Student ideas regarding entropy and the second law of thermodynamics in an introductory physics course

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Abstract

The concept of entropy is often introduced in the context of the second law of thermodynamics, which specifies that the entropy of an isolated system must always increase in any real process. To date, there have been no published investigations that specifically probed physics students' thinking regarding this concept. We report here a two-year study of students in an introductory calculus-based physics course in which they responded to a variety of questions that dealt with entropy changes of an arbitrarily defined system and that system's surroundings. We present free-response, multiple-choice, and interview data that reflect students' thinking as to how entropy must change during an arbitrary real process. We found that pre-instruction fewer than 10% of all students were able to give completely correct responses to relevant questions posed in both general and concrete contexts, and nearly two thirds of all students showed evidence of conservation-type reasoning regarding entropy. These outcomes persisted even after instruction that attempted to address these conceptual issues. However, we found that targeted instruction that *specifically* guided students to recognize that entropy is not a conserved quantity has appeared to yield improved performance on qualitative questions related to this concept.

I. INTRODUCTION

A. Motivation and overview

In this paper we report an investigation of student thinking regarding certain aspects of the second law of thermodynamics and of the law's relationship to changes in entropy. We examine both the knowledge elements students bring to bear and the difficulties they encounter when studying these concepts. The ultimate goal of this work is to lay the framework for the creation of instructional materials and strategies that can help students improve their understanding of second-law concepts.

The second law of thermodynamics (in its various forms) limits the direction of any naturally occurring processes to that which causes an overall increase in entropy, that is, that the entropy of the system plus that of the surroundings must increase. It is this key idea that helps explain the course of natural phenomena in all contexts.

The concepts of entropy and the second law of thermodynamics are considered to be key elements of the introductory curriculum for undergraduate students in a wide variety of science and engineering fields. Even in non-technical contexts, ideas related to entropy and the second law are often introduced with respect to issues of energy efficiency and conservation. A central idea is that, even under ideal conditions (e.g. in a reversible cycle), there exists an upper limit on the amount of usable work that can be gained from a given amount of thermal energy. This concept has broad implications and yet may easily lead to misunderstandings and confusion. For example, efficiencies lower than 100% may often be tied to ideas of "imperfection" or inadequate design. There are many different aspects of the relationships among entropy, the second law of thermodynamics, and physical processes involving energy transfers. Some investigators have made preliminary studies of student thinking regarding the energy "degradation" aspect of the second law and notions of the unidirectionality of natural processes such as heat flow.[1] Most recently, an investigation has been reported in which student thinking regarding second-law constraints on heat-engine efficiencies was carefully probed.[2] In this present paper, we explore student thinking regarding the idea that net entropy increase is a necessary outcome of any natural process.

The idea that entropy increases with time has traditionally been introduced in a variety of different ways, depending on the course and the context. It is often discussed in association with terms such as “system,” “surroundings” (or “surrounding environment”), “isolated system,” or “universe,” as well as in connection with the phrase “spontaneous process.” The meaning of entropy itself is often associated with both macroscopic and microscopic notions of disorder, although precise definitions are often omitted.

At this introductory level most of the emphasis is on changes in entropy and examples are typically drawn from heat transfer. Students are often asked to calculate entropy changes that occur during irreversible heat transfers, though to do so often requires the use of a reversible process that leaves the system in the same final state. The distinction between reversible and irreversible processes is a concept that is in itself quite subtle and difficult.

In this paper we will explore, in a variety of contexts both before and after instruction, student thinking related to entropy changes in natural processes. We will also describe development and initial testing of research-based instructional materials developed in the course of our investigation, and we will report preliminary data regarding student learning gains arising from use of these materials.

B. Previous research on student thinking regarding entropy

There is a growing collection of published work dealing with student understanding of thermodynamics at the introductory university level, particularly in connection with student thinking regarding the first law of thermodynamics[3,4] and the ideal gas law.[5,6] There have also been several brief reports regarding student conceptions in upper-level thermal physics courses.[7-13] However, there is very little previous research on student understanding of entropy and the second law of thermodynamics at the introductory university level.

Kesidou and Duit[1] interviewed 15- and 16-year-old students who had received four years of physics instruction, asking them questions that involved concepts related to both the first and second laws of thermodynamics. They reported that after instruction, most students had ideas that processes tend to go in one direction only and that energy is in some sense "used up" (or, becomes less available). However, Kesidou and Duit stated that these student notions were largely based on intuitive ideas about everyday life, and were not phrased within a framework characterized by deep understanding. On the other hand, neither the second law of thermodynamics nor its consequences had been part of the curriculum studied by the students. Nonetheless, Kesidou and Duit concluded that student difficulties with heat and temperature impede student learning of second law concepts.

Ben-Zvi[14] reported on student use of curricular materials she developed that dealt with energy and the quality of energy. She asserted that, in a college-level course for non-science majors, only one-quarter of the students had developed some understanding of entropy concepts. Specifically, Ben-Zvi stated that these students recognized that in processes involving energy transfer, “each transformation is accompanied by some of it being converted to heat and thus the ability to perform work decreases.”

In the context of chemistry, Granville[15] reported that, in part due to ambiguities in the usage of the symbol “S,” chemistry students sometimes became confused when applying the principle commonly stated as “$\Delta S > 0$ for a spontaneous process.” Granville noted that in some contexts discussed in the introductory chemistry course the letter S refers to the entropy of the system *plus* that of the surroundings or—equivalently—to the entropy of an isolated system; it is in this context that the cited “principle” is valid. In other contexts, however, S is used to refer to the entropy of the system *only*. Perhaps understandably, this inconsistent usage can lead to significant student confusion.

Thomas and Schwenz[16] investigated “prevalent alternative conceptions” on equilibrium and thermodynamics among 16 college-level physical chemistry students. Among the findings they reported was a strong tendency for students to believe, incorrectly, that the second law of thermodynamics *required* the entropy of “the system” to increase even in a context where other evidence showed that this would not be the case.

A very recent study by Sözbilir and Bennett[17] probed the thinking of university students enrolled in physical chemistry courses in Turkey. The authors reported student difficulties arising from misinterpretations of mathematical equations, along with students' inability to integrate new knowledge with their existing knowledge. In particular, they showed that many students formed inaccurate ideas regarding the connection between entropy changes of a system and the accompanying entropy changes in the surroundings.

Cochran and Heron[2] investigated student thinking on entropy and its role in constraining allowed heat-engine efficiencies. They found that, for the most part, students did not perceive any connection between constraints on engine efficiencies and increases in total entropy of the system and its surroundings. They developed curricular materials that guided students to make this connection explicit, in order to better understand and analyze efficiency constraints for arbitrary heat engines. Their investigation is, apart from brief reports in conference proceedings, the only published study to date of student learning related to entropy and the second law of thermodynamics in university-level physics courses.

II. CONTEXT OF THE INVESTIGATION

A. Sample characterization

The bulk of this study was conducted with students in a second-semester (of a two-semester sequence) calculus-based introductory physics course at Iowa State University (ISU). The calculus-based physics sequence at Iowa State usually enrolls 700-800 students per calendar year; most of these are engineering majors but a few physics majors and computer-science majors are included. The course content varies slightly among semesters as the individual instructor has some flexibility in choice of topics. However, in general, the first-semester course covers kinematics, dynamics, and fundamentals of electrostatics and electric circuits, while the second semester typically covers magnetism, AC circuits, waves, fluids, and thermal physics.

Additional data were collected in a sophomore-level physics course at the University of Washington that covers a wide range of topics on thermal physics. (This course was taught by one of the authors [DEM] in Winter 2006.) The students in this course are primarily physics majors, all of whom have completed UW's introductory calculus-based physics courses or an equivalent course. However, this thermal-physics course is, for most of them, their first exposure to thermodynamics in the context of university-level physics. Unless otherwise specified, the data described in this paper were from the ISU sample.

B. Student background on entropy

In order to assess students' previous exposure to entropy concepts, we conducted a brief background survey in the fall of 2006. The survey was distributed before any instruction on entropy or thermodynamics had begun. We found that of 272 students, 64% self-reported having studied entropy in a previous course, and at least that many reported taking a specific course where entropy was discussed as part of the instruction (primarily in one of a number of introductory chemistry courses). It is important to note that in many chemistry textbooks, students are typically introduced to entropy and the second law of thermodynamics in the context of "spontaneous" processes; it is emphasized that in such processes the entropy of the universe must increase. Chemistry texts are very explicit in the use of the formulation "system plus surroundings equals universe" (more so than many current physics books).[18]

III. QUESTIONS USED TO PROBE STUDENT THINKING

A. Entropy increase in natural processes[19]

We investigated students' thinking regarding the second-law concept of entropy increase in natural processes, as well as the role of "system" and "surroundings" in the application of that concept. The second law of thermodynamics states that the total entropy of the universe will always increase due to the occurrence of any real process. In this context, the universe can be divided by a boundary into two arbitrarily defined regions, a "system" and its "surroundings" (or, "surrounding environment"). A system is nothing other than a particular contiguous region of interest that is arbitrarily defined and enclosed by a boundary. The "surroundings" comprise everything outside that boundary.[20] The second-law statement regarding increasing entropy is often closely associated with students' introduction to the entropy concept itself, and this statement has even been referred to by some authors as the most general statement of the second law of thermodynamics.[21]

B. General-context question

The general-context question (see Fig. 1) relates to an arbitrary system, along with its surroundings with which the system can exchange energy. The context is any naturally occurring process; no further details are offered regarding either the system or the process. Students are asked whether the entropy of the system will increase, decrease, or remain the same during the process, or whether this is not determinable with the given information. That same question is posed regarding the entropy of the surroundings, as well as the total entropy of the system *plus* the entropy of the surroundings.

