Visualization Tool for Three-Dimensional Relationships and the Right-Hand Rule

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The need to develop an understanding of spatial relationships in three dimensions is one of the major challenges faced by introductory physics students. It arises, for example, when grappling with three-dimensional coordinate systems and with the vector ("cross-") product, when dealing with the concepts of torque and angular momentum, and perhaps most prominently when studying relationships involving magnetic fields and forces. A variety of so-called "right-hand rules" are important and widely used tools for working with such concepts. They are applicable both to the standard right-handed *x-y-z* coordinate system (where $\hat{x} \times \hat{y} = \hat{z}$), to a wide variety of concepts involving magnetic fields and forces, and to other phenomena in which vector products are involved. In this paper we describe a simple and inexpensive visualization tool that may be used to help learn and work with these important rules.

Greenslade[1] has described the evolution of the modern right-hand rule from a variety of mnemonic devices that originated shortly after Oersted's discovery in 1820 of the force exerted on a compass needle by a current-carrying wire. Various physical models made of cardboard, wires, and other materials were constructed, and a variety of visualization "rules" were developed and popularized in early textbooks. The right-hand rule in its more modern form began to appear in textbooks quite commonly beginning around 1900.

Although the right-hand rule is an important mnemonic technique, physics instructors are well aware of the various difficulties accompanying its use. For one thing, there are so *many* right-hand rules – different rules, for instance, for determining the direction of magnetic force on a current-carrying conductor, for finding the direction of magnetic field produced by a long straight wire, and for finding the direction of magnetic field produced by a current loop. Many of these – although not all – can be reduced to remembering a variety of vector cross-product relations, and then learning the rule for finding the direction of vector $\vec{C}$ where $\vec{C} = \vec{A} \times \vec{B}$. Adding to the confusion, however, is the multiplicity of standard techniques that are employed to *find* that direction, including what one might call the "thumb-forefinger-middle finger" rule,[2] the "thumb-and-wrist-twisting" rule,[3] and so on. Whatever their particular advantages may be, each of these techniques poses the need to remember which finger is associated with which quantity, and offers an additional anatomical challenge of manipulating the hand and fingers into the position needed to address a particular physical system.

It is not unusual to watch students attempting to apply the right-hand rule become so fixated in their hand manipulation that they actually switch or forget which finger (or hand orientation) they initially had associated with a particular axis or physical quantity. Another pitfall is that students attempting to apply the right-hand rule may choose an initial orientation from which it is difficult to move their body so as to obtain proper alignment of their hand and fingers with the corresponding physical entity. In this situation, a surprising number of students will rotate their arm or wrist to the maximum attainable in that body position, and then declare the result to be the correct vector-product direction. Many students apparently don't distinguish between the vector they are attempting to find, and the practical limits of their physical flexibility in operating the right-hand rule. Certain normal variations in flexibility or body structure may also prove to be an issue when students mistake the maximum contortion of their body with the resultant direction given by the right-hand rule.

A variety of pedagogical tools to assist students in learning vector relationships have been described in the literature. Rather bulky physical models that represent three-dimensional vector relations have been described by Francis,[4] and Wunderlich et al.[5] have discussed a somewhat similar model to demonstrate Cartesian- and polar-coordinate systems. A number of early devices developed to help with magnetic-field problems are discussed in the article by Greenslade.[1] Van Domelen[6] has recently described a device specifically designed to assist students with the right-hand rule. This consists of a transparent rectangular box constructed of plastic, inside of which are set three colored arrows oriented in three fixed, perpendicular directions.

For some years we have been using a simple device that is very quick and easy to construct and is very inexpensive, and yet has proved quite helpful to students in working with the right-hand rule. It consists of an ordinary 3″ × 5″ index card, folded in half along its short axis, on which arrows are drawn to represent three perpendicular directions in space. These arrows may represent, for instance, x, y, and z coordinate axes, directions of current (*I*), magnetic field (*B*) and magnetic force (*F*), etc. One might also design a card on which the three arrows are simply labeled $\vec{A}$, $\vec{B}$, and $\vec{C}$ to illustrate the vector product $\vec{A} \times \vec{B} = \vec{C}$. In the simplest version of this device one simply folds a blank card in half, draws one arrow along the fold, then draws a two-sided arrow along the long axis of the card intersecting the first arrow, labeling each arrowhead appropriately. A more professional-looking version can be generated by copying the accompanying figures (Fig. 1) and printing them out on card stock. The photographs (Figs. 2 and 3) illustrate the use of these cards.

