Exploring Student Difficulties in Mathematics Used in Introductory Physics

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Overview

- We’ve administered approximately 4000 math diagnostics to introductory algebra- and calculus-based physics courses since 2016
- We asked students high school level mathematics problems stripped of physics context
- Our most recent diagnostic included new questions including basic fractions
- We continue to focus on analyzing student difficulties with symbolic problems

“Symbolic” Versus “Numeric”

- Torigoe and Gladding (2007; 2011) investigated differences in students’ responses to physics problems in both numeric and symbolic form
  - “**Numeric**” and “**symbolic**” refer to the nature of the constant coefficients
- Starting in 2018, we asked paired problems in both symbolic and numeric form
  - Our problems were stripped of physics context and asked as pure math problems
  - Focused on trigonometry and algebra

Students' Difficulties with Symbols

Torigoe and Gladding (2011):

Numeric version:

A car can go from 0 to 60 m/s in 8 s. At what distance *d* from the start at rest is the car traveling 30 m/s?

Symbolic version:

A car can go from 0 to v_1 in t_1 seconds. At what distance *d* from the start at rest is the car traveling ($v_1/2$)?

Kinematic Equation Problem

Numeric:

$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = 60$$

$$\Delta t = 8$$

$$v = 30$$

$$d = ?$$

(Multiple Choice)

Correct answer: $d = 60$

Symbolic:

$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{v_1}{t_1}$$

$$v = \frac{v_1}{2}$$

$$d = ?$$

(Multiple Choice)

Correct answer: $d = \frac{v_1 t_1}{8}$

Correct-Response-Rate Differences

First Semester algebra-based (PHY 111) Tempe Campus:

Semester	Numeric Correct	Symbolic Correct	Difference
Spring 2018	81% (N=223)	37% (N=215)	**44%**
Fall 2018	78% (N=145)	31% (N=140)	**47%**

First Semester calculus-based (PHY 121) Tempe Campus:

Semester	Numeric Correct	Symbolic Correct	Difference
Spring 2018	89% (*N* =902)	72% (*N* =889)	**17%**
Fall 2018	85% (*N* =157)	64% (*N* =165)	**21%**
Spring 2019	90% (*N* =50)	71% (*N* =45)	**19%**

Differences in Procedure

Numeric:

Example of student work

5. What is the numerical value of d?

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{\Delta v}{\Delta t}$

$\Delta v = 60$

$\Delta t = 8$

$v = 30$

$$30^2 = 0^2 + 2\left(\frac{\Delta v}{\Delta t}\right)d$$

$$900 = 0 + 2\left(\frac{60}{8}\right)d$$

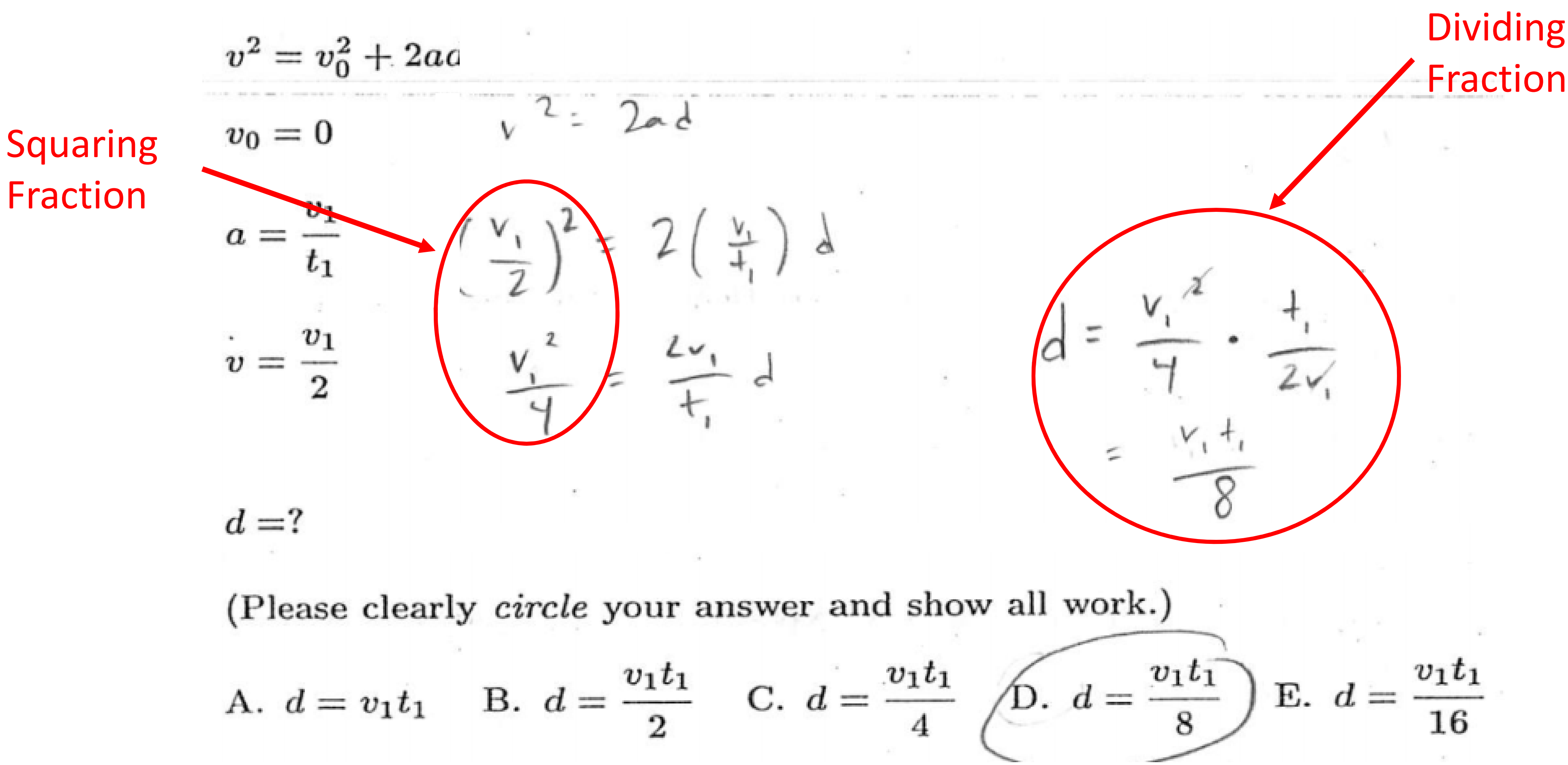
$$900 = 15d$$

$$\boxed{d = 60}$$

Arithmetic used on both sides of equation

Differences in Procedure

Symbolic:



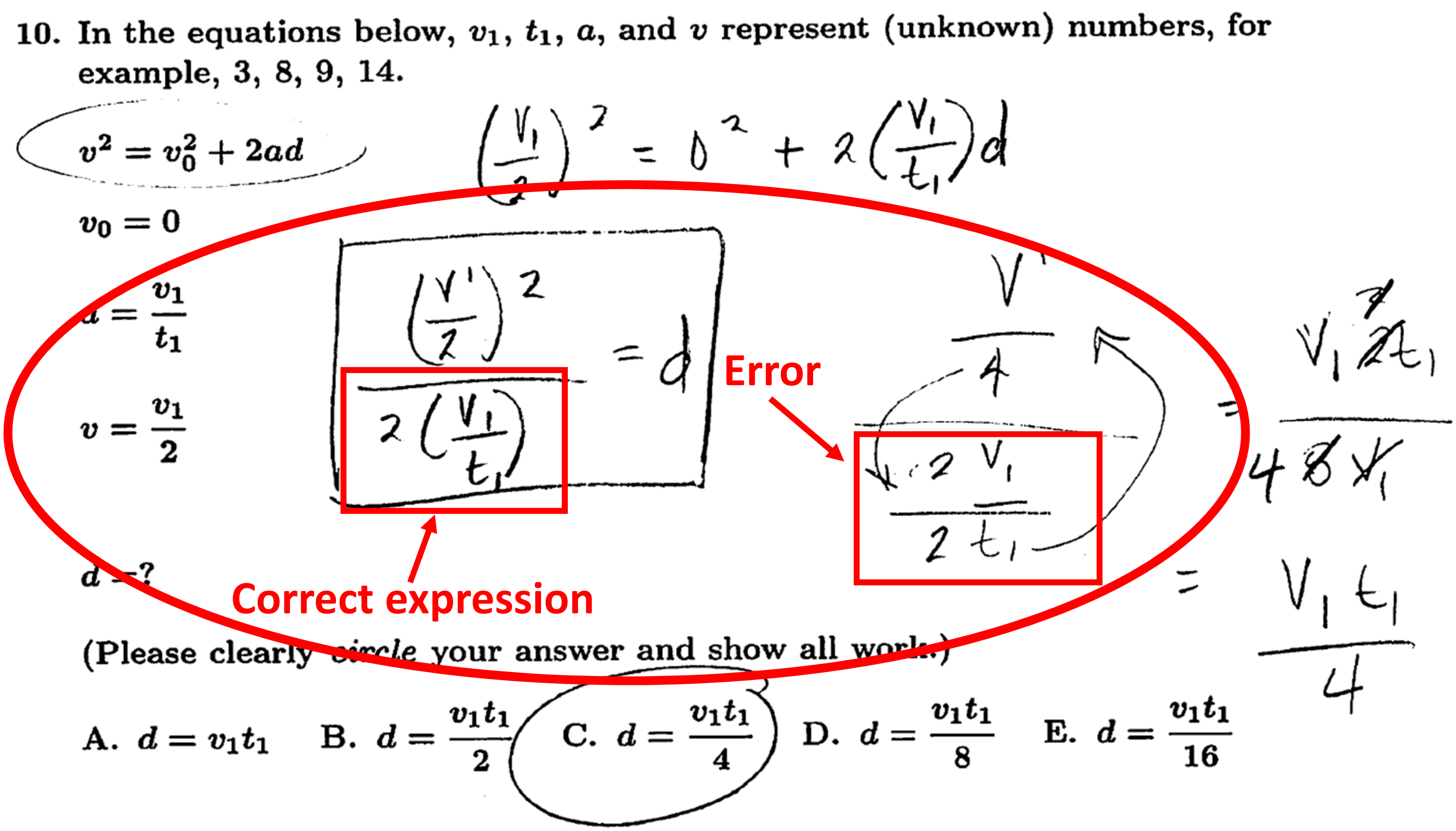
Errors Observed

- **Numeric version:** substituting the wrong value into original equation, e.g., Δv for v.

- **Symbolic version:** incorrectly squaring and multiplying/dividing fractions.

Students seem to struggle with the additional steps and complexity in the symbolic version.