The correct answer is that neither the change in entropy of the system nor that of its surroundings is determinable from the given information, because no specific information is provided about the system or the process. The only physical constraint is that the total entropy of the system plus the entropy of the surroundings must increase as a consequence of the second law of thermodynamics.

C. Concrete-context question

The concrete-context question (see Fig. 2) relates to an object placed in a thermally insulated room that contains air. The object and the air are initially at different temperatures and are allowed to exchange energy with each other. Students are asked whether the entropy of the object will increase, decrease, or remain the same during the process, or whether this is not determinable with the given information. That same question is posed regarding the entropy of the air in the room, as well as the entropy of the object *plus* the entropy of the air. A fourth part of the question asks specifically about the entropy change of the universe. Since the object and the air in the room are initially at different temperatures, the higher temperature entity (either object or air) will transfer energy in the form of heat to the lower temperature entity and thus undergo an entropy decrease. Whichever entity gains energy will undergo an entropy increase. However, the question does *not* specify whether the object temperature is initially higher or lower than that of the air in the room, and so there is insufficient information to determine the sign of entropy change either of the object or of the air. As in the general-context question, the only specification that can be made is that the total entropy of object plus air in the room must increase.

D. Spontaneous-process question

This multiple-choice question describes four processes that involve a change in the entropy of a system and its surrounding environment. In Version *A* (see Fig. 3), students are asked to decide which of the processes can actually occur “in the real world.” Version *B* of this question (see Fig. 4) includes an (incorrect) answer option (response *d*) that corresponds to the total entropy either increasing *or* remaining the same. Version *A* of the question does not include an answer option that combines those two possibilities, that is, no answer that corresponds to the option $\Delta S \geq 0$.[22]

There is no constraint on the change in entropy of either the system or the environment considered by itself, so the entropy of either one may increase *or* decrease. However, the sum of the two entropy changes must be positive, which means that processes II and IV (in Version *A*) or I and IV (in Version *B*) are possible but all the other processes are disallowed.

IV. STUDENTS’ REASONING REGARDING THE PRINCIPLE OF INCREASING ENTROPY

A. Prevalence of correct responses before instruction

We administered the general-context question during four different offerings of the second-semester calculus-based introductory physics course at ISU; in three of those four courses we also administered the concrete-context question. The questions were administered together before any instruction on entropy and the second law of thermodynamics took place; Table I shows the proportion of students who provided correct responses.

For the general-context question, somewhat less than half of all students (42%) answered correctly that neither the entropy change of the system nor that of the surroundings would be determinable from the given information. A smaller proportion of students (19%) gave the correct “increases” answer for the entropy change of the system plus surroundings. Almost no one (4%) gave a correct response for all three parts of the question.

The concrete-context question yielded similar results. Half of all students stated that the change in entropy of the object and that of the air in the room would not be determinable. About 90% of students who gave a “not determinable” response on part (a) (system/object) also gave a “not determinable” response on part (b) (surroundings/air), on both the general-context and concrete-context questions.

The proportion of students (14%) who gave a correct response on part (c) (that is, that entropy of the object plus entropy of the air in the room increases) was similar to the proportion (19%) who gave a correct response on the corresponding part (c) for the general-context question. A similar proportion correctly stated that the entropy of the universe would increase. The proportion of students (5%) who gave correct answers for all of the first three parts (*a*, *b*, and *c*) of the concrete-context question was nearly identical to the proportion who gave an "all-correct" answer on the general-context question (that is, correct answers on all three parts). Before instruction, only 44% of those students who gave an all-correct answer to the concrete-context question were *also* able to give correct answers to parts *a*, *b*, and *c* of the general-context question. After instruction in Spring 2005 this overlap proportion was virtually unchanged at 41%. We will see later (in Section VI below) that after use of specially designed curricular materials, this overlap proportion increased sharply.

B. Students' idea that overall entropy remains the same

Before instruction had begun on the second law of thermodynamics, a clear majority of students gave answers consistent with a belief that entropy is a conserved quantity (see Table II). On the general-context question, two thirds (67%) of all students responded that the entropy of the system plus the entropy of the surroundings stays the same. A statistically identical proportion (71%) of students responded on the concrete-context question that the entropy of the object plus the entropy of the air in the room stays the same. Of those students that gave a "total entropy remains the same" response for the general-context question, approximately 80% gave a similar response on the concrete-context question.

These consistent (yet incorrect) responses on similar questions in two different contexts suggest that at this point, most students had a fairly well-defined point of view regarding entropy conservation. This impression is strengthened by further analysis of students' responses as shown below.

i. Two categories of reasoning

When the answers of those students who gave these "entropy remains the same" responses are analyzed in more detail, we find that on both the general- and the concrete-context questions more than 75% of these students fall into one of two specific categories. (These categories are referred to as *A* and *B*, respectively, in Table II.) On the general-context question, the first category (*A*) consists of students who believe that the change in entropy of the system is not determinable and the change in entropy of the surroundings is not determinable, but the entropy of the system *plus* that of the surroundings remains the same (26% of all responses). Two thirds (65%) of the students who fall within this category specifically cited some type of conservation rule as their reasoning for entropy remaining the same. Many are unclear about what exactly is being conserved, but entropy, energy, and heat are the quantities most often mentioned. On this same question, students in the second category (*B*) claim that the system's entropy and the surroundings' entropy *do* change in some specified manner, but display an analogous chain of reasoning for the total change, i.e.: the system's entropy increases [decreases] and the surroundings' entropy decreases [increases], but the entropy of the system *plus* that of the surroundings remains the same. (Among these category-*B* students, a majority of 58% stated that the entropy of the *surroundings* would increase, while also claiming that of the system would decrease.) The proportions of students in categories *A* and *B* were virtually identical.

ii. Comparison of responses on general- and concrete-context questions

The results for the concrete-context question are very similar to those for the general-context question, including the breakdown into categories *A* and *B*. Most students (71%) stated that the entropy of the object plus the entropy of the air in the room (hereafter referred to as the "total entropy") would not change during a spontaneous process. More than half (60%) of all responses on the concrete-context question included a series of answers consistent with total entropy being conserved during a spontaneous process (see the last row in Table II).[23] The proportions of students who fell into categories *A* and *B* were similar, although not identical, to those found on the general-context question. In contrast to the *A*/*B* parity observed on the general-context question, category *A* was significantly more popular than category *B* on the concrete-context question (38% vs. 22%). (Categories *A* and *B* were defined similarly to those on the general-context question, except that "object" and "air in room" were substituted for "system" and "surroundings," respectively.) Among those students falling into category *B* on the concrete-context question, 71% stated that the entropy of the surroundings would increase rather than decrease, an even larger majority than did on the general-context question.

C. Post-instruction responses

i. Comparison of correct responses pre- and post-instruction

After all instruction was complete in Spring 2005, we were able to administer free-response questions to students during one week of lab classes. We compared students' responses on both the general-context and the concrete-context questions to their pre-instruction responses on the same questions given at the beginning of that semester, for a "matched sample" of students that consisted of exactly the same group both pre- and post-instruction. There was little difference in the proportion of correct responses before and after instruction. For example, correct responses on the "total entropy change" question [part (c)] increased from 24% to 35% on the general-context question, and from 20% to 23% on the concrete-context question.[24] The proportion of students with all three parts *a*, *b*, and *c* correct increased from 4% to 8% in the general context, and from 7% to 13% in the concrete contexts. (Detailed data and further discussion are given below in Section VI.)

ii. Pre- and post-instruction comparison of responses reflecting conservation ideas

Responses that are related to "conservation" thinking on the general- and concrete-context questions are shown in Table III, where students' post-instruction responses are compared to their pre-instruction responses during that same semester. The sample is "matched," meaning that exactly the same students are represented in the pre- and post-instruction categories. These students had completed our Entropy State-Function Worksheet during recitation.[25]

General-context question. On the general-context question many students continued to state, post-instruction, that entropy of the system plus that of the surroundings stays the same (48% of all responses, compared to 61% pre-instruction); more than 80% of this group fell into one of our two conservation categories A and B. There was a relatively small but statistically significant decrease from pre- to post-instruction (from 53% to 39%, $p < 0.05$) in the proportion of all responses that fell into one of the two conservation categories on the general-context question. For unknown reasons, there was a disproportionately larger decrease in those who fell into category A (i.e., "entropy of system is not determinable, entropy of surroundings is not determinable, total entropy remains the same"). Among those students in category B, the claim that the surroundings entropy would increase (rather than decrease) retained exactly the same majority support it had among the pre-instruction group.

Interview data were obtained from 18 student volunteers who agreed to participate in one-on-one interviews after all instruction was complete in the spring of 2005. Our interview data confirmed many of the student ideas which we observed in the free-response data. Seven of eighteen students provided some type of conservation argument in their answer to the general-context question, while none of eighteen gave a correct response for all three parts of this question.