Two black dots are placed on the cards to assist students in orienting the axes. For the Current-Magnetic Field-Force ["*I-B-F*"] card, the angle formed by lines connecting the dots to the centerfold should be smaller than 180° (see Fig. 4). For the Cartesian-axes card, the fold in the card should form a 90° angle at all times (Fig. 5). The cards can easily be flattened again to be stored in the inside pocket of a student's notebook.

The *I-B-F* card is used to find the direction of the magnetic force on a current-carrying conductor in a magnetic field.[7] First the student can orient the current arrow *I* in the direction along which the current flows. Then the entire card should be rotated and the fold-angle adjusted, with the current arrow staying fixed in orientation, until the magnetic-field arrow *B* is pointing along the direction of the external magnetic field. The force arrow *F* then shows the direction of the magnetic force, so long as the card doesn't become "bent backwards" and the dots are connected by an arc smaller than 180°. (If the angle exceeds 180°, the actual direction of the force will of course be opposite to the direction of the *F* arrow.)

The Cartesian-axes card can be set down ahead of time or rotated as needed in order to help the student remember the relative spatial orientation of *x*, *y*, and *z* axes, as well as distinguishing between the +*x* and –*x* directions. It can also help clarify the meaning of "*x-y* plane," "*x-z* plane," etc.

In the classroom environment, use of these cards has proved popular among most students. (We also allow their use on quizzes and exams.) We have found that the cards are most helpful when used in conjunction with other standard right-hand rule techniques.

Typically, students are first asked to solve the problem utilizing the cards, and then asked to try and replicate that result with one of the standard right-hand rule mnemonics using fingers and hands. It is also useful to ask students to relate the reversal of the force direction that can occur when using the *I-B-F* card to the negative sign resulting from an angle greater than 180° between the current and magnetic-field vectors, when using the equation $F = ILB \sin\theta$.

It is easy to come up with other possible uses of a folded index card to illustrate three-dimensional spatial relationships. Indeed, one might assign students the exercise of devising their own methods for illustrating such relationships with the use of the cards. We are exploring the possibility that other simple low-tech devices – perhaps somewhat more elaborate than a folded index card! – can assist students in learning physics principles in which three-dimensional vector concepts and spatial reasoning are involved.

Acknowledgment

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References

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4. Brother Columban Francis, "Vector Visual Aids," *Phys. Teach.* **5,** 119–122 (March 1967).

5. Francis J. Wunderlich, Michael J. Hones, Donald E. Shaw, "Three-dimensional vector demonstrator," *Phys. Teach.* **14,** 232–233 (April 1976).

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7. It is interesting to compare this card to a device developed by Roget to achieve a similar objective. It was described by Noad in an early textbook, and is illustrated in the article by Greenslade (Ref. 1, Fig. 3): Henry M. Noad, *A Manual of Electricity* (Lockwood and Co., London, 1859), p. 643.

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Figure Captions

1. Copy masters for the Current-Magnetic Field-Force ["*I-B-F*"] card and the Cartesian-axes card. These may be copied directly onto card stock. Although different colors are used here to distinguish *I*, *B*, and *F*, monochrome cards are completely satisfactory.

