Investigating Student Difficulties Solving Systems of Equations

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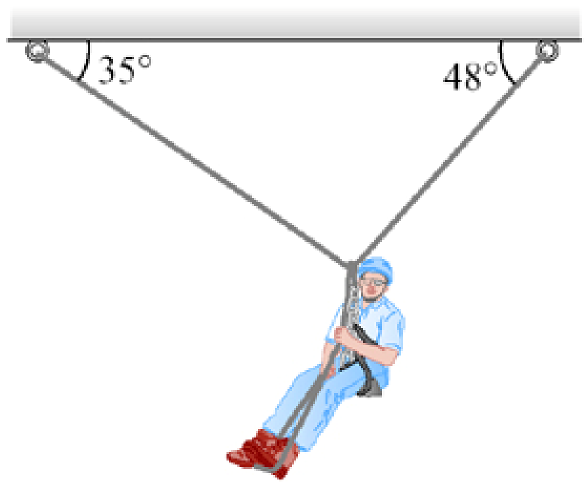
Necessity



Students are frequently asked to solve systems of equations in introductory physics courses

Rope tension problems, such as the figure to the left, are very popular

These types of problems require solving systems of equations

Importance of mathematical understanding

It is well established that many students are ill prepared for the math aspect of introductory physics courses

When students do not follow the math, the physics can be lost as well

It is important to ensure that students understand the mathematics that is going on behind the physics to gain a complete understanding

Three types of problems

Numeric

$$3x = 2y$$
$$5x + y = 26$$

- Easiest to solve for students
- Not very likely to appear in a physics course

Symbolic

$$\mathrm{a} \cdot x = \mathrm{b} \cdot y$$
$$\mathrm{c} \cdot x - \mathrm{a} \cdot y = \mathrm{b}$$

- Very adaptable, values can be inserted later
- Much more difficult for students to solve

Trigonometric

$$x \cdot \cos(20°) = y \cdot \cos(70°)$$
$$x \cdot \cos(70°) + y \cdot \cos(20°) = 10$$

- Actual problem that could arise from a rope tension problem
- Also difficult for students to solve

Student success rates

All three of these questions were asked in a preinstruction diagnostic on the first recitation day of an algebra based, second semester physics course

123 students

Numeric

- 66-75% correct

Symbolic

- 21-23% correct

Trigonometric

- 30-35% correct

Interviews

To see in depth how students solved these problems, I interviewed 44 students

Primarily students from a second semester, calculus based physics course

- Additional interviews are planned for students in algebra based courses

The main methods for solving systems of equations

SUBSTITUTION

$$y = \frac{3}{2}x$$

$$5x + \frac{3}{2}x = 26$$

$$x\left(5 + \frac{3}{2}\right) = 26$$

$$x = \frac{26}{5 + 1.5} = \boxed{4 = x}$$

$$y = \frac{3}{2} \cdot 4 = \boxed{6 = y}$$

ELIMINATION

$$5x + y = 26$$

$$3x = 2y$$

$\times 2$

$$3x - 2y = 0$$

$$+\ 10x + 2y = 52$$

$$13x = 52$$

$$x = 4$$

$$3x = 2y$$

$$\frac{3(4)}{2} = y$$

$$y = 6$$

Basic steps of each method

SUBSTITUTION

Isolate

Substitute

Factor

Divide

Insert first value into initial equation to solve for second value

$$y = \frac{3}{2}x$$

$$5x + \frac{3}{2}x = 26$$

$$x\left(5 + \frac{3}{2}\right) = 26$$

$$x = \frac{26}{5 + 1.5} = \boxed{4 = x}$$

$$y = \frac{3}{2} \cdot 4 = \boxed{6 = y}$$

Basic steps of each method

ELIMINATION

$$5x + y = 26$$
$$3x = 2y$$
$$3x - 2y = 0$$
$$\times 2 \quad (5x + y = 26) \rightarrow 10x + 2y = 52$$
$$+ \quad 10x + 2y = 52$$
$$13x = 52$$
$$x = 4$$

$$3x = 2y$$
$$\frac{3(4)}{2} = y$$
$$y = 6$$

Arrange/multiply equations

Add equation/eliminate variable

Divide

Insert first value into initial equation to solve for second value

Student use rates in interviews (N=44)

SUBSTITUTION

79% overall

If students used substitution for the numeric problem, they always used substitution for the symbolic and trigonometric problems

ELIMINATION

17% overall

27% on numeric

13% on symbolic/trigonometric

Pros of both methods

SUBSTITUTION

Very adaptable

The exact same method can be used for any type of coefficients

Slightly, but not statistically significant, higher correct percentage in interviews

ELIMINATION

Works very well with numeric problems

Good for math class

Cons of both methods

SUBSTITUTION

Vulnerable to careless algebra errors

- ~20% of errors occurred during the factor step

Some preliminary evidence suggests that students sometimes become stuck at the factor step, and don't think to pull out the variable

ELIMINATION

Vulnerable to careless algebra errors

- ~60% of errors occurred during the multiply equations step

Becomes much more difficult with symbolic, trigonometric problems

It is not always immediately clear how to multiply the equations to cancel a variable

Conclusions

For students struggling to understand problems that involve systems of equations, it may be that math that is getting in the way

Substitution seems to be the superior method

- Useful with any coefficients
- The basic steps can be followed easily, without any additional complications

A quick fix to help struggling students:

- Four basic steps of substitution (isolate, substitute, factor, divide)
- Careless algebra errors: go slow and check work