$	$N = 447$
Greater *c*, Smaller ΔT	69%	73%
Correct with correct explanation	*59%*	--
Equal ΔT	0%	1%
Greater *c*, Greater ΔT	22%	26%

Appendix V. After all instruction, Object in Liquid, text multiple-choice vs. Object in Liquid, symbol multiple-choice. Rows lettered A-E correspond to student response rates on Object in Liquid, text multiple-choice (first three columns) and Object in Liquid, symbol multiple-choice (last three columns); see Figures 4a and 4b. In Spring 2004, the Text MC question was given on a midterm exam while the Symbol MC question was given on the final exam.

	Object in Liquid, Text MC			Object in Liquid, Symbol MC		
	Spring 2003	Spring 2004	Average Response	Spring 2004	Spring 2006	Average Response
	N = 299	*N* = 461	*N* = 760	*N* = 420	*N* = 311	*N* = 731
A.	4%	8%	6%	10%	6%	8%
B.	13%	16%	15%	9%	6%	8%
C.	13%	18%	16%	16%	14%	15%
D.	12%	13%	13%	7%	7%	7%
E.	57%	44%	51%	56%	68%	62%

Appendix VI. After all instruction, Object in Liquid, text multiple-choice: Spring 2003, Spring 2004; Object in Liquid, symbol multiple-choice: Spring 2004, Spring 2006.* See Table IV footnote for definitions of response categories

	Object in Liquid, Text MC			Object in Liquid, Symbol MC		
	Spring 2003	Spring 2004	Average Response	Spring 2004	Spring 2006	Average Response
	$N = 299$	$N = 461$	$N = 760$	$N = 420$	$N = 311$	$N = 731$
Greater c, Smaller ΔT	71%	60%	66%	66%	73%	70%
Equal ΔT	12%	13%	13%	8%	7%	8%
Greater c, Greater ΔT	17%	27%	22%	26%	20%	23%
Heat transfers are not equal	17%	25%	21%	19%	12%	16%
Heat transfers are equal	83%	75%	79%	81%	88%	85%

*In Spring 2004, the Text MC question was given on a midterm exam while the Symbol MC question was given on the final exam.

Appendix VII. Object in Liquid, free-response explanation breakdown, semester by semester.

	Spring 2003	Fall 2005	Spring 2006
	$N = 359$	$N = 427$	$N = 250$
Correct (Greater *c*, Smaller Δ*T*)	64%	61%	64%
With correct explanation	*55%*	*51%*	*54%*
With incorrect explanation:			
temperature change is larger because initial temperature is higher (or lower)	3%	1%	1%
Other explanations	6%	7%	8%
Incorrect (Equal Δ*T*)	21%	25%	20%
…because energy transfers are equal	8%	6%	2%
…because system goes to equilibrium/common final temperature	6%	7%	7%
…because masses are equal	3%	5%	4%
other explanations	3%	8%	7%
Incorrect (Greater *c*, Greater Δ*T*)	14%	14%	16%
… because specific heat is directly proportional to magnitude of temperature change	6%	4%	6%
"correct" explanation and incorrect answer	1%	1%	1%
other explanations	9%	8%	8%

Figures

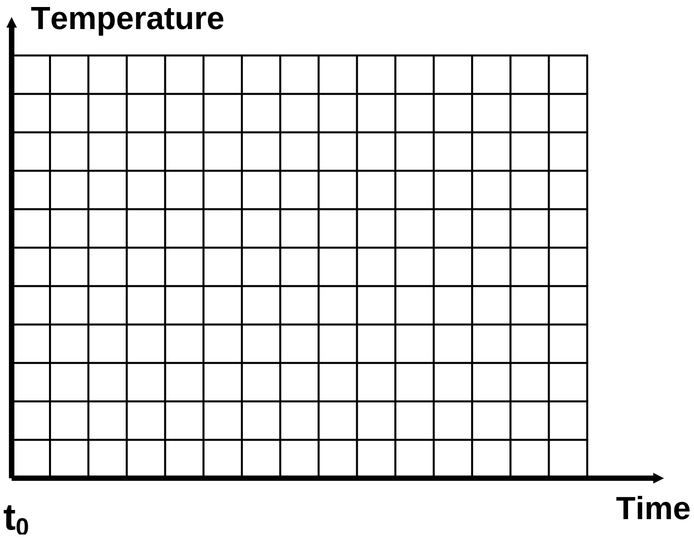
Figure 1. Object in Liquid, free response question. Four different version of this question were used, varying both the pair of substances (water and copper, or bromoform and aluminum) and the identity of the substance which was specified to have the higher initial temperature.