2. The angle of the fold in the *I-B-F* card can vary between 0° and 180°.

3. Use of the Cartesian-axes card.

4. The angle between the $\vec{I}$ and $\vec{B}$ vectors must be less than 180°.

5. The fold in the Cartesian-axes card should form a 90° angle.

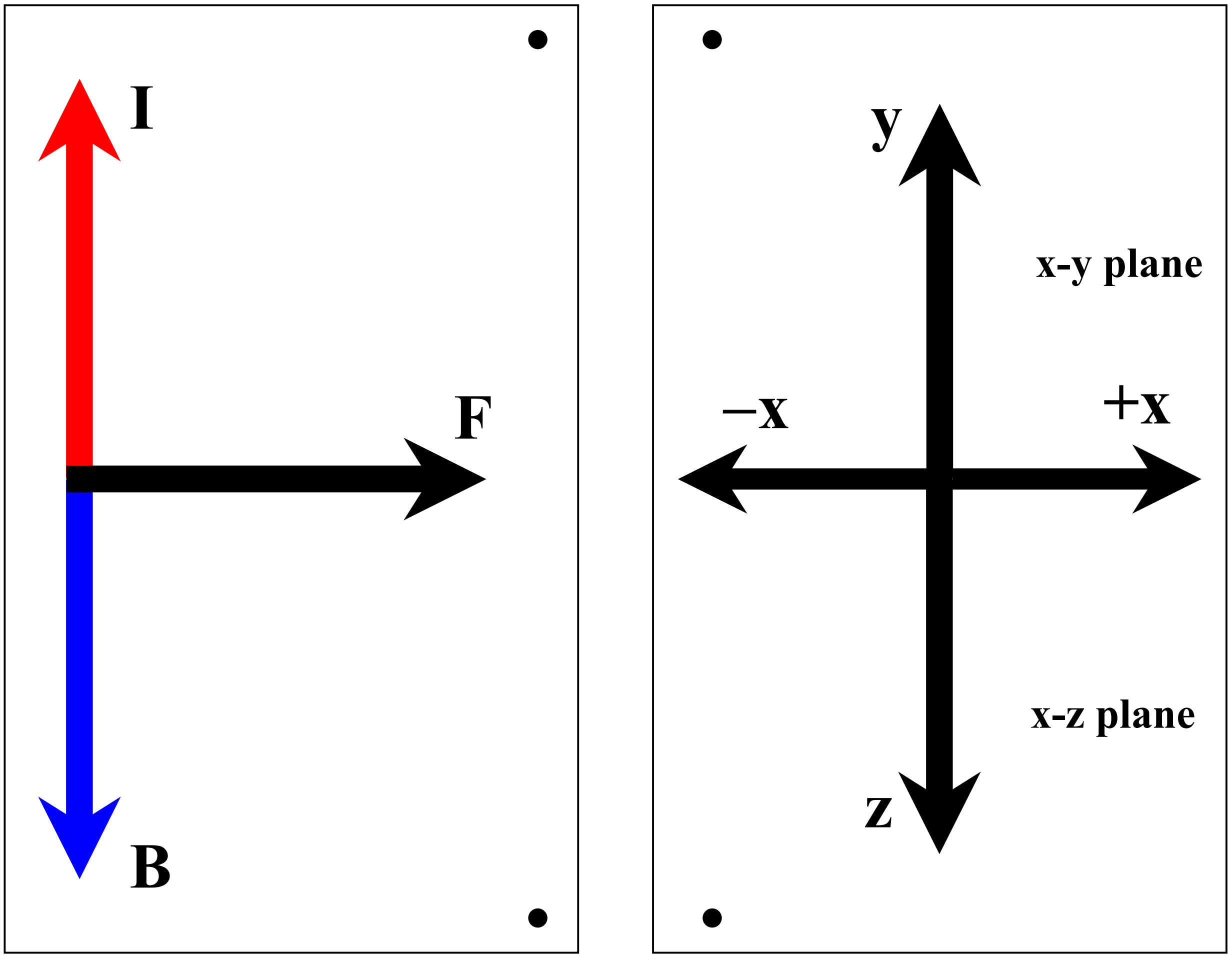

Figure 1

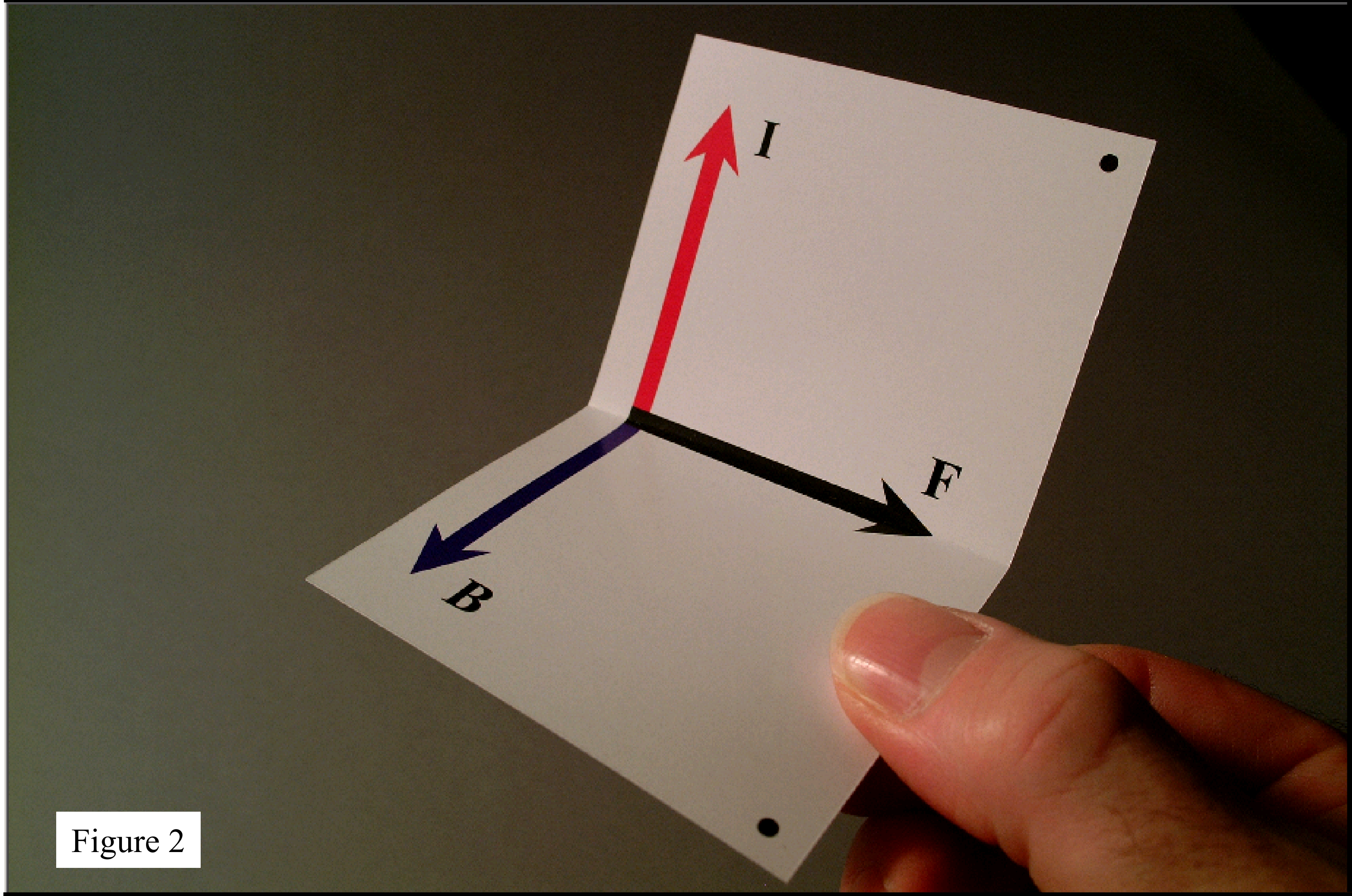

