Introductory and advanced students' difficulties with thermodynamic work

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We use the Survey of Thermodynamic Processes and First and Second Laws-Long (STPFaSL-Long), a research-based survey instrument with 78 items at the level of introductory physics, to investigate introductory and advanced students' difficulties with work. We analyze data from 12 introductory and advanced physics classes at four different higher education public institutions in the US in which the survey was administered in-person to more than 1000 students. The specific concepts discussed include (1) recognizing and applying the path-dependent nature of work, and (2) interpreting work as area under the curve on a PV diagram. We find that not only introductory but also advanced physics students have some common difficulties with these concepts even after traditional lecture-based instruction. Our results are consistent with prior research findings but extend them to large numbers of students at both introductory and advanced levels and to several new problem contexts not previously investigated.

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I. INTRODUCTION AND GOAL

Since many physics courses for science and engineering majors focus on helping students with both conceptual understanding and problem solving, research-based conceptual multiple-choice surveys administered before and after instruction can be invaluable for assessing course success in improving students' understanding over the course of instruction [1-7]. With respect to the specific topic of this paper, we note that prior research suggests that both introductory and upper-level students have many persistent difficulties with introductory thermodynamics concepts [2-26], implying that conceptual surveys in thermodynamics could potentially play an important role in guiding improvements in instruction.

Here we discuss use of a validated, research-based 78-item multiple-choice survey instrument called the Survey of Thermodynamic Processes and First and Second Laws-Long (STPFaSL-Long). This survey was administered to students in traditionally taught undergraduate physics courses at both the introductory and upper level, as well as to physics graduate students in their first-year, first-semester. Survey data from upper-level students can be helpful in assessing the evolution of students' thinking throughout their study of physics. Our objective was to investigate the extent to which students had learned basic concepts related to various thermodynamic variables; in this paper, we will focus on students' difficulties with work. Our data were obtained from 12 different traditionally taught courses from four different universities in the US. The details pertaining to the development, validation and administration of this survey can be found in Ref. [27]. In addition to administering the written survey in various courses, we interviewed 11 introductory and 6 upper-level students individually using a think-aloud protocol to get a deeper insight into students' thought processes as they responded to the survey questions.

Earlier we had investigated student difficulties using the Survey of Thermodynamic Processes and First and Second Laws-Short survey with 33 items [28-30] in which questions involving multiple variables (e.g., heat transfer, work, and internal energy) are combined into single survey items; this can make it impossible to disentangle specific student difficulties related to individual thermodynamic variables. In contrast, each item in the STPFaSL-Long survey discussed here focuses on one specific variable, increasing the value of the survey in designing instructional tools targeted at student conceptual difficulties with particular variables.

Many of the conceptual difficulties with work discussed here have already been documented, generally in small samples of upper-level students or larger samples of calculus-based introductory students [2-25]. By contrast, our survey instrument was administered to a broad sample of more than 1000 students in 12 different introductory and upper-level physics courses from four different universities, including both algebra-based and calculus-based courses, incorporating survey items that employed an unusually wide variety of different problem contexts. Thus, our results expand the scope of previously reported findings that were generally restricted to students either from introductory or advanced courses but not both, and which used diagnostic instruments containing a relatively small number of items. Our investigation thus sheds light on the robustness of previous findings regarding these student difficulties, assessing their prevalence in diverse student populations and their persistence across a wider variety of problem contexts.

II. METHODOLOGY

This research utilized the STPFaSL-Long, a validated survey instrument with 78 items that focuses on introductory thermodynamics concepts. The details of the development and validation of this survey can be found in Ref. [27] and the survey can be found in [31]. Most items (problems) on the survey have four possible answer choices but 22 out of 78 items are true or false (T or F) questions.