Example of Multiplication Error



10. In the equations below, v_1, t_1, a, and v represent (unknown) numbers, for example, 3, 8, 9, 14.

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{v_1}{t_1}$

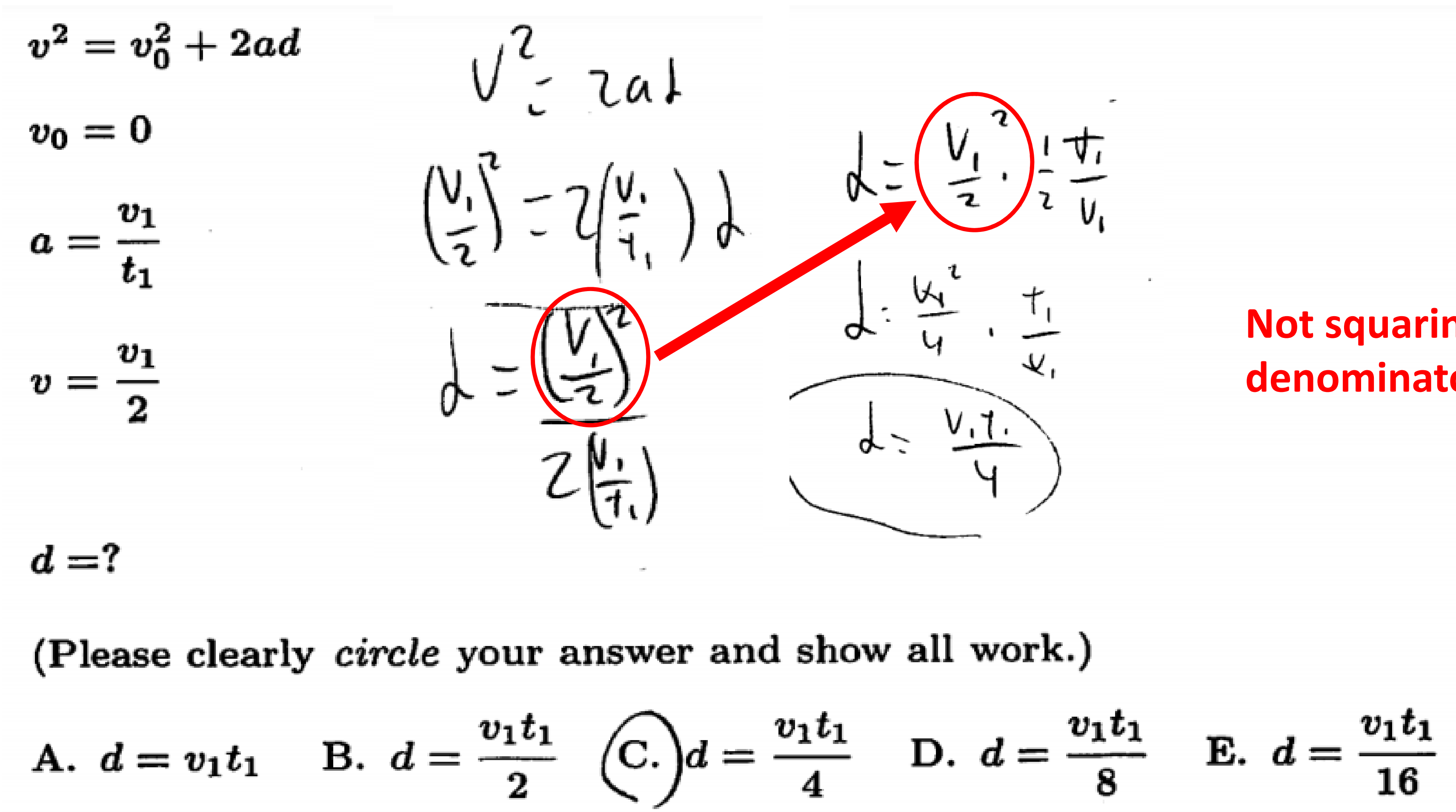
$v = \frac{v_1}{2}$

$d = ?$

(Please clearly *circle* your answer and show all work.)

A. $d = v_1 t_1$ B. $d = \frac{v_1 t_1}{2}$ C. $d = \frac{v_1 t_1}{4}$ D. $d = \frac{v_1 t_1}{8}$ E. $d = \frac{v_1 t_1}{16}$

