Introductory Physics Students' Mathematical Preparedness and Conceptual Understanding of Force

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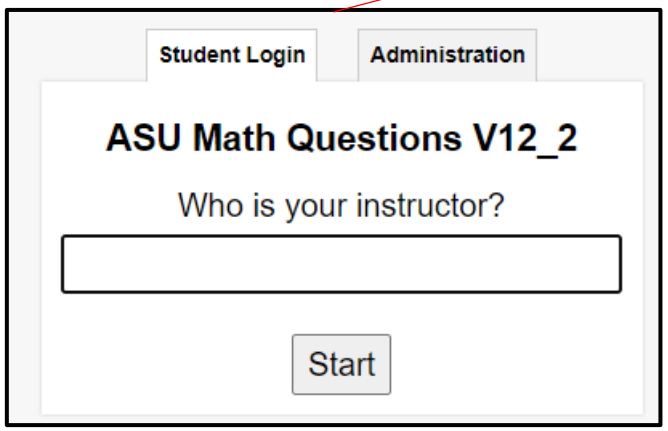
Our newest mathematics diagnostic

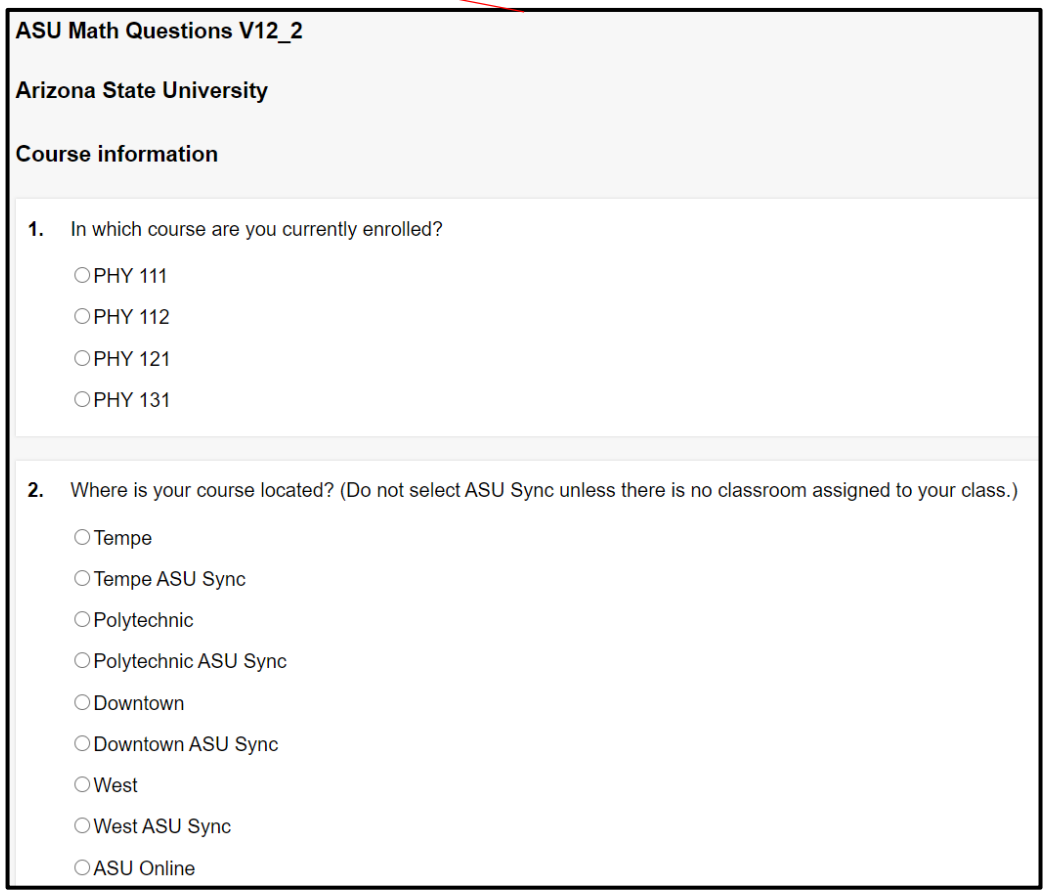
- Online
- Identical math items found on our previous hand-written version
  - Trigonometry, geometry, graphing, and algebra
- 100% multiple choice
  - Free-response items were reformatted to multiple-choice by analyzing student responses from four years of data
- 4 physics items testing conceptual understanding of force
  - Newton's second and third laws

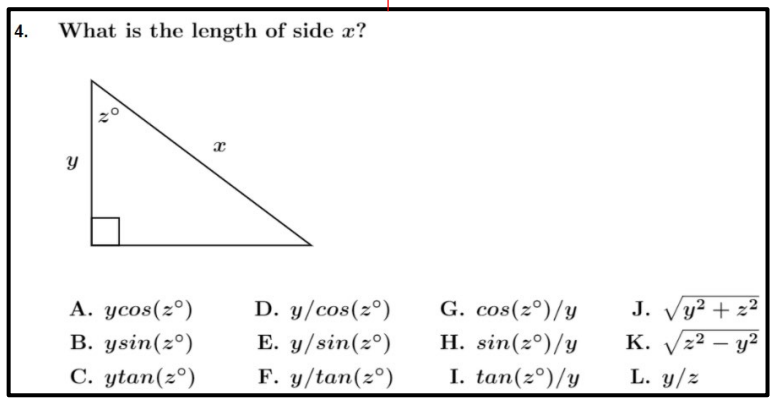
Our newest mathematics diagnostic

Identifying information

Multiple-choice items (15 math, 4 physics)