This investigation uses data obtained from administration of the survey both before and after instruction in relevant concepts. In particular, the written data analyzed here were taken by administering the survey in proctored in-person classes as a pre-test (before instruction) and post-test (after students had learned the relevant concepts), but before students' final exam in the course. Students were given some extra credit for completing the survey. Students completed the survey in class on Scantrons during a 50-minute class period. We discuss analysis of student difficulties in the written post-test data from three groups of students: 550 students in the introductory algebra-based (Int-alg) physics course, 492 students in the introductory calculus-based (Int-calc) physics course, and 89 students in their upper-level thermodynamics and statistical mechanics course.

Students in the Int-calc courses were typically engineering majors with some physics, chemistry, and math majors, while students in the Int-alg courses were mainly biological science majors and/or those interested in health-related professions. Students included in the upper-level group were typically physics majors in thermodynamics courses or Ph.D. students in their first year, first-semester of their graduate program, who generally had not taken any graduate-level thermodynamics. (Since the survey was administered as a pre-test to this latter group of students, they were presumed to have taken upper-level undergraduate thermodynamics.) The interview data are from 11 introductory and 6 upper-level students from one institution who volunteered after an opportunity to participate in this study was announced. Each interview lasted between 1-2 hours in one sitting depending upon students' pace. The interviews used a semi-structured think-aloud protocol. Students were asked to think-aloud as they answered the survey questions and were not disturbed except to urge them to keep talking if they became quiet. Only at the end did we ask them for clarifications of points they had not made clear, particularly if they had not answered the question correctly.

Lastly, 349 students from two Int-calc courses were asked to answer the survey questions at the beginning of the semester (pre-test) electronically on Qualtrics and provide their reasoning for each question. While many students did not provide meaningful reasoning, some students provided short but insightful responses. We will only discuss these written explanations for survey items on which most of the interviewed students provided correct responses, such that the interviews (in those cases) did not provide sufficient insight into reasons for student difficulties.

III. RESULTS AND DISCUSSION

On the overall survey, the standard deviation in scores for all introductory groups ranged from 10%-12% while the standard deviation in scores of the upper-level students was 14%. These standard deviations provide a measure of the scale of performance differences between courses that could be considered meaningful.

We discuss two types of difficulties related to "work done by the system" (W): difficulties related to understanding that W is a path-dependent (and not a state) variable (survey items 7 and 46), and difficulties with interpreting W as area under the curve in a PV diagram (items 7, 10, 42, 43, 57, 58 and 59). Correct-response rates on these questions post-instruction are shown in Table I.

TABLE I. Response rates for items related to work for upper-level (Upper), and introductory calculus-based (Int-calc) and algebra-based (Int-alg) physics students. Correct responses are boldfaced and underlined. Item 46 is a true/false question. Due to sample-splitting for validation purposes, the sample size of the introductory groups varied from item to item, from 320 to 491 for Int-calc and from 332 to 549 for Int-alg.

Item #	A	B	C	D	Level
7	31%	**<u>51%</u>**	16%	2%	Upper
	31%	**<u>46%</u>**	21%	2%	Int-calc
	28%	**<u>18%</u>**	51%	3%	Int-alg
46	10%	**<u>90%</u>**	-	-	Upper
	26%	**<u>74%</u>**	-	-	Int-calc
	32%	**<u>68%</u>**	-	-	Int-alg
10	9%	**<u>88%</u>**	3%	0%	Upper
	18%	**<u>72%</u>**	9%	1%	Int-calc
	30%	**<u>58%</u>**	10%	2%	Int-alg
42	9%	7%	**<u>84%</u>**	0%	Upper
	15%	10%	**<u>74%</u>**	1%	Int-calc
	38%	14%	**<u>46%</u>**	2%	Int-alg
43	**<u>83%</u>**	12%	4%	0%	Upper
	<u>74%</u>	16%	9%	1%	Int-calc
	<u>60%</u>	21%	17%	2%	Int-alg
57	3%	**<u>89%</u>**	7%	1%	Upper
	10%	**<u>64%</u>**	24%	1%	Int-calc
	15%	**<u>52%</u>**	33%	1%	Int-alg
58	6%	**<u>87%</u>**	7%	1%	Upper
	10%	**<u>61%</u>**	27%	2%	Int-calc
	13%	**<u>44%</u>**	41%	2%	Int-alg
59	9%	**<u>78%</u>**	12%	1%	Upper
	17%	**<u>54%</u>**	26%	3%	Int-calc
	14%	**<u>38%</u>**	44%	3%	Int-alg