Example of Squaring Error



$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{v_1}{t_1}$

$v = \frac{v_1}{2}$

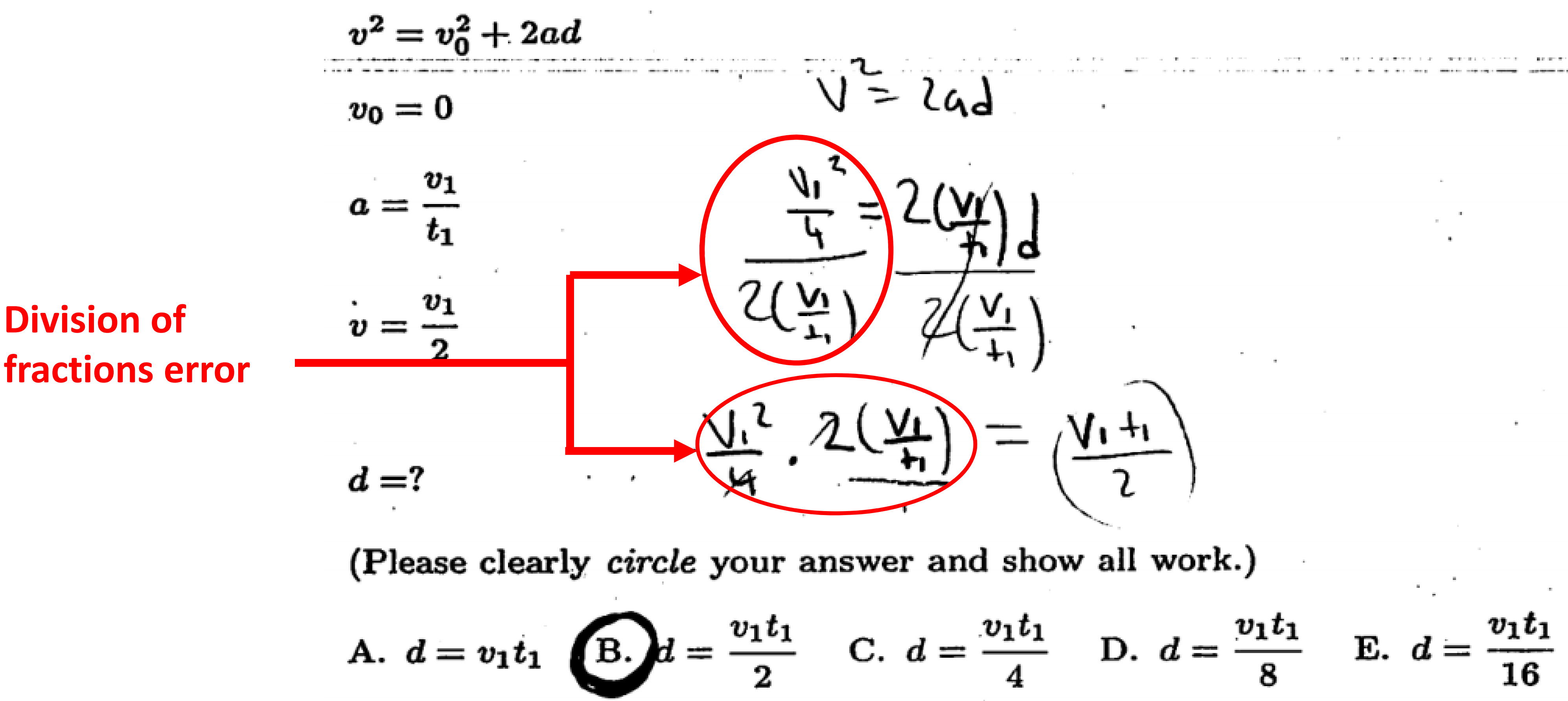
$d = ?$

(Please clearly *circle* your answer and show all work.)

A. $d = v_1 t_1$ B. $d = \frac{v_1 t_1}{2}$ C. $d = \frac{v_1 t_1}{4}$ D. $d = \frac{v_1 t_1}{8}$ E. $d = \frac{v_1 t_1}{16}$

Not squaring the denominator

Example of Division Error



Testing Operational Skills

- To help better understand the errors occurring on the symbolic problem, we asked basic questions to test math skills at the middle-school level

- We wanted to reduce the complexity of the problem in order to isolate errors

  - “Complexity” implies, among other things, the number of steps involved in the problem

- We administered three problems on the manipulation of fractions which were directly related to the kinematic equation algebra problem