> "[S1] I think for the irreversible process… I actually started with step (c). I was thinking that the entropy of the system plus surroundings equals zero, so it would remain the same. I know these two would be opposite of each other… I wasn't 100% sure, but I was thinking the system would decrease, and the surroundings would increase"

> "[S2] … [c] it remains the same because the surroundings and system is like the universe and entropy of the universe is constant"

Concrete-context question. The concrete-context question yielded post-instruction responses that, in every category, were virtually unchanged from their pre-instruction values (see Table III). However, if we compare these same students' responses on the general- and concrete-context questions given after instruction, some differences between the contexts become apparent. After instruction, responses that claimed total entropy would remain the same were offered more frequently on the concrete-context question (71%) than they were by the same students on the general-context question (48%). Of those students that gave a "total entropy remains the same" response for the concrete-context question, 61% gave a similar response on the general-context question. Before instruction, this overlap in similar responses between the two contexts had been 80%. This decreasing consistency of incorrect responses suggests that, after instruction, students' thinking may have been less closely characterized by the notion of entropy conservation than had been the case before instruction.

On both questions, a substantial proportion of all students still fell into one or the other of the two conservation categories regarding the "total entropy remains the same" responses. However, after instruction, the concrete-context question yielded a higher proportion of conservation arguments (59% of all students) than did the general-context question (39% of all students, difference significant at $p = 0.001$). In addition, the proportion of correct responses (that is, "total entropy increases" responses) after instruction in the concrete context (23%) was lower than that in the general context (36%), a difference that is statistically significant ($p = 0.02$).[26]

Spontaneous-process question. Two different versions of the spontaneous-process question were administered after all instruction on thermodynamics was complete in the Fall 2004 and Spring 2005 semesters. After administering Version *A* (Fig. 3) in the Fall 2004 course, we conducted seven interviews in which we asked this question in a free-response format. During these interviews, we asked students to identify which of the situations could actually occur in a real process. Four of the seven students stated that total entropy must either increase *or* remain the same. We therefore re-cast the multiple-choice options to reflect this change in Version *B* (Fig. 4), administered in the Spring 2005 course. (We were unable to administer this question again in the fall-semester course due to logistical difficulties.) Responses to both versions of this question are shown in Table IV.

It is unclear to what extent the students in the Fall 2004 course would have preferred an "increases *or* remains the same" answer but in both semesters, after instruction, over half of all students gave a response consistent with a belief that entropy should (or at least could) remain unchanged during a spontaneous process.[27] (The proportion of correct responses was not significantly different on the two versions of the question.)

V. STUDENTS' REASONING REGARDING "SYSTEM" AND "SURROUNDINGS"

A. Student idea that "system" and "surroundings" are not arbitrary distinctions

On the general-context question, the most common pre-instruction responses were that the changes in entropy of the system and of the surroundings were not determinable with the given information. For the pre-instruction concrete-context question, a similar proportion of students (~50%) responded that the changes in entropy of the object and of the air in the room were not determinable (see Table V).

If we look at those pre-instruction responses in which students made a specific directional choice (that is, either increases *or* decreases) we find that in the general context, students' preferred answer was that the entropy of the system would increase (26%), rather than decrease (19%) or remain the same (10%); the difference between the "increases" and "decreases" responses is significant over our four semesters of data ($p < 0.05$ using a one-tailed paired two-sample t-test). Similarly, more students expected the entropy of the surroundings to increase rather than decrease or remain the same. This preferential response is also statistically significant ($p = 0.001$). In contrast, for the pre-instruction concrete-context question, we do not see the same preferential response regarding changes in the entropy of the object (17% increases, 19% decreases). However students do show a significant preference regarding the entropy of the air in the room ($p < 0.001$), with responses that entropy of the air would increase (27%) nearly triple those that stated entropy of the air would decrease (9%). At the outset of our study we expected students would disproportionately expect entropy to increase rather than decrease, calling to mind the often-heard phrase "entropy never decreases." Our findings have shown that while this may be true in a variety of circumstances, there are contexts in which that expectation does not hold up.

The matched-data sample from the Spring 2005 course show that responses *before* and *after* instruction are mostly, but not entirely, consistent with each other (see Table VI). In most cases students seem to have a preference for the "entropy increases" responses (compared to "decreases" or "remains the same") both before and after instruction. (However, this pattern does not hold for the object in the concrete context; see below.)

According to a test for binomial proportions[28] on the post-instruction "entropy of the system" general-context question (and considering only those students who made a directional choice), the "increases" response is more common than the "decreases" response ($p < 0.001$). Similarly, the post-instruction general-context response that entropy of the surroundings *increases* is more popular than the response that entropy of the surroundings *decreases* ($p < 0.01$), and a similar preference is expressed on the concrete-context question for the entropy of the air in the room ("increases" preferred over "decreases," $p < 0.0001$). Both before and after instruction, students show a statistically significant preference for stating that entropy of the system, the surroundings, and the air in the room would increase. However, for the case of the object in the concrete-context question, the matched sample shows no significant difference either before or after instruction between the proportions of "increases" and "decreases" responses. This is consistent with our finding from the larger three-semester pre-instruction data sample.

Interviews were conducted throughout our study; 18 were carried out after all instruction was complete in the spring of 2005. In this group, seven of the 18 students used some type of “entropy can never decrease” argument. This is particularly noteworthy because, prior to our study, we thought that students might be attracted to the general notion that “entropy increases” and over-apply it. And, in fact, this “entropy increases” answer *was* a popular response when dealing with “the system”; the seven students in this sub-sample did say that the system entropy must increase. However, their answers for entropy of the surroundings varied among “not determinable” (four), “remains the same” (two), and “increases” (one). At the same time, all seven students stated either that the entropy of the system plus that of the surroundings would increase, or that it would never decrease. Two typical responses are given here:

> “[S3] Entropy of the system will increase because it’s irreversible and you have to have an increase in entropy if it’s irreversible… second one [the entropy of the surroundings] I wasn’t sure of… entropy must either stay the same or increase…Because you can’t achieve order from disorder, but it can go the other way around.”

> “[S4] [For “surroundings”] I said remain equal or increase, and that depends on whether the heat is transferred to the system.”
>
> I: Could it decrease?

> “[S4] It [the surroundings] would always remain the same or increase. [Part c] remain[s] the same because the universe can’t possibly become more ordered… it’s one of the laws of thermodynamics.”

B. Student idea that entropy change depends on “size”

Approximately 15% of the explanations on part (d) of the concrete-context question (i.e., the “entropy of the universe”) included a claim that the entropy of the universe is “unaffected” by the process, or that the universe is “isolated” from the process. A small subsection of these students (roughly 5% of the total population) specifically argued that during this process, the entropy of the universe would be unaffected “because it’s too big.” Students’ explanations were not sufficiently clear or complete to allow us to determine the extent to which “remains the same” answers were based on the “unaffected because it’s too big” argument, or how many students might have perceived, mistakenly, that the term “universe” was intended to *exclude* the object, air, and room.

We developed a “metal in the ocean” question in an attempt to provide clearer evidence of student thinking on this issue. The problem describes a 1cm^3 block of hot metal being thrown into an ocean. (It was noted that the hot metal was initially at a higher temperature than the ocean.) The students are asked to consider the entropy change of the metal, the ocean, and the ocean plus the metal, after several hours have elapsed.

The question was first developed in spring 2006 and has only been used during the post-instruction interviews from that semester. Out of 20 students interviewed post-instruction in spring 2006 (all of whom had received research-based instruction in both tutorials and lectures), 100% correctly stated that the entropy of the metal will decrease during the process. Seventeen of the twenty stated that the entropy of the ocean would increase, and all but one of those 17 students correctly stated that the total entropy of metal plus ocean would increase. The one remaining student stated that the entropy of the metal would decrease and the entropy of the ocean would increase, but that the total would not be determinable because it could either increase or stay the same.

The most surprising finding was that three out of the 20 students claimed that although the metal would decrease in entropy, the entropy of the ocean would remain the same. Their explanation hinged on some type of ocean-size argument, and led to their conclusion that the total entropy of metal plus ocean would actually *decrease*. Excerpts from interviews with one of these three students are given below:

> "[S5] …entropy of the metal is going to decrease because it's losing heat, once it reaches equilibrium it will have lost entropy because it's also lost heat; the entropy of the surroundings I think means the ocean, then the ocean remains the same, it's a law or it's a frame of reference… a very small change in entropy into a very large

> surroundings isn't going to result in any measurable change in entropy in the surroundings because of the size difference between the two… It [the change in entropy of the metal cube plus the surroundings] would decrease because the entropy in the ocean is going to remain the same but the entropy of the very hot piece of metal will decrease drastically to come in equilibrium with the ocean…"
>
> "…In the object in the room the object was large enough to create a change in entropy in the room; then there would be enough to determine if it's the same. In *this* problem there wasn't a noticeable change in entropy of the ocean but there was in the metal."

Although our study did not assess the full extent of this error among our sample, we feel that this issue is suitable for future study.

C. Commentary

We documented specific student difficulties regarding the entropy changes in a spontaneous process. Both before and after instruction, most students failed to recognize the correct answers on questions regarding the change in entropy during a naturally-occurring process. These questions deal with entropy changes in a system and its surroundings, as well as the total entropy change of the system plus the surroundings. The most common responses suggest belief in a conservation principle that requires total entropy to remain the same. Among those students who assert a direction for entropy change even when none can be specified (e.g., stating that the surroundings' entropy increases, or stating that it decreases), a significantly higher proportion of students claim that entropy will increase rather than decrease for both system *and* surroundings. The exception to this occurs on a question involving the entropy change of an object placed in an otherwise empty (though air-filled) room; in this case students show no preference for believing in either increasing or decreasing entropy of the object. For the most part, however, we found that students' responses to questions posed both in a general context (using the terms "system" and "surroundings") and in a concrete context (referring to an object placed in an air-filled room) were very similar in the two different contexts.[29]

Among the other student ideas that we discussed in this section, one involves a belief that entropy change depends on system "size" in some poorly defined manner. Although this issue was only identified for one question, it is possible that this notion may be leading students along incorrect lines of reasoning on other questions as well.