A. Difficulties with W as path-dependent variable

Several prior investigations have studied student understanding of thermodynamic concepts related to changes in variables in cyclic processes that start and end in the same state [8-10, 16, 32]; our findings are consistent with those previously reported. Fig. 1a shows the diagram for item 7 related to a counterclockwise cyclic process. The net work done by the system, W, is path dependent and is negative for the actual path traversed even though the initial and final states of the system are the same.

Table I shows that on item 7, more than half (51%) of the Int-alg students said that the net work done by the gas for one complete cycle would be zero (option C), very similar to results reported in Ref. [8]. This response was also given by 21% of the Int-calc students. All three student groups had ≈30% sign errors (option A), and this was the most common error among the Int-calc and upper-level students. (It is interesting to consider that 56% of calculus-based students who responded to a long, descriptive cyclic-process question during interviews reported in Ref. [10] asserted that net work done by the system would be zero; that question lacked a PV diagram.)

Our interview data shed additional light on students' thinking. One of the interview responses on item 7 illustrated a common reasoning pattern: "If it's one complete cycle, I think it's [work] going to have to be zero because this is the positive work and this is the negative work." (The idea that negative work would cancel positive work in a cyclic process was also often expressed in the interviews reported in Ref. [10].) In response to item 46, which was a true or false question about whether W "is determined by the state of the system and not by the process that led to the state," one student stated, "…I think for work, it should be yes because we can measure the pressure and volume [in a state on which W depends]." It is striking that, in many of their responses, students invoked relationships between only two of the three variables (internal energy E, Q and W) that are incorporated in the first law of thermodynamics, leading them to make incorrect inferences. This "variable-exclusion" thinking pattern related to thermal phenomena was previously described by Rozier and Viennot in Ref. [33], and in Ref. [8].

In Ref. [10], interview responses regarding heat and work done in a cyclic process reflected thinking analogous to that found in our investigation, i.e., that the identical values of temperature, pressure, and volume in initial and final states implied that both net heat transferred, and net work done would be zero. Interviews in both studies also suggested that the sign errors in W often arose either from confusion about whether net work was done *on* or *by* the system, or by reasoning errors in connecting the sign of the work done to the sign of the net heat transfer. A finding that the "work done on or by" confusion may extend to the context of PV diagrams also seems implicit in results reported in Ref. [8]. This could be considered a somewhat surprising result in that arrows depicting the direction of a process on a PV diagram offer a convenient mnemonic with, for example, arrows pointing to the right (higher volume) signifying that work is being done *by* the system during an expansion process.

B. Difficulty in correctly interpreting W as area under the curve on a PV diagram

One of the most useful problem-solving aids in introductory thermodynamics is the "work done equals (signed) area under the curve" interpretation for processes represented on PV diagrams. Student difficulties with this interpretation have been widely reported and analyzed in detail; see, for example, Refs. [8-10, 34]. Table I shows results of the survey items that ask students to examine PV diagrams to determine whether work done during a process is positive, negative, or zero. These include item 7 (Fig. 1a), item 10 (Fig. 1b), items 42 and 43 (Fig. 1c), and items 57, 58 and 59 (Fig. 1d). For all of these, PV diagrams were provided with the problem statement and the item could be answered merely by a correct application of the "area under the curve" interpretation. Nonetheless, correct-response rates varied widely depending on the specific problem context; they were lowest by a wide margin on item 7 for all student groups.