Online vs hand-written diagnostic

Correct-response rate comparison

Trigonometry, algebra, geometry, and graphing

Item comparison: "Greek vs ABC"
Hand-written vs online

Hand-written

Online

Greek letter coefficients

Solve for θ.

$$\gamma\theta + \eta = \lambda\theta + \omega$$

Solve for θ.

$$\gamma\theta + \eta = \lambda\theta + \omega$$

A. $\frac{\eta + \omega}{\gamma - \lambda}$ C. $\frac{\gamma - \lambda}{\omega - \eta}$ E. $\frac{\eta - \omega}{\gamma\lambda}$ G. $\boxed{\frac{\omega - \eta}{\gamma - \lambda}}$ I. $\frac{\eta - \omega + \gamma}{\lambda}$

B. $\boxed{\frac{\eta - \omega}{\lambda - \gamma}}$ D. $\frac{\lambda - \gamma}{\eta - \omega}$ F. $\frac{\omega - \eta}{\gamma\lambda}$ H. $\frac{\omega - \eta}{\gamma + \lambda}$ J. $\frac{\omega - \eta + \lambda}{\gamma}$

English alphabet coefficients

Solve for x.

$$ax + b = cx + d$$

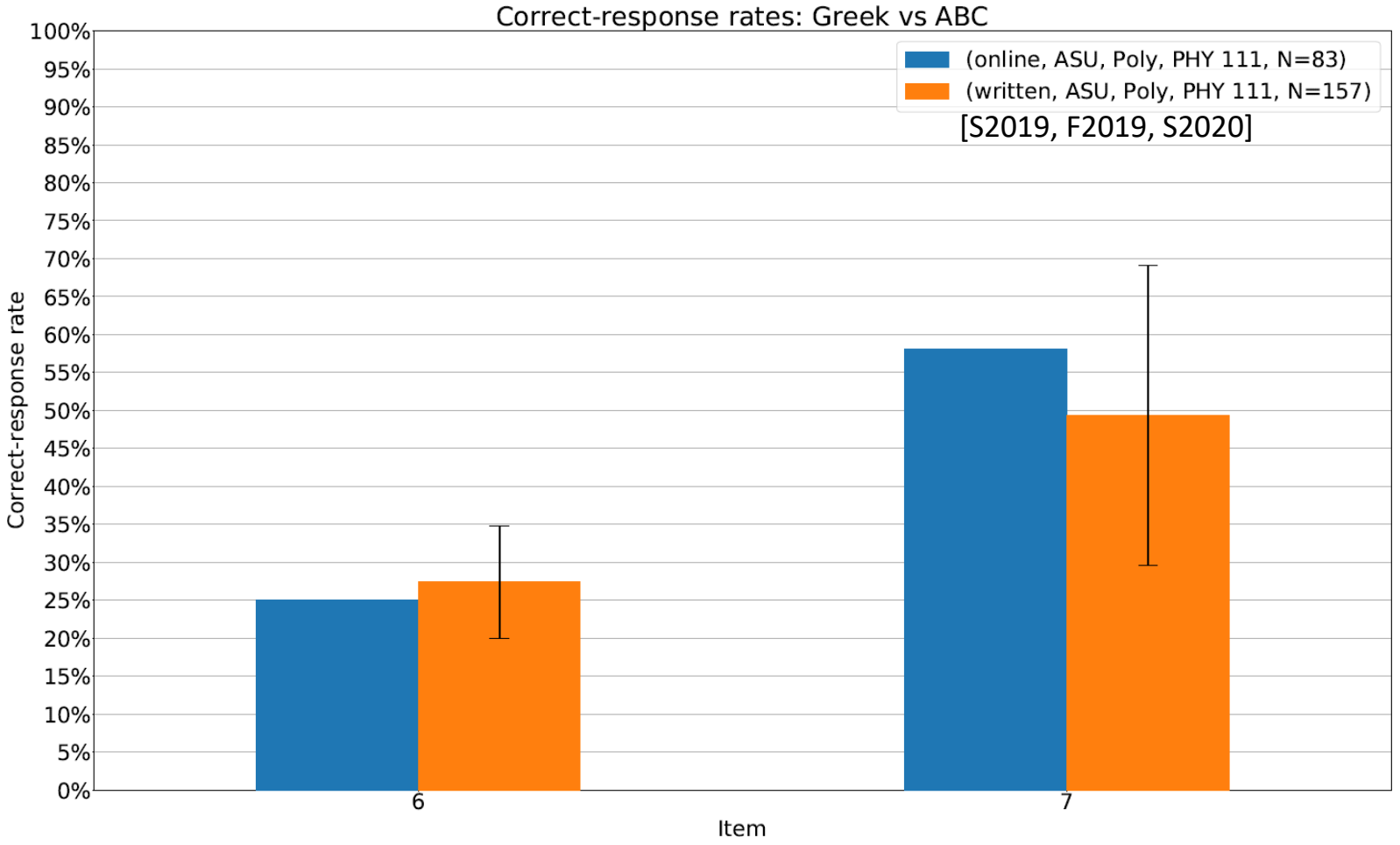
Solve for x.

$$ax + b = cx + d$$

A. $\frac{b + d}{a - c}$ C. $\frac{a - c}{d - b}$ E. $\frac{b - d}{ac}$ G. $\boxed{\frac{d - b}{a - c}}$ I. $\frac{b - d + a}{c}$

B. $\boxed{\frac{b - d}{c - a}}$ D. $\frac{c - a}{b - d}$ F. $\frac{d - b}{ac}$ H. $\frac{d - b}{a + c}$ J. $\frac{d - b + c}{a}$

Item comparison: “Greek vs ABC”
Hand-written vs online



Solve for θ.