VI. CURRICULUM DEVELOPMENT

A. Entropy spontaneous-process worksheet

Based on our finding that many students over-generalized the notion of conservation to questions regarding total entropy change during real processes, we sought to develop curricular materials that might help students address this difficulty. Our strategy was to guide students to consider a physical situation which would allow them to affirm their understanding of energy conservation[30] and, at the same time, would challenge the notion that entropy is conserved. It was also important to choose a system and process in which the outcome of entropy increase would be easy to deduce.

We developed a tutorial based on a set of two large, insulated metal blocks, connected by a thin, insulated metal rod of negligible heat capacity; we refer to this as our "Entropy Spontaneous-Process Worksheet" tutorial. The two blocks are initially at different temperatures, and students are asked to consider net changes in energy and entropy of the two blocks during the heat-transfer process. Dimensions of the blocks and rod are specified, and temperature changes of the blocks are shown to be so small as to be negligible during the time interval under consideration. The relationship $\boldsymbol{\Delta S} \equiv \int_{\textit{Initial State}}^{\textit{Final State}} \frac{\boldsymbol{\delta Q_{rev}}}{\boldsymbol{T}}$ simplifies, for the constant-temperature blocks (which act as thermal reservoirs), to $\Delta S = \frac{Q}{T}$, where Q is the heat transfer to the block and T is the temperature of that block. (Heat transfers to the thin rod are stated to be negligibly small.) An excerpt from the tutorial worksheet is shown in Appendix IX.

At the very beginning of the tutorial, students are asked questions concerning the change in entropy of the low-temperature block, and the net change in entropy of both blocks together. Students are asked whether there are any conserved quantities for this process, and whether energy and/or entropy are conserved. As our data show that most students tend to apply an inappropriate conservation argument to questions of this type, we wanted to elicit these difficulties at the beginning so that students could address and resolve them over the course of the tutorial.[31]

Students are asked to consider the magnitudes and signs of heat transfers to the two blocks; they are led to recognize that these heat transfers are equal in magnitude and opposite in sign, and that net energy change is zero. Students are then asked to consider the relative magnitudes and signs for the entropy changes of each block, as well as the net change in entropy. Students are guided to realize that the entropy increase of the cooler block is larger in magnitude than the entropy decrease of the warmer block, and so the *net* change in entropy is positive.

The tutorial continues by guiding students to explore relationships among the system, surroundings, and universe (that is, system *plus* surroundings). Our goal is to get students to realize that regardless of how the “system” and “surroundings” are defined—e.g., no matter which block is taken to be the "system" and which the "surroundings"—the total entropy of system *plus* surroundings will always increase during this process.

Students are led to consider the net entropy change in an imaginary process where heat transfer occurs spontaneously from the low temperature block *to* the high temperature block. Students come to recognize that, although this process would result in a net entropy decrease, it cannot actually occur since heat transfer is never observed to occur in the given direction. Finally, students are asked to consider a limiting case for entropy change as the temperatures of the two cubes approach each other arbitrarily closely. Students are guided to realize that in this situation, net entropy change becomes infinitesimally small even when the net amount of heat transfer is the same as it was in the previous examples; this is stated to be an approximation to an ideal "reversible" process.

B. Results

In the spring of 2006 we administered our Entropy Spontaneous-Process Worksheet tutorial (see Appendix IX) to all students ($N \approx 200$) who attended recitation during the week in which entropy was covered in class. Post-instruction testing took place on the mid-term exam that covered all thermodynamics topics (using multiple-choice questions), and also during one week of laboratories conducted two weeks after the mid-term was complete (using free-response questions). As seen in Tables VII (general-context question) and VIII (concrete-context question), student performance gains (pretest to post-test) on both questions are much better in the Spring 2006 course, on each sub-part, when compared to the matched sample in the Spring 2005 course. (For a complete breakdown of the matched sample data see Appendix VI.) The most substantial gains are in correct answers for part *c* of both questions, that is, the question about the "system + surroundings" (68% correct post- vs. 21% correct pre-instruction) and the corresponding question about the "object + air in the room" (69% correct post- vs. 16% pre-instruction). There was also a dramatic improvement in the proportion of students answering all three parts *a*, *b*, and *c* correctly (55% and 53%, respectively, on the general- and concrete-context questions, post-instruction, compared to only 6% pre-instruction). Moreover, we found that after tutorial instruction, in comparison to before this instruction, a much higher proportion of students who were able to answer *a*, *b*, and *c* correctly on the concrete-context question also got "all-correct" answers on the general-context question: This "overlap" proportion rose from 45% to 90%, (pre- to post-instruction), thus indicating far greater consistency in correct-answer responses after use of the tutorials.[32] Previously, in 2005, no shift in this overlap proportion was observed even after instruction had taken place. At the same time, the overlap proportion for the incorrect "total entropy remains the same" responses in 2006 decreased significantly from 83% to 69%, suggesting that even those students who gave incorrect responses did not do so as consistently as before.

A third version of the Spontaneous Process multiple-choice question was designed that was virtually identical to Version *B* (which had been used in the Spring 2005 course). The proportion of correct responses (61%) on this question was significantly higher after instruction with the Spontaneous-Process Worksheet than it had been without use of that worksheet in 2004 (30% correct) and 2005 (27% correct).

It is important to note that there were other substantial changes made in the content of instruction during the Spring 2006 course, in comparison to the Spring 2005 course. The same instructor taught both courses and the form of instruction was consistent, but the instructor drastically modified the approach used in his lectures on entropy. In these lectures, he modeled some of the same steps that were used in the worksheet tutorial, and incorporated a number of related questions which he posed to the class using electronic "clickers."[33]

We also administered our Entropy Spontaneous-Process Worksheet in a sophomore-level physics course at the University of Washington (UW). Before instruction the UW students performed at a level similar to that of the ISU students, although a higher proportion of UW students gave "all-correct" answers (i.e., correct on parts *a*, *b*, and *c*) on the general- and concrete-context questions (13% and 19%, respectively, $N = 32$) than did ISU students in the two matched samples (≈ 6%). We found that the students' post-instruction performance was significantly better than that of students in the Spring 2005 ISU course, with a high proportion of students giving all-correct (*a*, *b*, and *c*) answers for the general-context (63%) and concrete-context (69%) questions. These high post-instruction proportions are consistent with post-instruction performance in the Spring 2006 course at ISU. (For detailed data, see Appendices VII & VIII.)

C. Student performance related to "universe = system + surroundings" concept

We attempted to assess students' thinking regarding the commonly used terminology in which an arbitrarily defined system and that system's surroundings are together taken to define "the universe." Our concrete-context question shed light on this by asking for the change in entropy inside the insulated room as well as the change in entropy of the universe. The question does not explicitly ask about a "system" or "surroundings," but students had received instruction through the Entropy Spontaneous-Process Worksheet on these concepts and therefore might be expected to give consistent answers. The proportion of responses for each possible answer of the "entropy of the object + air in the room" question and the "entropy of the universe" question are statistically equivalent both before and after instruction in the Spring 2005 course (in which the instruction relied on our tutorial-style worksheet that addressed the state-function property of entropy).

However, after instruction with the Entropy Spontaneous-Process Worksheet, student responses to the "entropy of the object + air in the room" question and "entropy of the universe" question were statistically different (see Appendix X). The proportion of students who claimed incorrectly that the entropy of the universe would stay the same (53%) was far higher than those who gave the same answer on part *c* of the concrete-context question (24%). Student explanations that justified the "entropy of the universe remains the same" response often described the universe as being isolated from the room, which was contrary to the meaning employed in the worksheet. Despite the substantial improvement in overall understanding as measured by our free-response and multiple-choice questions (see Tables VII and VIII and discussion in Section IVB above), it seems that use of our Entropy Spontaneous-Process Worksheet actually increased student difficulties in interpreting consistently the meaning of "universe" in the context used here. This is evidently something that will need to be addressed in future versions of this tutorial worksheet.

VII. CONCLUSION

We conducted an extensive analysis of student thinking regarding certain aspects of the principle of increasing entropy including, in particular, those that relate to the meaning of "system" and "surroundings." Analysis of data from four semesters of classes demonstrated that before instruction, students have well-defined and consistent lines of thinking and reasoning. These lines include the popular notion that total entropy remains unchanged during a real process, implicitly based on an assumption that entropy is a conserved quantity. These ideas can lead to difficulties in understanding the role of entropy in the second law of thermodynamics.

Before instruction fewer than 10% of the students were able to correctly respond to questions on entropy changes, and there was very little dependence on whether these questions were posed in a general or in a concrete context.[29] Nearly two thirds of all students showed evidence of conservation-type reasoning regarding entropy.

When students were not given information about a specific process, most responded correctly that the sign of the entropy change would not be determinable in that case. However, about one third of students did claim, despite the lack of required information, that the entropy change would have a specific sign; most of this group asserted that entropy would increase. (There was a concrete context in which this preference was not consistently observed, specifically, in the case of an object in a room containing air.) It appeared as if many students were attempting to reconcile simultaneously two popular ideas, (1) the common perception that "entropy always increases," and (2), a belief that *total* entropy must be a conserved quantity.