It is not too surprising that item 7 (Fig. 1a) was the most difficult as that is the only one of this group that involves a cyclic process; we already discussed difficulties with work done in cyclic processes in the preceding section. One notable difference between the Int-calc and Int-alg students on item 7 is that a sign error ($W>0$) was the most common incorrect response given by the Int-calc group, while more than half (51%) of the Int-alg students responded that work done would be zero. One student justified a sign-error response ($W>0$) during the interviews as follows: "I'm thinking that it's [work] positive. I was thinking about the equation work equals PV and then, yeah, it would be like positive. That's positive work done on the system." [The interviewer then asked the interviewee how they know if work done *by* the system is positive or negative.] "I was just thinking about the pressure and volume, that's all."

Aside from difficulties on the cyclic-process item, several other findings on this set of problems do not appear to have been reported previously; we enumerate them here:

1. Introductory students were significantly more likely to give a correct ($W>0$) response on the isobaric expansion (#43, Fig. 1c; 60-74% correct) than on the adiabatic expansion (#58, Fig. 1d; 44-61% correct). Some introductory students who provided explanations on the pre-test with an incorrect sign of work for item 58 employed reasoning related to the PV diagram such as, "The line is going down," "arrow is going down," "based on chart direction," "the pressure decreased."
2. Less than half (46%) of the Int-alg students realized that that $W=0$ in the isochoric process (item #42; Fig. 1c, Process 1); more than one third thought instead that $W>0$, perhaps misled by the upward-pointing arrow. Such thinking was sometimes expressed during the interviews, e.g., one student said, "Process 1…since there is a net, I would assume that the work done will be positive for process 1. Since the initial here is going up to the pressure to the final point 1." Similar explanations were provided by some students on their pre-test papers, for example: "It increases pressure, so it increases work"; "P*V is positive:, "P is positive, V is the same"; "The pressure is increasing so the work is positive"; "work has to be done to increase pressure." Some of those who incorrectly thought that the work done by the gas is negative in the isochoric process provided reasoning such as this: "Using $W= -P\Delta V$, it would be negative work done on the system," and "work is being done on the gas making the gas have a higher pressure and more energy."
3. All student groups had greater difficulty comparing the amount of work done in an isothermal vs. *adiabatic* expansion (item #59) than in an isothermal vs. *isobaric* expansion (item #10). Error rates on the isothermal vs. adiabatic comparison (item #59) were 18-20% higher for the introductory students.
   a. Students may have found the isobaric process easier to analyze in general; see #1 above.
   b. Students who answered $W_1=W_2$ on these problems often argued that this was because ΔV was the same for both processes, but there were more such errors on #59. On item #10, one student reasoned, "I'm thinking work equals $P\Delta V$. They both start at the same P and then both end at the same V_f, so I'm thinking that the work done would be equal." Another student stated, "…the start volume is the same and the end volume is the same, so the work done is the same." On item #59, one upper-level student said, "So the volume changes the same amount in both of them so they are equal, but not zero."
   c. There were more sign errors on #59. Written pretest responses suggested that the larger ΔP of the adiabatic process may have been a particularly distracting feature, for example: "The adiabatic process experiences a larger change in P and volume"; "The pressure of the adiabatic system is less than that isothermal system"; "There is a greater difference in pressure loss, the process with a greater pressure loss experienced more work done." Since the PV diagram shows that there is more "area under the curve" in the isothermal process, it is evident that the area interpretation was often inadequate to counter the "intuitive" reasoning triggered by other features of the diagram.