$\gamma\theta + \eta = \lambda\theta + \omega$

Solve for x.

$ax + b = cx + d$

Item comparison: “Kinematics multiple-choice”
Hand-written vs online

Hand-written & online

Symbolic coefficients

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \dfrac{\Delta v}{\Delta t}$

$\Delta v = 60$

$\Delta t = 8$

$v = 30$

$d = ?$

A. $d = 30$ B. $\boxed{d = 60}$ C. $d = 120$ D. $d = 240$ E. $d = 480$

Numeric coefficients

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

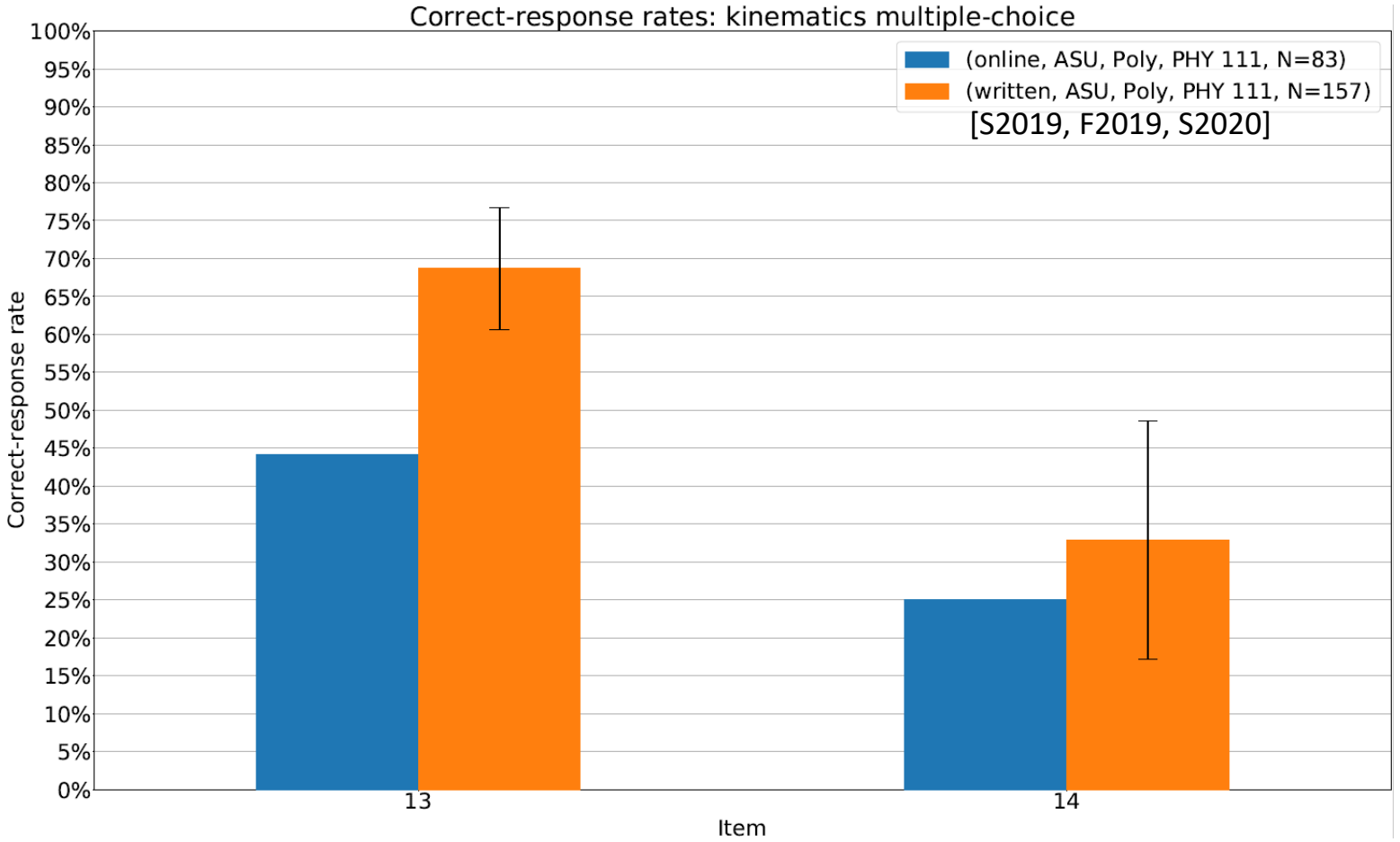
$a = \dfrac{v_1}{t_1}$

$v = \dfrac{v_1}{2}$

$d = ?$

A. $d = v_1 t_1$ B. $d = \dfrac{v_1 t_1}{2}$ C. $d = \dfrac{v_1 t_1}{4}$ D. $\boxed{d = \dfrac{v_1 t_1}{8}}$ E. $d = \dfrac{v_1 t_1}{16}$