Results from matched samples of students assessed through pre- and post-instruction testing showed that some of these student difficulties can persist despite instructor awareness of the difficulties and deliberate attempts to overcome them. We subsequently developed a research-based tutorial worksheet that explicitly addressed some of these student difficulties. Early indications are that instruction using this worksheet is effective in improving students' performance on questions regarding the principle of entropy increase in spontaneous processes, at least in processes that involve heat transfer.[34]

Acknowledgments

We are very grateful for the assistance and input of Howard Shapiro, Thomas Greenbowe, and Steve Kawaler. We also thank the Physics 222 recitation instructors for their effort, and are very appreciative of the assistance given by Thomas Stroman in analyzing portions of the data. We are very grateful to Donald Mountcastle for a careful and thorough reading of the manuscript and for his many valuable suggestions. We are also grateful for valuable feedback from our collaborators John Thompson and Brandon Bucy. This material is based upon work supported by the National Science Foundation under PHYS #0406724 and PHYS #0604703.

Table I. Pre-instruction correct responses as a proportion of all responses on general- and concrete-context questions. Figures shown are mean values and 95% confidence intervals (i.e., "±" values), based on score variances among the four samples for the general-context question and three samples for the concrete-context question; see detailed data in Appendices I and II.

	Pre-Instruction, General Context, Cumulative Results		Pre-Instruction, Concrete Context, Cumulative Results
	N = 1184 (four samples)		*N* = 609 (three samples)
a. Entropy change of system is not determinable	42 ± 10%	*a*. Entropy change of object is not determinable	50 ± 11%
b. Entropy change of surroundings is not determinable	42 ± 6%	*b*. Entropy change of air in the room is not determinable	49 ± 3%
c. Entropy of the system + surroundings increases	19 ± 5%	*c*. Entropy of the object + air in the room increases	14 ± 9%
		d. Entropy of the universe increases	15 ± 18%
All correct (parts *a*-*c*)	**4 ± 1%**	**First three parts correct (parts *a*-*c*)**	**5 ± 3%**

Table II. Responses related to total (net) entropy change on the general- and concrete-context questions (see Figs. 1 and 2), before any instruction on entropy. Figures shown are mean values as a proportion of all responses, and 95% confidence intervals based on score variances among the four samples for the general-context question and three samples for the concrete-context question. (See detailed data in Appendices III and IV.) Figures in the first row (*"Total entropy…remains the same"*) correspond to students who answered "remain the same" to part (c) of each question, respectively; "system + surroundings" refers to the general-context question, while "object + air in the room" refers to the concrete-context question. Figures in the second row (*"A. Entropy change of (system* and *surroundings)…"*) correspond to students who responded "not determinable" to parts (a) and (b), but "remain the same" to part (c) of each question, respectively. Figures in the third row (*"B. Entropy of (system/object) increases [decreases]…"*) correspond to students who answered *either* "increase" *or* "decrease" to part (a), but gave the opposite answer (i.e., "decrease" or "increase") to part (b), and who *also* answered "remain the same" to part (c) of each question, respectively. Figures in the last (fourth) row correspond to students who fell into either category *A or* category *B*.

[Table II]

	Pre-Instruction, General Context, Cumulative	Pre-Instruction, Concrete Context, Cumulative
	N = 1184 (four samples)	*N* = 609 (three samples)
Total entropy *[of (system + surroundings)/(object + air in the room)]* remains the same	**67 ± 8%**	**71 ± 7%**
A. Entropy change of (system *and* surroundings)/(object *and* air) not determinable, but total entropy remains the same	26 ± 12%	38 ± 8%
B. Entropy of (system/object) increases [decreases] and entropy of (surroundings/air) decreases [increases], but total entropy remains the same	25 ± 10%	22 ± 6%
Students with one of these notions of entropy conservation (sum of A and B above)	*51 ± 7%*	*60 ± 13%*

Table III. Pre- and post-instruction responses related to overall entropy change as a proportion of all responses, given by students on the general-context (Fig. 1) and concrete-context (Fig. 2) questions in Spring 2005. The same group of students (the “matched sample”) responded to the questions both pre-instruction and post-instruction. (See detailed data in Appendix V.) There were no significant differences between general- and concrete-context question responses pre-instruction, but such differences did appear post-instruction as indicated by the “†” symbol. (*P*-values are calculated using the binomial proportions test.) There were no significant differences between pre- and post-instruction responses on the concrete-context question, although such differences did occur on the general-context question.

[Table III]

	Pre-instruction, General Context	Post-instruction, General Context	Pre-instruction, Concrete Context	Post-instruction, Concrete Context
	Spring 2005 Matched Sample $N = 127$			
Total entropy *[of (system + surroundings)/(object + air in the room)]* remains the same	**61%**§	**48%**§†	**69%**	**71%**†
A. Entropy change of (system *and* surroundings)/(object *and* air) not determinable, but total entropy remains the same	34%*	16%*†	39%	36%†
B. Entropy of (system/object) increases [decreases] and entropy of (surroundings/air) decreases [increases], but total entropy remains the same	19%	24%	21%	23%
Students with one of these notions of entropy conservation (sum of A and B above)	53%§	39%§†	60%	59%†

* Significant difference ($p < 0.01$) between *pre*- and *post*-instruction responses on the general-context question.

§ Significant difference ($p < 0.05$) between *pre*- and *post*-instruction responses on the general-context question.

† Significant difference ($p \leq 0.001$) between *concrete*-context and *general*-context responses on post-instruction questions.

Table IV. Post-instruction responses on versions *A* and *B* of the spontaneous-process question. (Only Version *B* contains the option of total entropy either increasing *or* remaining the same.) Response descriptions in column one are characterizations of the numerical response options in the original question.

	Fall 2004 Post-Instruction (Version *A*)	Spring 2005 Post-Instruction (Version *B*)
	N = 539	*N* = 341
A. Total entropy remains the same	54%	36%
B. Total entropy increases and system entropy increases	5%	12%
C. Total entropy decreases and system entropy increases	7%	2%
Answers B & C†	4%	--
Total entropy increases and system entropy can increase *or* decrease *[correct]*	30%	27%
Total entropy increases *or* remains the same*	--	23%

†Version *A* only.

*Version *B* only.

Table V. Pre-instruction responses related to “system” and “surroundings,” as a proportion of all responses. Uncertainties reflect the 95% confidence interval based on response rates and standard deviations observed in four different courses for the general-context question, and three different courses for the concrete-context question. (See Appendices I and II for detailed data tables.)

	Pre-Instruction, General Context, Cumulative Results	Pre-Instruction, Concrete Context, Cumulative Results
	N = 1184 (four samples)	*N* = 609 (three samples)
Entropy of…	*System…*	*Object…*
increases	26 ± 3%	17 ± 2%
decreases	19 ± 4%	19 ± 3%
remains the same	10 ± 4%	6 ± 7%
is not determinable [correct]	42 ± 6%	50 ± 5%
Entropy of…	*Surroundings…*	*Air in room…*
increases	28 ± 2%	27 ± 2%
decreases	14 ± 2%	9 ± 1%
remains the same	11 ± 1%	6 ± 3%
is not determinable [correct]	42 ± 4%	49 ± 1%

Table VI. Pre- and post-instruction responses related to "system" and "surroundings," general- and concrete-context questions, Spring 2005. The same group of students (the "matched sample") responded both pre-instruction and post-instruction. (See detailed data in Appendix V.)

	Pre-Instruction, General Context	Post-Instruction, General Context	Pre-Instruction, Concrete Context	Post-Instruction, Concrete Context
	Spring 2005 Matched Sample $N = 127$			
Entropy of…	System…		*Object…*	
increases	28%	35%†	20%	17%†
decreases	14%	20%	17%	23%
remains the same	3%*	9%*†	2%	3%†
is not determinable [correct]	51%*	35%*†	55%	57%†
Entropy of…	*Surroundings…*		*Air in room…*	
increases	29%	31%	25%	29%
decreases	10%	17%†	10%	6%†
remains the same	8%	10%	6%	7%
is not determinable [correct]	47%	39%†	51%	57%†

*Significant difference ($p < 0.05$) between *pre-* and *post-*instruction rates on general-context question, according to binomial proportions test.

† Significant difference ($p < 0.05$) between *concrete*-context and *general*-context rates on post-instruction questions, according to binomial proportions test.

Table VII. Correct responses as a proportion of all responses on the general-context question, matched samples, Spring 2005 and Spring 2006. The Spring 2005 class used the Entropy State-Function Worksheet, while the Spring 2006 class used the Entropy Spontaneous-Process Worksheet.

	Pre-instruction	Post-instruction with Entropy State-Function Worksheet	Pre-instruction	Post-instruction with Entropy Spontaneous-Process Worksheet
	Spring 2005 *N* = 127		Spring 2006 *N* = 191	
a. Entropy change of system not determinable	51%	35%*	42%	74%*
b. Entropy change of surroundings not determinable	47%	39%*	42%	75%*
c. Entropy of (system + surroundings) increases	25%	36%*	21%	68%*
All Correct (*a-c*)	**5%**	**8%***	**6%**	**55%***

*Statistically significant difference on post-instruction responses, $p < 0.0001$ using a test for binomial proportions.

Table VIII. Correct responses as a proportion of all responses on the concrete-context question, matched samples, Spring 2005 and Spring 2006. The Spring 2005 class used the Entropy State-Function Worksheet, while the Spring 2006 class used the Entropy Spontaneous-Process Worksheet

[Table VIII]

	Pre-instruction	Post-instruction with Entropy State-Function Worksheet	Pre-instruction	Post-instruction with Entropy Spontaneous-Process Worksheet
	Spring 2005 *N* = 127		Spring 2006 *N* = 191	
a. Entropy change of object not determinable	55%	57%*	53%	73%*
b. Entropy change of air in the room not determinable	51%	57%*	52%	73%*
c. Entropy of (object + air in the room) increases	20%	23%†	16%	69%†
d. Entropy of universe increases	26%	26%*	15%	44%*
***a*, *b*, and *c* correct**	**7%**	**13%†**	**6%**	**53%†**

* Statistically significant difference on post-instruction responses, $p < 0.001$, using a test for binomial proportions.