Fractions – 3 Problems

Symbolic Multiplication

(1)

$$2\left(\frac{a}{b}\right) = ?$$

A. $\frac{2a}{b}$ B. $\frac{2a}{2b}$ C. $\frac{a}{2b}$

Symbolic Division

(2)

$$\frac{a/b}{c^2/d} = ?$$

$\frac{ad}{bc^2}$

A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

Exponent

(3)

$$\left(\frac{a}{3}\right)^3 = ?$$

A. $\frac{a^3}{3}$ B. $\frac{a}{27}$ C. $\frac{a^3}{27}$

(1) Correct Response Rates (multiplication)

$$2\left(\frac{a}{b}\right) = ?$$

- ASU *Tempe* campus averages:

  1st semester calculus-based course, (*N*=95): **96%**

- ASU *Polytechnic* campus averages:

  1st semester calculus-based course, (*N*=69): **75%**

(2) Correct Response Rates (division)

$$\frac{a/b}{c^2/d} = ?$$

- ASU *Tempe* campus averages:

  1st semester calculus-based course, (*N*=95): **92%**

- ASU *Polytechnic* campus averages:

  1st semester calculus-based course, (*N*=69): **68%**

(3) Correct Response Rates (exponent)

$$\left(\frac{a}{3}\right)^3 = ?$$

- ASU *Tempe* campus averages:

  1st semester calculus-based course, (*N*=95): **100%**

- ASU *Polytechnic* campus averages:

  1st semester calculus-based course, (*N*=69): **91%**

Weak Operational Skills, or Carelessness?

- We define “non-operational errors” as all errors that occur when the student has knowledge of the mathematical operations needed to solve steps of a problem but fails to do so correctly, e.g. , from not checking their work, not accessing previously learned skills, or not exercising sufficient care.
- Knowledge of how to solve each fraction problem is essential to work the kinematic equation problem correctly
- With certain assumptions, we can then estimate the percentage of students that solved the kinematic equation problem incorrectly because of “non-operational errors”

Measuring “Non-Operational Errors”

- We hypothesize, based on analysis of thousands of written diagnostics, that the majority of errors on the symbolic version were due to errors on one or more of the three “fraction” operations

- Therefore, we assume that, if a student responds incorrectly to any of the three fraction problems, they would probably give an incorrect response to the kinematics problem; we call this an “operational” error

- We define any other error as a “non-operational” error

Calculation Example

Percentage of errors that are “operational” errors

Percentage of errors that are “non-operational” errors

Total incorrect response rate (kinematics problem)

$$40\% + X = 60\% \Rightarrow \boxed{X = 20\%}$$

Calculation Example

Percentage of errors that are "non-operational" errors

$$\frac{\boldsymbol{X}}{\mathbf{60\%}} \sim \boxed{\mathbf{35\%}}$$

Total incorrect response rate (kinematics problem)

Proportion of errors that are "non-operational" errors

Table of Results: “Non-Operational Errors”

1st semester algebra-based course:

Campus	Semester	Number of Students	Error on any of the three fraction problems	All errors on kinematics problem	Total "non-operational error"	Proportion of errors that are "non-operational"
Polytechnic	Spring 2019	28	20%	55%	35%	**~60%**

1st semester calculus-based course:

Campus	Semester	Number of Students	Error on any of the three fraction problems	All errors on kinematics problem	Total "non-operational error"	Proportion of errors that are "non-operational"
Polytechnic	Spring 2019	36	50%	60%	10%	**~15%**
Tempe	Spring 2019	45	10%	30%	20%	**~65%**

2nd semester calculus-based course:

Campus	Semester	Number of Students	Error on any of the three fraction problems	All errors on kinematics problem	Total "non-operational error"	Proportion of errors that are "non-operational"
Tempe	Spring 2019	98	10%	25%	15%	**~60%**

Summary and Relation to Interviews

- Students' tend to self-correct their errors during problem-solving interviews approximately 50% of the time, consistent with findings on the written diagnostics
- Therefore, we conclude that students often posses the operational tools necessary to solve a problem, but make non-operational mistakes due to "carelessness", inability to access relevant knowledge, etc.
- We see the possibility of significant improvement through implementation and improvement of work-checking strategies