Item comparison: “Kinematics multiple-choice”
Hand-written vs online



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$d = ?$

$v^2 = v_0^2 + 2ad$

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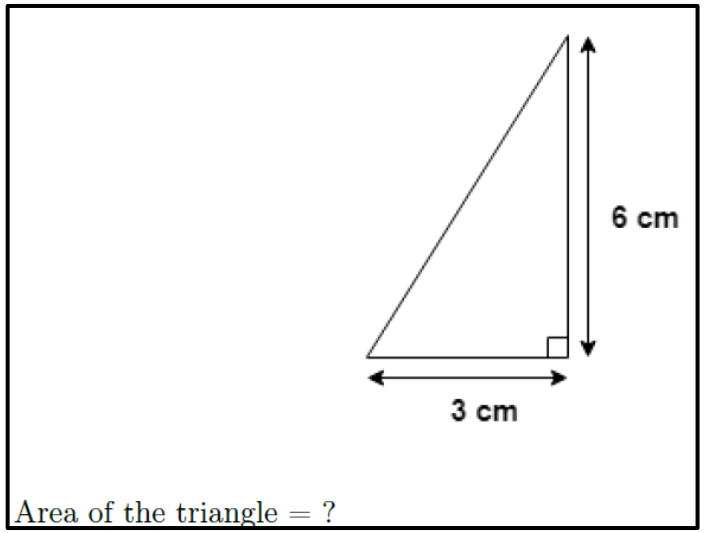
$a = \dfrac{v_1}{t_1}$