† Statistically significant difference on post-instruction responses, $p < 0.0001$ using a test for binomial proportions.

Figure 1. General-context question.

For each of the following questions consider a system undergoing a naturally occurring (spontaneous) process. The system can exchange energy with its surroundings.

a) During this process, does the entropy of the **system** [S_{system}] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

b) During this process, does the entropy of the **surroundings** [$S_{surroundings}$] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

c) During this process, does the entropy of the system *plus* the entropy of the surroundings [$S_{system} + S_{surroundings}$] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

Figure 2. Concrete-context question.

An object is placed in a thermally insulated room that contains air. The object and the air in the room are initially at different temperatures. The object and the air in the room are allowed to exchange energy with each other, but the air in the room does not exchange energy with the rest of the world or with the insulating walls.

a) During this process, does the entropy of the **object** [S_{object}] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

b) During this process, does the entropy of the **air in the room** [S_{air}] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

c) During this process, does the entropy of the object *plus* the entropy of the air in the room [$S_{\text{object}} + S_{\text{air}}$] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

d) During this process, does the entropy of the **universe** [S_{universe}] *increase*, *decrease*, *remain the same*, or is this *not determinable* with the given information? ***Explain your answer.***

Figure 3. Spontaneous process question version *A*.

A subsystem *A* is in thermal contact with its environment *B*, which together comprise an isolated system. Consider the following situations:

I. Entropy of system increases by 5 J/K; entropy of the environment decreases by 5 J/K.
II. Entropy of system increases by 5 J/K; entropy of the environment decreases by 3 J/K.
III. Entropy of system increases by 3 J/K; entropy of the environment decreases by 5 J/K.
IV. Entropy of system decreases by 3 J/K; entropy of the environment increases by 5 J/K.

Which of the above four situations can actually occur in the real world?

A. I only
B. II only
C. III only
D. II and III only
E. II and IV only **[*correct*]**

Figure 4. Spontaneous-process question version *B*.

A subsystem *A* is in thermal contact with its environment *B* and they together comprise an isolated system that is undergoing an irreversible process. Consider the following situations:

I. Entropy of system increases by 5 J/K; entropy of the environment decreases by 5 J/K.
II. Entropy of system increases by 5 J/K; entropy of the environment decreases by 3 J/K.
III. Entropy of system increases by 3 J/K; entropy of the environment decreases by 5 J/K.
IV. Entropy of system decreases by 3 J/K; entropy of the environment increases by 5 J/K.

Which of the above four situations can actually occur?

A. I only
B. II only
C. III only
D. II and IV only **[*correct*]**
E. I, II, and IV only

Appendix I. Itemized response data, general-context question, pre-instruction, all semesters

	Fall 2004 Pre-Instruction	Spring 2005 Pre-Instruction	Fall 2005 Pre-Instruction	Spring 2006 Pre-Instruction	Pre-Instruction General Context Cumulative Results
	N = 406	*N* = 171	*N* = 360	*N* = 247	*N* = 1184
Entropy of system…					
increases	30%	26%	24%	24%	26 ± 4%
decreases	19%	14%	25%	18%	19 ± 7%
remains the same	9%	5%	13%	13%	10 ± 6%
is not determinable [*correct*]	39%	50%	35%	43%	42 ± 10%
Entropy of surroundings…					
increases	26%	26%	31%	28%	28 ± 4%
decreases	16%	11%	14%	14%	14 ± 4%
remains the same	12%	9%	11%	11%	11 ± 2%
is not determinable [*correct*]	42%	47%	38%	42%	42 ± 6%
Entropy of system + surroundings…					
increases [*correct*]	19%	23%	16%	19%	19 ± 5%
decreases	2%	1%	3%	2%	2 ± 1%
remains the same	67%	60%	69%	71%	67 ± 8%
is not determinable	8%	12%	7%	4%	8 ± 5%
All Correct	**5%**	**4%**	**4%**	**5%**	**4 ± 1%**

Appendix II. Itemized response data, concrete-context question, pre-instruction, all semesters

	Spring 2005 Pre-Instruction Concrete Context	Fall 2005 Pre-Instruction Concrete Context	Spring 2006 Pre-Instruction Concrete Context	Pre-Instruction Concrete Context Cumulative Results
	N = 155	*N* = 207	*N* = 237	*N* = 609
a. Entropy of object…				
increases	19%	15%	17%	17 ± 5%
decreases	16%	20%	21%	19 ± 6%
remains the same	3%	14%	3%	6 ± 16%
is not determinable [*correct*]	54%	45%	52%	50 ± 11%
b. Entropy of air in the room…				
increases	25%	27%	28%	27 ± 3%
decreases	8%	10%	10%	9 ± 2%
remains the same	7%	9%	3%	6 ± 8%
is not determinable [*correct*]	48%	48%	50%	49 ± 3%
c. Entropy of object + air in the room…				
increases [*correct*]	17%	11%	15%	14 ± 9%
decreases	1%	5%	4%	3 ± 6%
remains the same	68%	71%	74%	71 ± 7%
is not determinable	5%	4%	3%	4 ± 2%
d. Entropy of universe…				
increases [*correct*]	23%	9%	14%	15 ± 18%
decreases	1%	2%	0%	1 ± 2%
remains the same	61%	73%	72%	69 ± 17%
is not determinable	6%	7%	8%	7 ± 3%
***a*, *b*, and *c* correct**	**6%**	**3%**	**5%**	**5 ± 3%**

Appendix III. Responses related to overall entropy change, general-context question, pre-instruction, all semesters

	Fall 2004 Pre-Instruction	Spring 2005 Pre-Instruction	Fall 2005 Pre-Instruction	Spring 2006 Pre-Instruction	Pre-Instruction General Context Cumulative Results
	N = 406	*N* = 171	*N* = 360	*N* = 247	*N* = 1184
Total entropy *[of (system + surroundings / (object + air in the room)]* remains the same	67%	60%	69%	71%	67 ± 8%
A. Entropy of (system *and* surroundings)/ (object *and* air) not determinable, but total entropy remains the same	27%	33%	16%	29%	26 ± 12%
B. Entropy of (system/object) increases [decreases] and entropy of (surroundings/air) decreases [increases], but total entropy remains the same	30%	16%	31%	25%	25 ± 10%
C. Students with one of these notions of entropy conservation (sum of A and B above)	57%	49%	46%	53%	51 ± 7%

Appendix IV. Responses related to overall entropy change, concrete-context question, pre-instruction, all semesters

	Spring 2005 Pre-Instruction	Fall 2005 Pre-Instruction	Spring 2006 Pre-Instruction	Pre-Instruction Concrete Context Cumulative Results
	N = 155	*N* = 207	*N* = 237	*N* = 609
Total entropy *[of (system + surroundings)/ (object + air in the room)]* remains the same	68%	71%	74%	71 ± 7%
A. Entropy of (system *and* surroundings)/ (object *and* air) not determinable, but total entropy remains the same	37%	35%	41%	38 ± 8%
B. Entropy of (system/object) increases [decreases] and entropy of (surroundings/air) decreases [increases], but total entropy remains the same	19%	22%	24%	22 ± 6%
C. Students with one of these notions of entropy conservation (sum of A and B above)	56%	57%	65%	60 ± 13%

Appendix V. Itemized response data, general- and concrete-context questions, pre- and post-instruction, spring 2005 matched sample

Spring 2005 Matched Sample *N* = 127	Pre-Instruction General Context	Post-Instruction General Context	Pre-Instruction Concrete Context	Post-Instruction Concrete Context
a. Entropy of…	*System…*		*Object…*	
increases	28%	35%	20%	17%
decreases	14%	20%	17%	23%
remains the same	3%	9%	2%	3%
is not determinable [*correct*]	51%	35%	56%	57%
b. Entropy of…	*Surroundings…*		*Air in the Room…*	
increases	29%	31%	25%	29%
decreases	10%	17%	10%	6%
remains the same	8%	10%	6%	7%
is not determinable [*correct*]	47%	39%	51%	57%
c. Entropy of…	*System + Surroundings…*		*Object + Air in the Room…*	
increases [*correct*]	25%	36%	20%	23%
decreases	1%	3%	0%	0%
remains the same	61%	48%	69%	71%
not determinable	10%	12%	5%	6%
d. Entropy of…	--		*Universe…*	
increases [*correct*]	--	--	26%	27%
decreases	--	--	1%	0%
remains the same	--	--	62%	65%
is not determinable	--	--	5%	5%
***a*, *b*, and *c* correct**	**5%**	**8%**	**7%**	**13%**

Appendix VI. Itemized response data, general- and concrete-context questions, pre- and post-instruction, spring 2006 matched sample