Figure 2

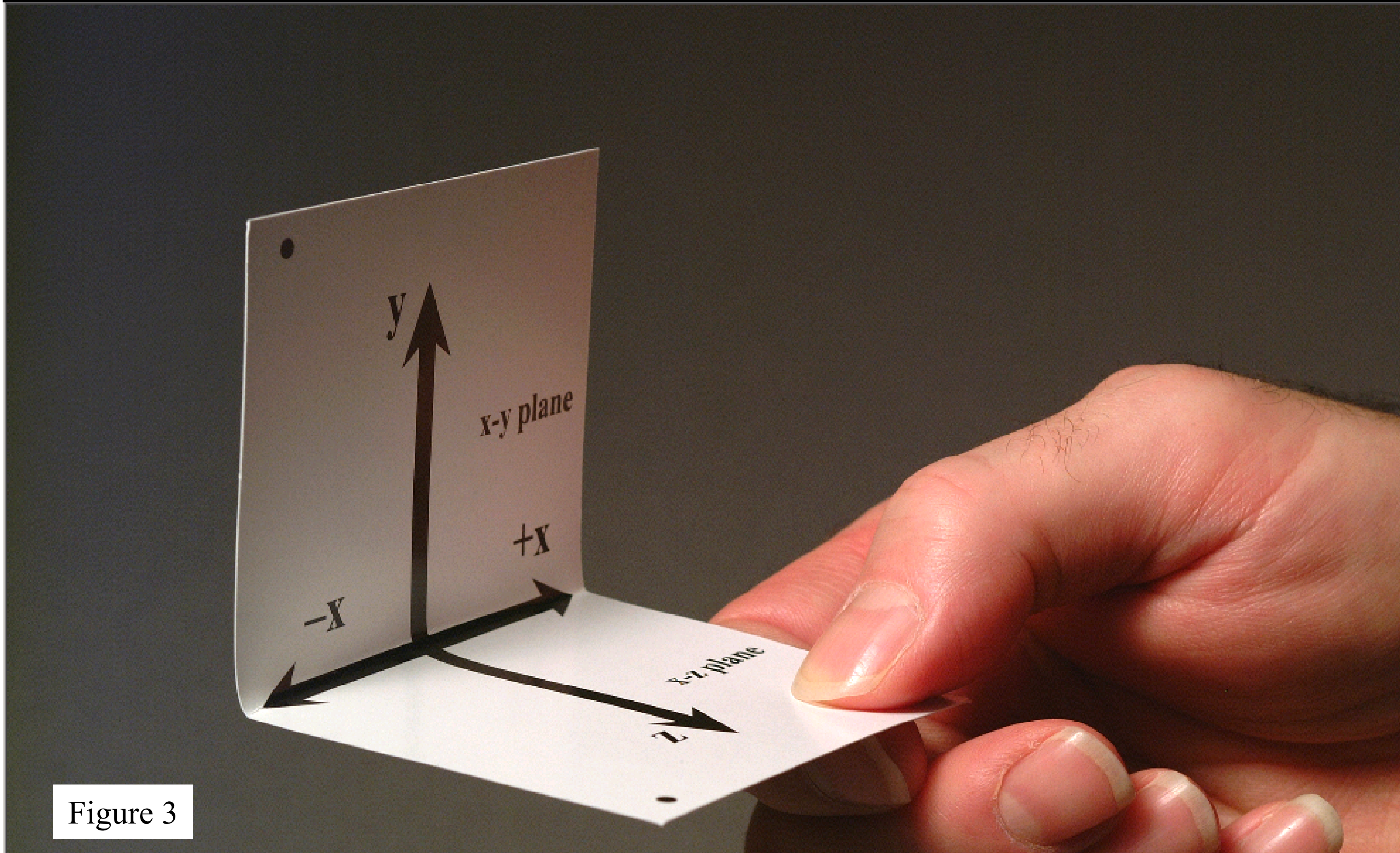

Figure 3

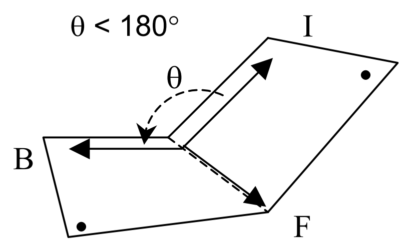

Figure 4

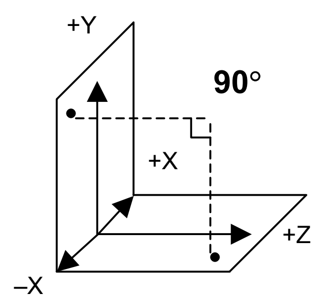

Figure 5