$v = \dfrac{v_1}{2}$

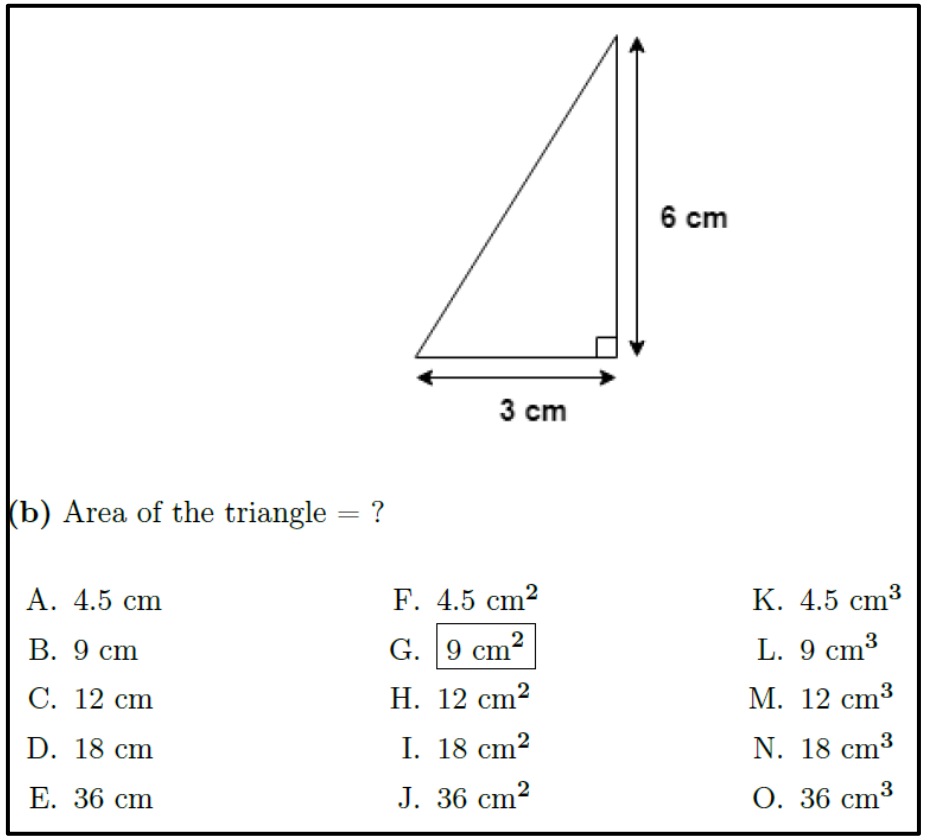
$d = ?$

Item comparison: Area of a triangle and circle
Hand-written vs online

Hand-written

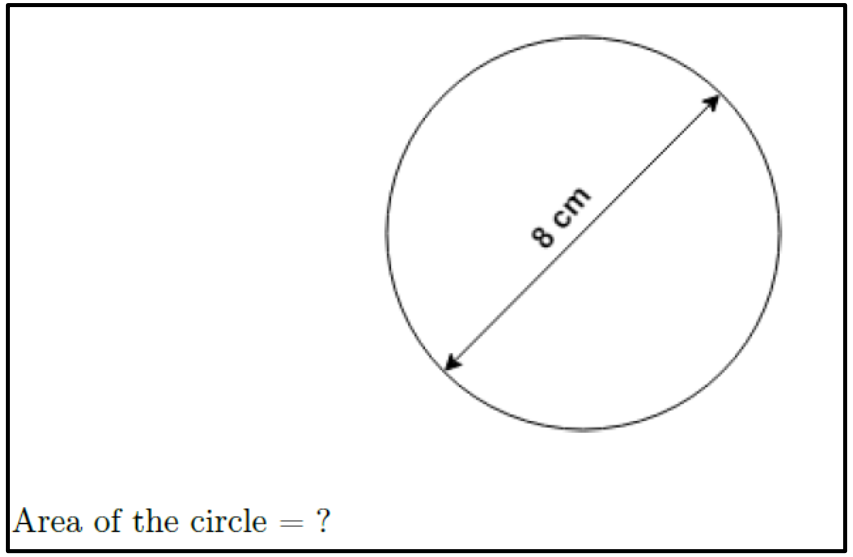


Online

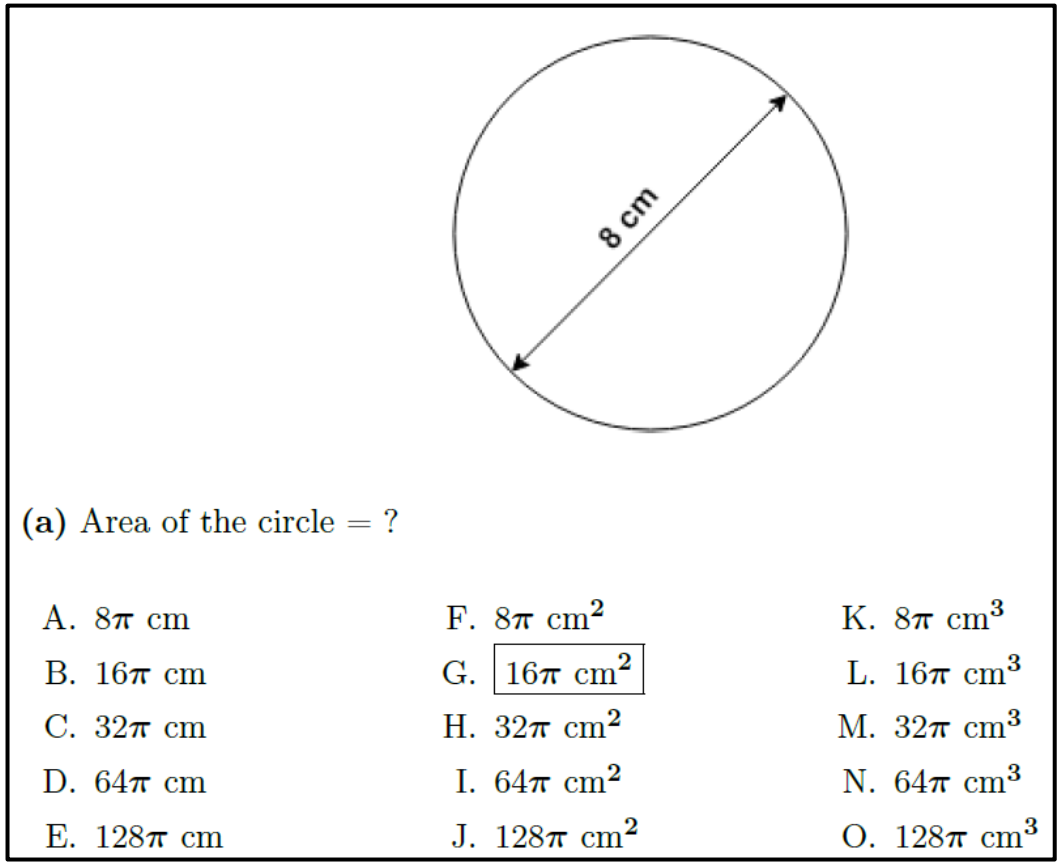


Item comparison: Area of a triangle and circle
Hand-written vs online

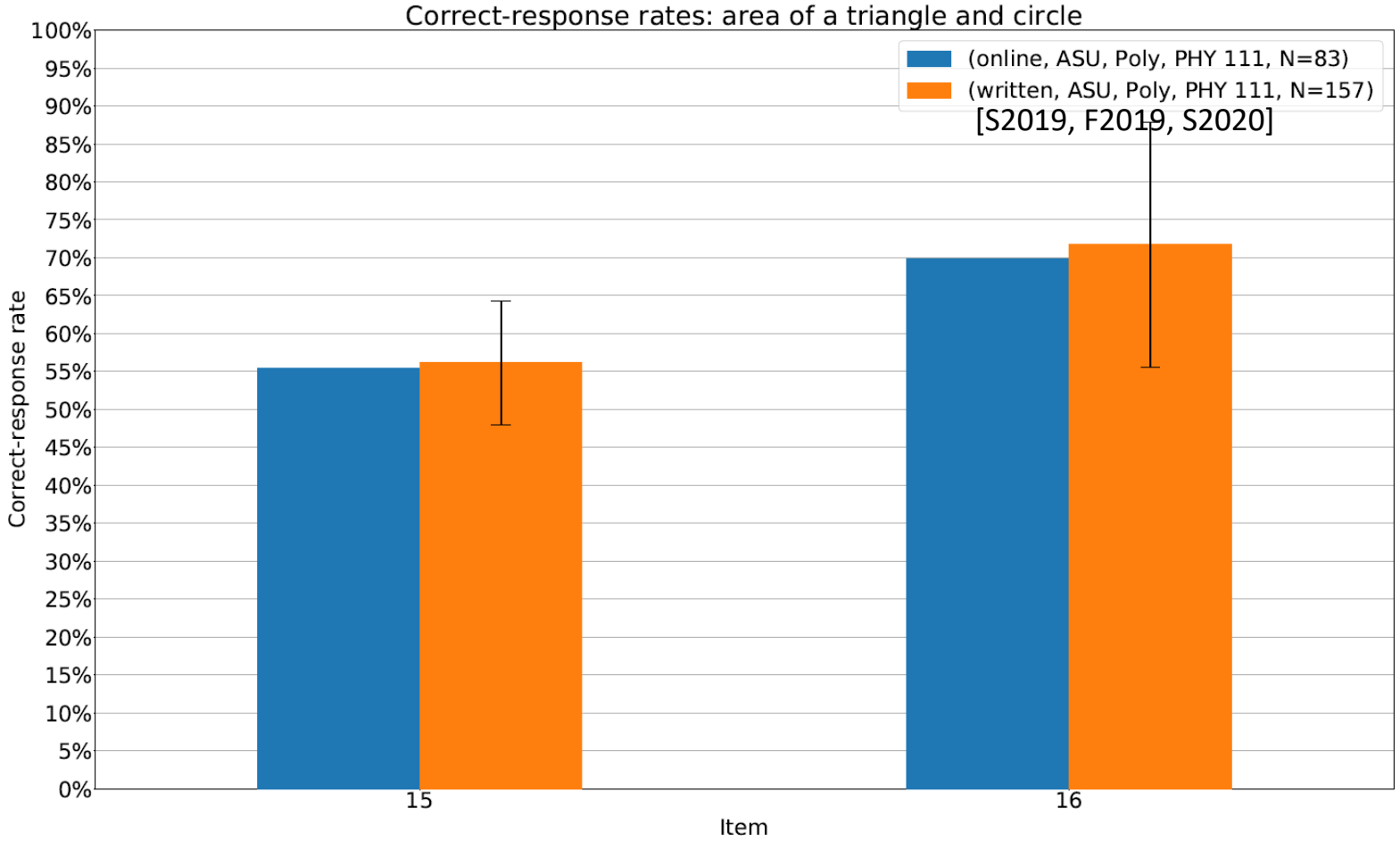
Hand-written

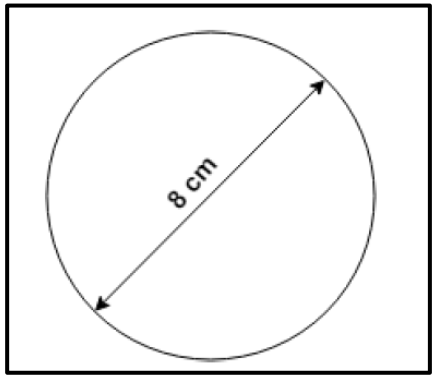