Spring 2006 Matched Sample *N* = 191	Pre-Instruction General Context	Post-Instruction General Context	Pre-Instruction Concrete Context	Post-Instruction Concrete Context
a. Entropy of…	*System…*		*Object…*	
increases	25%	13%	16%	15%
decreases	18%	12%	23%	12%
remains the same	14%	2%	3%	1%
is not determinable [*correct*]	42%	74%	53%	73%
b. Entropy of…	*Surroundings…*		*Air in the Room…*	
increases	28%	17%	29%	16%
decreases	14%	6%	8%	8%
remains the same	12%	3%	4%	2%
is not determinable [*correct*]	42%	75%	52%	73%
c. Entropy of…	*System + Surroundings…*		*Object + Air in the Room…*	
increases [*correct*]	21%	68%	16%	69%
decreases	2%	2%	4%	2%
remains the same	71%	21%	73%	24%
is not determinable	4%	8%	3%	5%
d. Entropy of…	--		*Universe…*	
increases [*correct*]	--	--	15%	44%
decreases	--	--	1%	1%
remains the same	--	--	73%	54%
is not determinable	--	--	8%	1%
***a*, *b*, and *c* correct**	**6%**	**55%**	**6%**	**53%**

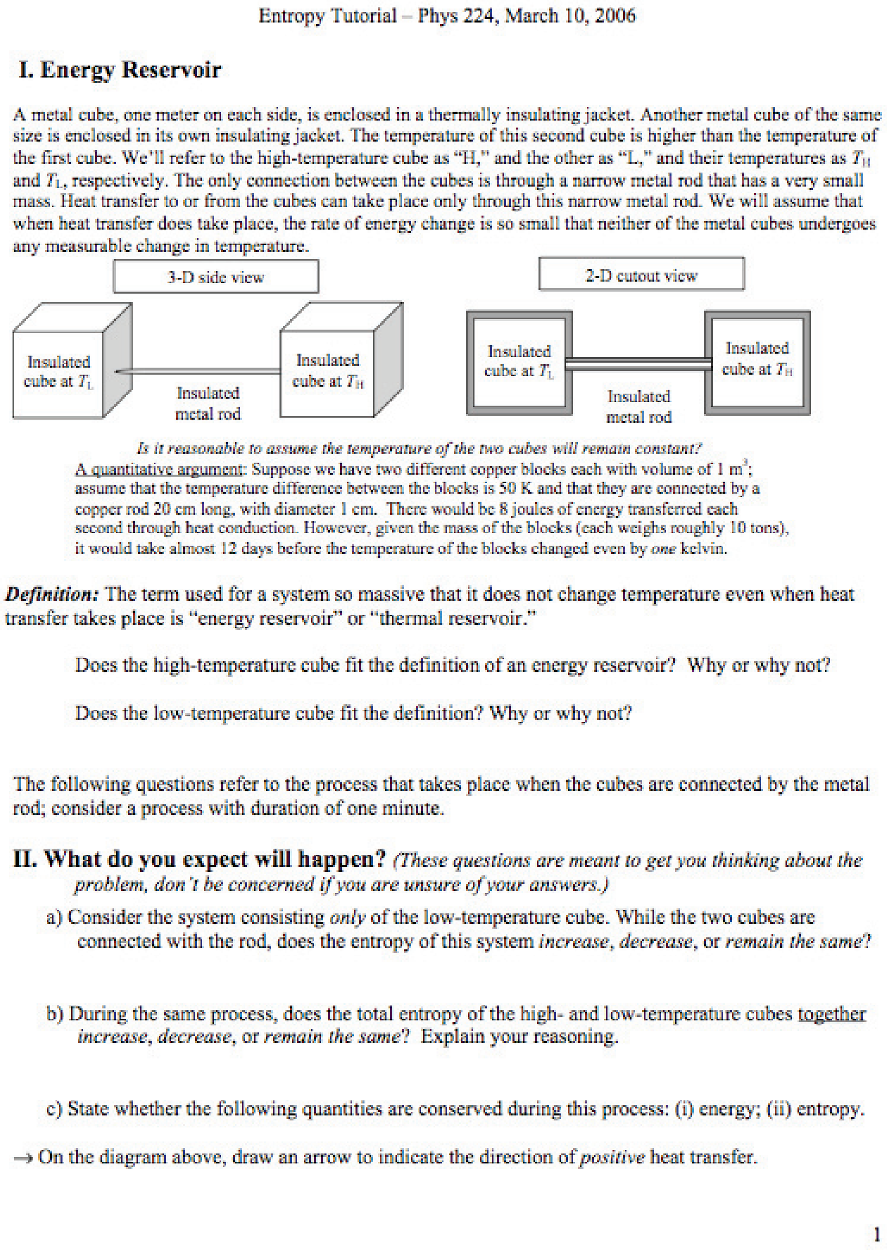
Appendix VII. Correct responses, pre- and post-instruction, general-context question, ISU 2005 and UW 2006

	Iowa State University Introductory Course		University of Washington Sophomore Course	
	Pre-Instruction	Post-Instruction with Entropy State-Function Worksheet	Pre-Instruction	Post-Instruction with Entropy Spontaneous-Process Worksheet
	Spring 2005 Matched Sample *N* = 127		Winter 2007 Matched Sample *N* = 32	
a. Entropy change of system not determinable	51%	35%	50%	84%
b. Entropy change of surroundings not determinable	47%	39%	53%	84%
c. Entropy of system + surroundings increases	25%	36%	34%	72%
All Correct	5%	8%	13%	63%

Appendix VIII. Correct responses, pre- and post-instruction, concrete-context question, ISU 2005 and UW 2006

	Iowa State University Introductory Course		University of Washington Sophomore Course	
	Pre-Instruction	Post-Instruction with Entropy State-Function Worksheet	Pre-Instruction	Post-Instruction with Entropy Spontaneous-Process Worksheet
	Spring 2005 Matched Sample *N* = 127		Winter 2007 Matched Sample *N* = 32	
a. Entropy change of object not determinable	55%	57%	47%	88%
b. Entropy change of air in the room not determinable	51%	57%	47%	88%
c. Entropy of object + air in the room increases	20%	23%	34%	78%
a, *b*, and *c* correct	7%	13%	19%	69%

Appendix IX. Entropy spontaneous-process tutorial, pp. 1-2



Entropy Tutorial – Phys 224, March 10, 2006

I. Energy Reservoir

A metal cube, one meter on each side, is enclosed in a thermally insulating jacket. Another metal cube of the same size is enclosed in its own insulating jacket. The temperature of this second cube is higher than the temperature of the first cube. We'll refer to the high-temperature cube as "H," and the other as "L," and their temperatures as $T_{\rm H}$ and $T_{\rm L}$, respectively. The only connection between the cubes is through a narrow metal rod that has a very small mass. Heat transfer to or from the cubes can take place only through this narrow metal rod. We will assume that when heat transfer does take place, the rate of energy change is so small that neither of the metal cubes undergoes any measurable change in temperature.

Is it reasonable to assume the temperature of the two cubes will remain constant?

A quantitative argument: Suppose we have two different copper blocks each with volume of 1 m^3; assume that the temperature difference between the blocks is 50 K and that they are connected by a copper rod 20 cm long, with diameter 1 cm. There would be 8 joules of energy transferred each second through heat conduction. However, given the mass of the blocks (each weighs roughly 10 tons), it would take almost 12 days before the temperature of the blocks changed even by *one* kelvin.

Definition: The term used for a system so massive that it does not change temperature even when heat transfer takes place is "energy reservoir" or "thermal reservoir."

Does the high-temperature cube fit the definition of an energy reservoir? Why or why not?

Does the low-temperature cube fit the definition? Why or why not?

The following questions refer to the process that takes place when the cubes are connected by the metal rod; consider a process with duration of one minute.

II. What do you expect will happen? *(These questions are meant to get you thinking about the problem, don't be concerned if you are unsure of your answers.)*

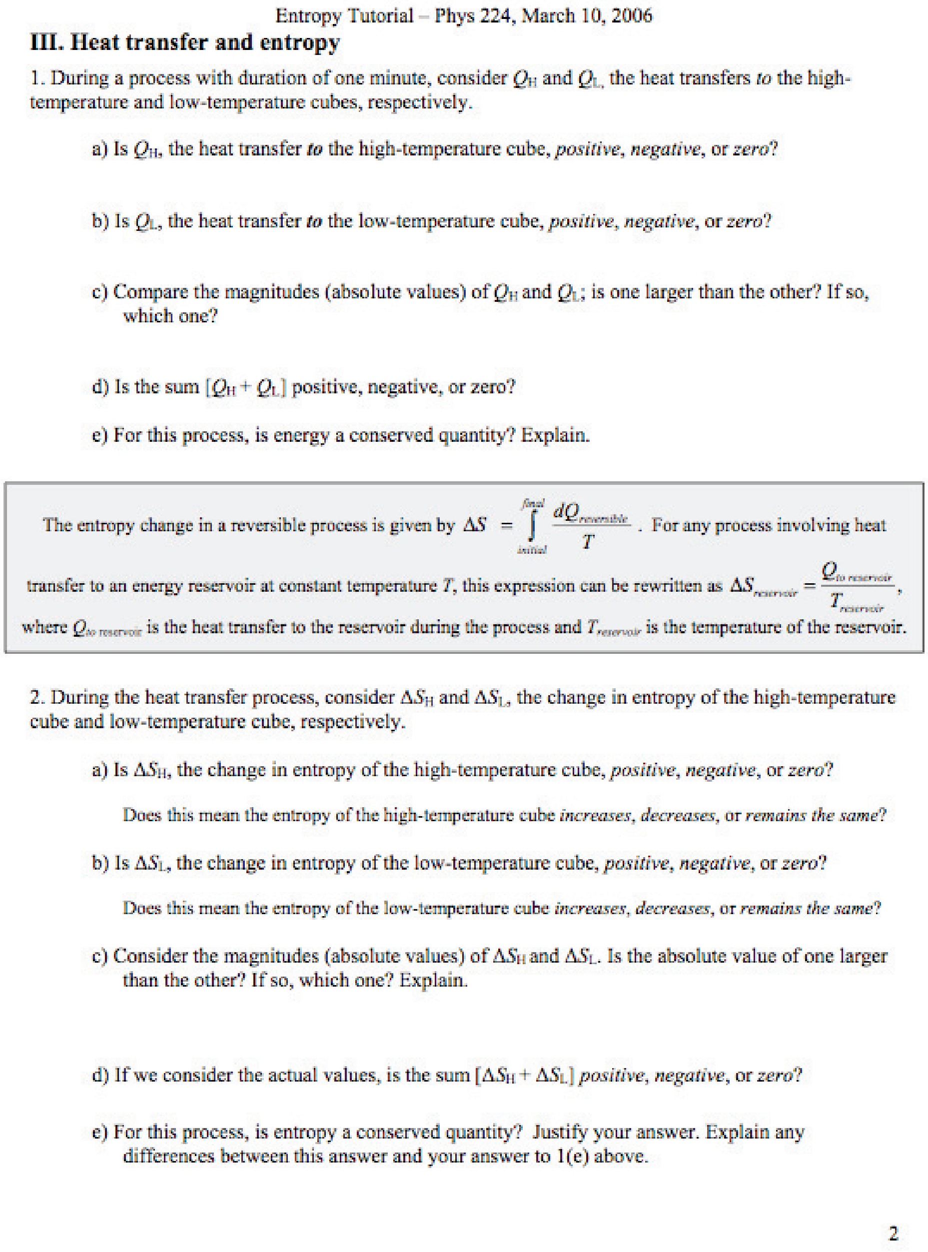
a) Consider the system consisting *only* of the low-temperature cube. While the two cubes are connected with the rod, does the entropy of this system *increase*, *decrease*, or *remain the same*?