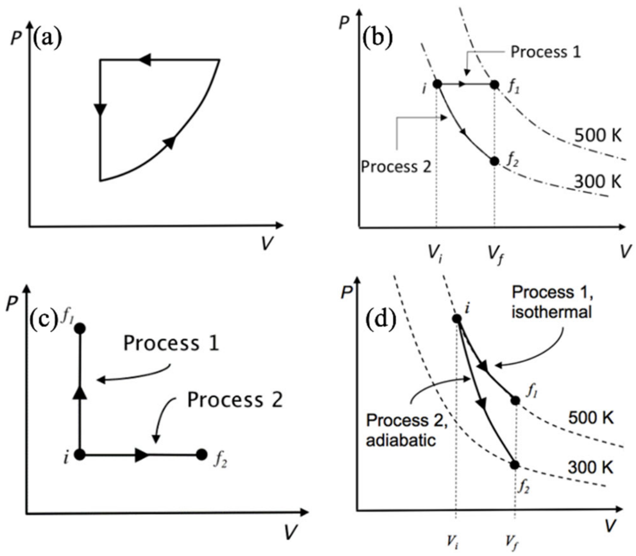

FIG. 1. Diagrams for selected survey items. (a) the diagram for item 7, in which students were asked whether the net work done by the gas for one cycle is positive, negative, or zero; (b) the diagram for item 10, in which students were asked to compare the work done in the two processes; (c) the diagram for items 42 and 43, in which students were asked whether work done in each process was positive, negative, or zero; and (d) the diagram for items 57-59. In 59 students were asked to compare the work done in the two processes, while in 57 (Process 1) and 58 (Process 2) the question was whether the work done was positive, negative, or zero.

It is notable that items 57, 58, and 59 related to processes that were explicitly identified both on the PV diagram and in the problem statement as either isothermal or adiabatic, while items 10, 42, and 43 omitted process descriptions from the diagram itself, putting them instead in a *description* of the diagram. It is conceivable that the additional "extraneous" information in the former group served to divert students' focus from the PV diagram that alone was adequate to answer the question. This is suggested, e.g., by the response one student provided in an interview: "I think the work would be zero because in adiabatic processes, there's no heat transfer so I don't think it would require work to reach the final state." Particularly when attempting to compare the work done in the two processes (item 59, in which work done in the isothermal process is greater), some students were misled by an inappropriate focus on the details of the processes. For example, during the interviews, one student said, "I'm going to say that the work done by the gas in the isothermal process is less than the work done by the gas in the adiabatic process because of the natural log involved in the isothermal work equation." As noted above, written explanations on the pre-test strongly indicated that the relatively larger change in pressure during the adiabatic process was a very distracting feature that helped to prompt sign errors.

IV. SUMMARY

Using a validated survey, we studied the challenges both introductory and upper-level students face after traditional lecture-based instruction with concepts related to work at the level covered in introductory physics courses. The findings presented here suggest that even though instructors may cover work topics in introductory physics courses, some work concepts remain difficult even for upper-level students.

Our survey results from more than a thousand students from 12 introductory and upper-level courses from four different universities are consistent with previously reported findings but extend them to different student populations using a wider variety of problem contexts. Thus, our research validates the previous findings in new contexts and points to the robustness of those findings. Since previous investigations have primarily focused either on introductory or advanced students but not both, our results can also help gain insight into how difficulties reported in previous studies with students at one level (e.g., introductory, or advanced) may persist or vary across different levels of students. For example, we confirmed previous findings that student errors are very common when finding net work done in cyclic processes, but noted that for a counterclockwise cycle, W=0 errors were most common among students in algebra-based courses, while sign errors were more common among upper-level students and students in calculus-based courses.

Our use of multiple problem contexts and diverse student groups also enables us to reveal specific areas in which previous findings can now be seen to be either robustly supported (e.g., with cyclic processes), or to need additional context for clarity and completeness (e.g., regarding distracting surface features in isothermal processes represented on PV diagrams). We investigated students' responses in problem contexts that have not been previously explored or which have received only brief mention in the extant literature, for example, asking students to compare the amount of work done in isothermal and adiabatic processes represented on the same PV diagram. The use of multiple problem contexts allowed us to gauge the relative severity of the learning difficulties and the degree to which they were linked either to fundamental conceptual confusion, or instead to specific surface features of the problem context. We were able to do these types of comparisons in diverse problem settings in a way that previous investigations could not address due to their relative paucity of problem types.

Previous investigations have shown that research-based curricula and pedagogies can often help students learn these concepts more effectively [15, 35]. The findings presented here regarding student difficulties in traditionally taught introductory courses can be used as baseline data, useful for comparison with courses in which innovative evidence-based curricula and pedagogies are used, in order to gauge the level of improvement in students' understanding. (More extensive tabulations of our survey results may be found in [36], along with findings related to heat and internal energy.)

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