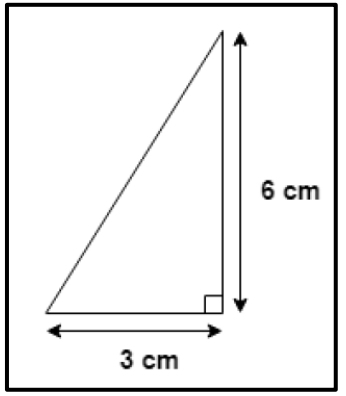
Online



Item comparison: area of a triangle and circle
Hand-written vs online

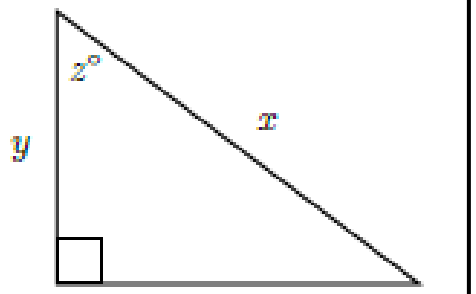




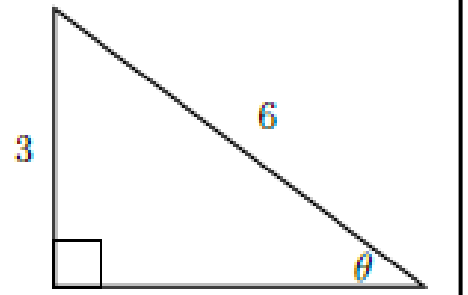


Other items
Hand-written vs online

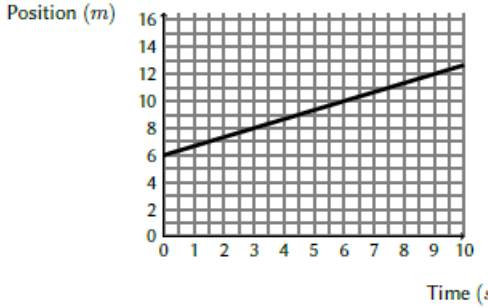
What is the length of side x?



What is the value of θ?



What is the slope of the graph below?