b) During the same process, does the total entropy of the high- and low-temperature cubes together *increase*, *decrease*, or *remain the same*? Explain your reasoning.

c) State whether the following quantities are conserved during this process: (i) energy; (ii) entropy.

→ On the diagram above, draw an arrow to indicate the direction of *positive* heat transfer.

1



III. Heat transfer and entropy

1. During a process with duration of one minute, consider Q_H and Q_L, the heat transfers *to* the high-temperature and low-temperature cubes, respectively.

a) Is Q_H, the heat transfer ***to*** the high-temperature cube, ***positive***, ***negative***, or ***zero***?

b) Is Q_L, the heat transfer ***to*** the low-temperature cube, ***positive***, ***negative***, or ***zero***?

c) Compare the magnitudes (absolute values) of Q_H and Q_L; is one larger than the other? If so, which one?

d) Is the sum $[Q_H + Q_L]$ positive, negative, or zero?

e) For this process, is energy a conserved quantity? Explain.

> The entropy change in a reversible process is given by $\Delta S = \int_{initial}^{final} \frac{dQ_{reversible}}{T}$. For any process involving heat transfer to an energy reservoir at constant temperature T, this expression can be rewritten as $\Delta S_{reservoir} = \frac{Q_{to\ reservoir}}{T_{reservoir}}$, where $Q_{to\ reservoir}$ is the heat transfer to the reservoir during the process and $T_{reservoir}$ is the temperature of the reservoir.

2. During the heat transfer process, consider ΔS_H and ΔS_L, the change in entropy of the high-temperature cube and low-temperature cube, respectively.

a) Is ΔS_H, the change in entropy of the high-temperature cube, ***positive***, ***negative***, or ***zero***?

Does this mean the entropy of the high-temperature cube ***increases***, ***decreases***, or ***remains the same***?

b) Is ΔS_L, the change in entropy of the low-temperature cube, ***positive***, ***negative***, or ***zero***?

Does this mean the entropy of the low-temperature cube ***increases***, ***decreases***, or ***remains the same***?

c) Consider the magnitudes (absolute values) of ΔS_H and ΔS_L. Is the absolute value of one larger than the other? If so, which one? Explain.

d) If we consider the actual values, is the sum $[\Delta S_H + \Delta S_L]$ ***positive***, ***negative***, or ***zero***?

e) For this process, is entropy a conserved quantity? Justify your answer. Explain any differences between this answer and your answer to 1(e) above.

Appendix X. Pre- and post-instruction, concrete-context question, entropy of “object + air in the room” vs. entropy of the “universe” responses, spring 2005 and 2006

	Pre-Instruction Spring 2005 Matched Sample		Post-Instruction with Entropy State-Function Worksheet Spring 2005 Matched Sample		Pre-Instruction Spring 2006		Post-Instruction with Entropy Spontaneous-Process Worksheet Spring 2006	
	N = 131		*N* = 131		*N* = 223		*N* = 231	
	Entropy of the object + air	Entropy of Universe	Entropy of the object + air	Entropy of Universe	Entropy of the object + air	Entropy of Universe	Entropy of the object + air	Entropy of Universe
Increases	20%	27%	23%	26%	15%	14%	68%*,	44%*, †
Remains the same	69%	62%	70%	66%	74%	72%	24%*	53%*†

* Statistically significant difference compared to *pre*-instruction response on same item ($p < 0.0001$)

† Statistically significant difference compared to “object + air” response on same question ($p < 10^{-6}$)

References

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[18] For example, N.J. Tro, *Chemistry, A molecular Approach* (Prentice Hall, New Jersey, 2008), p. 783-785.

[19] In general, we use the terms “natural,” “spontaneous,” “real,” and “naturally occurring” synonymously when modifying the word “process.”

[20] For practical purposes and to facilitate analysis, one ordinarily specifies a particular system along with a separate region or reservoir referred to as the “surroundings”; however, both this system and the specified surroundings are then isolated from the rest of the universe. See discussion of the “concrete-context” question in Section IIC.

[21] D. Giancoli, *Physics for Scientists and Engineers* (Upper Saddle River, New Jersey, 2000), 3rd ed., p. 539; D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics* (John Wiley and Sons, Inc., New York, 2001), 6th ed., p. 500; R.D. Knight, *Physics for Scientists and Engineers* (Addison Wesley, San Francisco, 2004), p. 566; R.A. Serway, *Physics for Scientists and Engineers* (Saunders College Publishing, Philadelphia, 1996), 4th ed., p. 629-632.

[22] The instructor for the course in which this question was first used employed the terminology “isolated system,” so the question was written to include that language in Version *A*. Student understanding of the term “isolated system” was not a focus of our research. We also found that the distracters in Version *A* may not fully represent student thinking; this was the motivation for creating Version *B*.

[23] It is conceivable that some students may simply confuse the word “entropy” with the word “energy.” The words are spelled similarly and sound similar, and the two concepts are closely linked. But while there may be some confusion regarding words, there is no significant evidence from their responses that students actually believe energy and entropy to be the same entity.

[24] These correspond to normalized gains, <g>, of 0.14 and 0.04 respectively, using Hake’s definition of normalized gain; see R.R. Hake, “Interactive engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses,” Am. J. Phys. **66**, 64-74 (1998).

[25] The Entropy State-Function Worksheet was created for use in the Spring 2005 course; it was targeted at difficulties regarding the state-function property of

entropy and the principle of increasing entropy. This tutorial worksheet guided students to evaluate and compare changes in P, V, T, and S for an ideal gas undergoing either an isothermal expansion or a free expansion. There is evidence that use of this tutorial helped raise students' correct-response rates on certain questions regarding entropy change; see Ref. 22, Chap. 5.

[26] The detailed data corresponding to these differences are shown in Tables VII and VIII below.

[27] Student responses are consistent with the most general form of the entropy inequality and therefore might be considered to be partially correct. However, the questions we presented explicitly referred to spontaneous, irreversible, or "naturally occurring" processes for which the total entropy always increases. Moreover, the patterns of student responses we have reported appear to be independent of the specific terminology used in a particular question. That is, whether one or another term is used, the proportion of student responses for each answer option remains essentially unchanged.

[28] J. P. Guilford, *Fundamental Statistics in Psychology and Education* (McGraw-Hill, New York, 1965), 4th ed., p. 185-187.

[29] Other studies have explored the role of context-dependence (or lack of it) of students' responses, mostly in the form of different problem representations such as mathematical, graphical, verbal, etc. See D. E. Meltzer, "Relation between students' problem-solving performance and representational format," Am. J. Phys. **73**, 463-478 (2005), and references therein.

[30] W.M. Christensen, "An investigation of student thinking regarding calorimetry, entropy, and the second law of thermodynamics" Ph.D. dissertation, Iowa State University (2007), UMI #3274888; Chapters 2 and 4.

[31] L. C. McDermott, "Bridging the gap between teaching and learning: The role of research," in *The Changing Role of Physics Departments in Modern Universities: Proceedings of the International Conference on Undergraduate Physics Education*. E. F. Redish, J. S. Rigden, Eds. 139-165. AIP Conference Proceedings 399. **(**AIP**,** Woodbury, New York, 2007).

[32] Analogous results were found in a different context as reported in D.E. Meltzer, "Analysis of shifts in students' reasoning regarding electric field and potential concepts," in *Proceedings of the 2006 Physics Education Research Conference,* edited by Laura McCullough, Leon Hsu, and Paula Heron (Syracuse, NY, 2006) [American Institute of Physics Conference Proceedings **883**, 177-189 (2007)].

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[34] In contrast to this, we and others have found that students' conceptual difficulties regarding processes that do not involve heat transfer, e.g. the free expansion of a gas, persist to some extent; see Refs. 8 and 12.