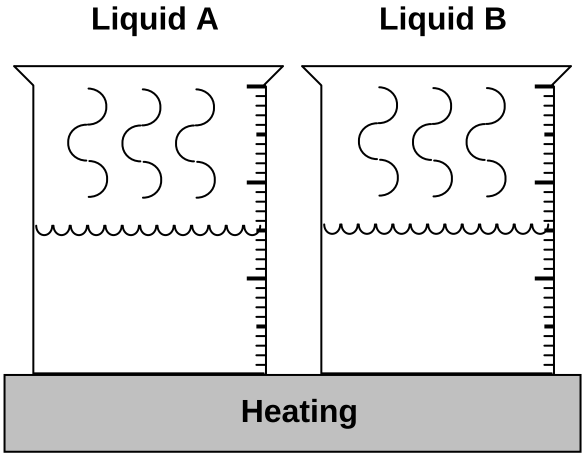
> The specific heat of water is greater than that of copper.
>
> A piece of copper metal is put into an insulated calorimeter which is nearly filled with water. The mass of the copper is the same as the mass of the water, but the initial temperature of the copper is higher than the initial temperature of the water. The calorimeter is left alone for several hours.
>
> During the time it takes for the system to reach equilibrium, will the temperature change (number of degrees Celsius) of the copper be more than, less than or equal to the temperature change of the water? Please explain your answer.

Figure 2. Two-Liquids, free response question. Three different versions of this question were used in which the ratios of specific heats of Liquids A and B were specified as being either 2, 3, or 4.

Suppose we have two ***separate*** containers: One container holds Liquid A, and another contains Liquid B. The mass and initial temperature of the two liquids are the same, but the ***specific heat*** of Liquid A is ***two times*** that of Liquid B. Each container is placed on a heating plate that delivers the ***same rate of heating*** in joules per second to each liquid beginning at initial time $\mathbf{t_0}$.

a) On the grid below, graph the temperature as a function of time for *each* liquid, A and B. Use a separate line for each liquid, even if they overlap. Make sure to clearly label your lines, and use proper graphing techniques.





The specific heat of A is greater than specific heat of B.

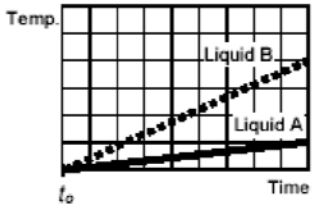
b) Please **explain** the reasoning that you used in drawing your graph. (Please continue on the back of the page.)

Figure 3. Two-Liquids, multiple-choice version.

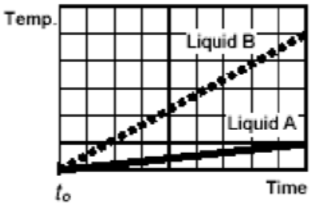
Suppose we have two *separate* containers: One container holds Liquid *A*, and another contains Liquid *B*. The mass and initial temperature of the two liquids are the same, but the specific heat of Liquid *A* is *four times* that of Liquid *B*. Each container is placed on a heating plate that delivers the same rate of heating in joules per second to each liquid beginning at initial time t_0.

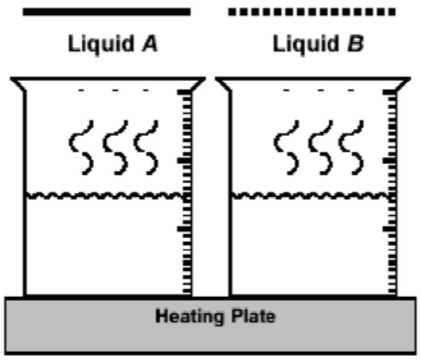
On the grids below are four graphs that represent the temperature-versus-time plots for liquid *A* and liquid *B*, with liquid A represented by a solid line and liquid *B* by a dashed line. Indicate the graph whose temperatures are plotted most accurately for liquid *A* versus liquid *B*.

A)



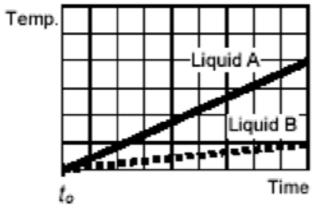
B)



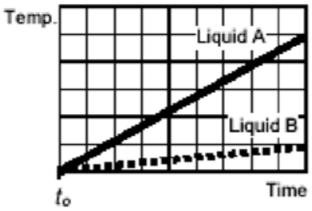


The specific heat of *A* is greater than the specific heat of *B*.