A. $\frac{1}{3}$ m/s because the object moves 1 meter in 3 seconds.

B. $\frac{1}{3}$ m/s because the line rises 1 box while it goes 3 boxes in the horizontal direction.

C. $\frac{2}{3}$ m/s because the object moves 2 meters in 3 seconds.

D. $\frac{2}{3}$ m/s because the line rises 2 boxes while it goes 3 boxes in the horizontal direction.

$$\frac{a/b}{c^2/d} = ?$$

A. $\frac{ac^2}{bd}$ B. $\boxed{\frac{ad}{bc^2}}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

$$\left(\frac{a}{3}\right)^3 = ?$$

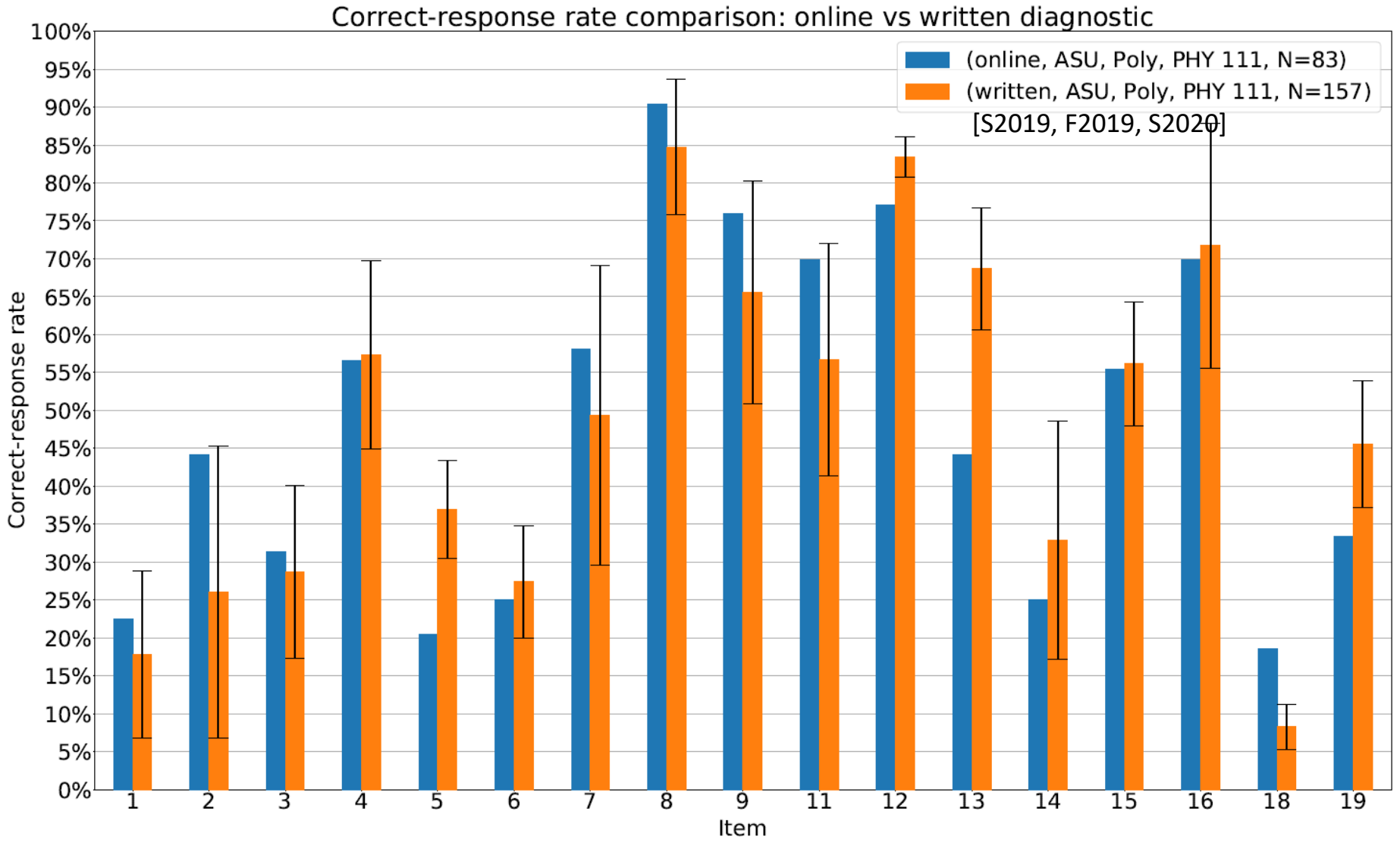
A. $\frac{a^3}{3}$ B. $\frac{a}{27}$ C. $\boxed{\frac{a^3}{27}}$

$$2\left(\frac{a}{b}\right) = ?$$

A. $\boxed{\frac{2a}{b}}$ B. $\frac{2a}{2b}$ C. $\frac{a}{2b}$

Comparing the hand-written diagnostic to the online version

Correct-response rates (math items only)



Online vs hand-written diagnostic

Comparing correlations and performance prediction

Trigonometry, algebra, geometry, and graphing

Comparing the hand-written diagnostic to the online version
Predicting course diagnostic performance

- Taking the course score on a single item and predicting the overall course score on the remaining items