C)



D)



E)

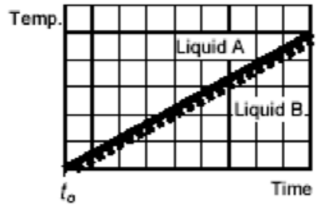


Figure 4a. Object in Liquid, multiple-choice “text-based” version

An object is immersed in a liquid within a sealed and insulated container. The mass of the object is the same as the mass of the liquid. The initial temperature of the object is lower than the initial temperature of the liquid, but the specific heat of the object is ***greater*** than that of the liquid. The calorimeter is left alone for several hours until it reaches equilibrium. Which of the following is true? *Note: Here, “temperature change” means “number of degrees Kelvin increased* ***or*** *decreased.”*

A. The energy transfer to the object is *not* equal to the energy transfer away from the liquid, and the temperature change of the object is greater than the temperature change of the liquid.

B. The energy transfer to the object is *not* equal to the energy transfer away from the liquid, and the temperature change of the object is less than the temperature change of the liquid.

C. The energy transfer to the object is equal to the energy transfer away from the liquid, and the temperature change of the object is greater than the temperature change of the liquid.

D. The energy transfer to the object is equal to the energy transfer away from the liquid, and the temperature change of the object is equal to the temperature change of the liquid.

E. The energy transfer to the object is equal to the energy transfer away from the liquid, and the temperature change of the object is less than the temperature change of the liquid.

Figure 4b. Object in Liquid, multiple-choice “equation-based” version

Object A has mass m_A, specific heat c_A, and the initial temperature $T_{initial\ A}$. Liquid B has mass m_B, specific heat c_B, and initial temperature $T_{initial\ B}$. Object A is immersed in Liquid B within a sealed and insulated container (i.e., a calorimeter). We are given the following information:

$m_A = m_B$
$c_A > c_B$
$T_{initial\ A} < T_{initial\ B}$ but after a long time, $T_{final\ A} = T_{final\ B}$
Which of the following is true? [Q is heat transfer; $\Delta T \equiv T_{final} - T_{initial}$]

A. $Q_{to\ A} \neq Q_{away\ from\ B}$; $|\Delta T_A| > |\Delta T_B|$

B. $Q_{to\ A} \neq Q_{away\ from\ B}$; $|\Delta T_A| < |\Delta T_B|$

C. $Q_{to\ A} = Q_{away\ from\ B}$; $|\Delta T_A| > |\Delta T_B|$

D. $Q_{to\ A} = Q_{away\ from\ B}$; $|\Delta T_A| = |\Delta T_B|$

E. $Q_{to\ A} = Q_{away\ from\ B}$; $|\Delta T_A| < |\Delta T_B|$

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[18] This is reminiscent of the confusion regarding relative temperatures that is due to physiological sensations caused by differing thermal conductivities; see Ref. 2.

[19] Four different versions of the Object in Liquid question were used. The different versions featured different substances as the object and liquid such that in one case the liquid had a higher specific heat, while in the other case the object had the higher specific heat. In addition, the identity of the substance with the higher initial temperature was varied; see Figure 1. Similarly, three different versions of the Two-Liquids question were used in which the ratio of c_A/c_B was either 2, 3, or 4; see Figure 2. Six separate tests were administered in which different versions of the two questions were paired on the same question sheet. No significant difference in student performance was measured across the different versions of the diagnostic questions.

[20] A free-response version and a multiple-choice version were administered, see Figs. 2 and 3.

[21] For the precise breakdown of each response frequency, see Appendix IV.

[22] A few students during the Spring 2004 midterm complained about the extensive "legalese" or wordiness of this question, and there were concerns that this wordiness might lead to student confusion. This led to our development of a nearly identical question with a more compact formulation. This version relied on symbolic notation (e.g. "ΔT_{liquid}" for "change in temperature of the liquid"). The text multiple-choice question

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was given on a midterm exam during Spring 2004, while the symbol multiple-choice question was given on a final exam in the same course. Responses in each category were very similar, with a discrepancy of ≤6% on each of the five categories (Appendix VI).

[23] There are option-by-option differences in responses on the text-based question in comparison to the symbol-based question (see Appendix V). However, when the multiple-choice responses are combined according to the categories described in the caption to Figure IV, we find a very similar pattern on the two versions (see Appendix VI). This pattern is also consistent with, although not identical to, that obtained on the free-response question (see Table IV).

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