$\frac{a/b}{c^2/d} = ?$

A. $\frac{ac^2}{bd}$ B. $\boxed{\frac{ad}{bc^2}}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

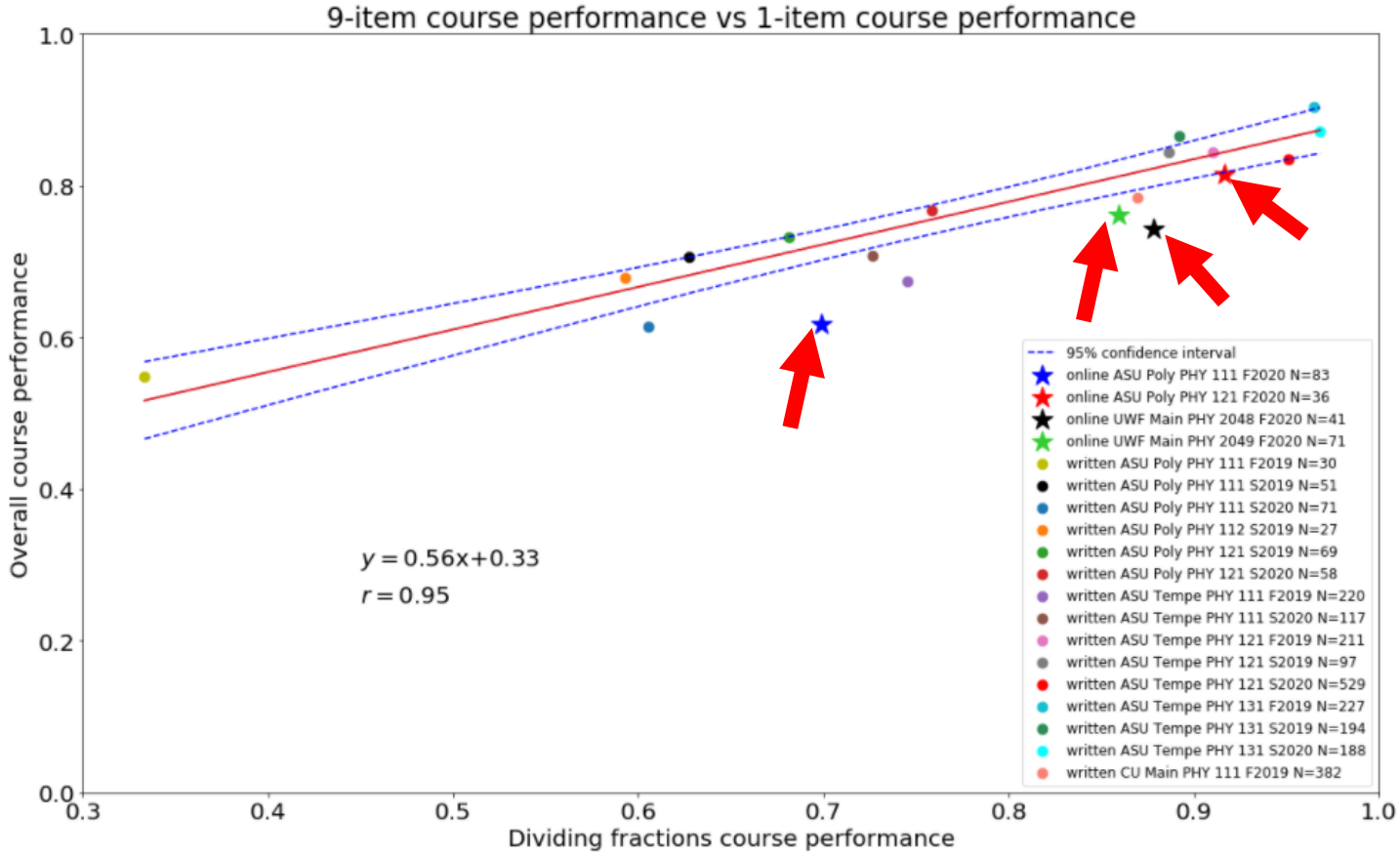
- 2 Trigonometry problems
- 1 graphing problem
- 2 geometry problems
- 4 algebra problems

?

Comparing the hand-written diagnostic to the online version

Predicting course diagnostic performance

Course performance on entire diagnostic (without the predictor item)



$$\frac{a/b}{c^2/d} = ?$$

A. $\frac{ac^2}{bd}$ B. $\boxed{\frac{ad}{bc^2}}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

Course performance on the predictor item

Comparing the hand-written diagnostic to the online version
Predicting course diagnostic performance

- Taking a student's response on a single item (incorrect or correct) and predicting the student's overall score on the remaining items

$$\frac{a/b}{c^2/d} = ?$$

A. $\frac{ac^2}{bd}$ B. $\boxed{\frac{ad}{bc^2}}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

- 2 Trigonometry problems
- 1 graphing problem
- 2 geometry problems
- 4 algebra problems

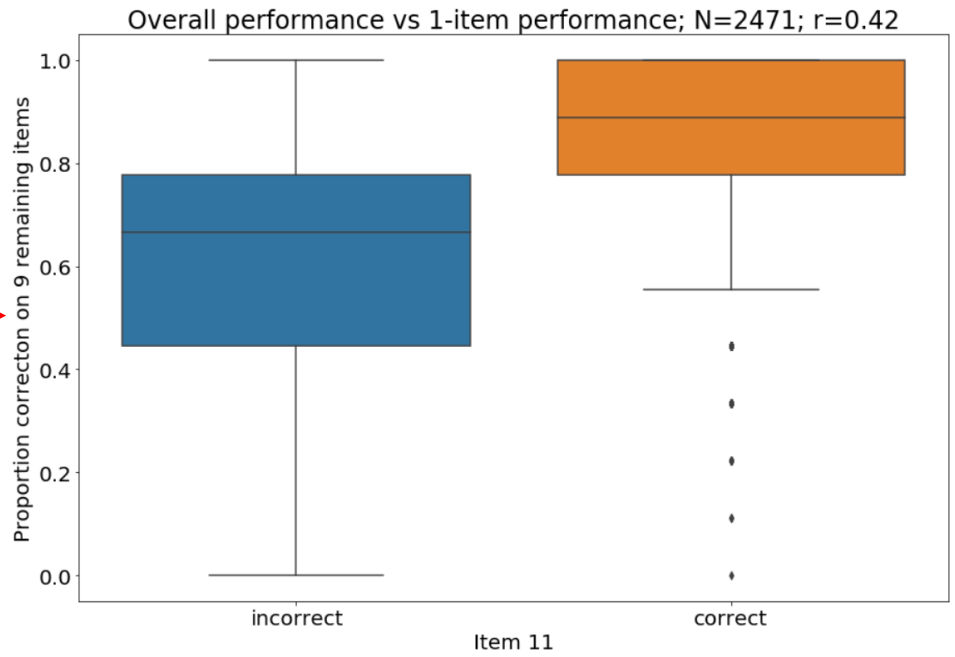
?

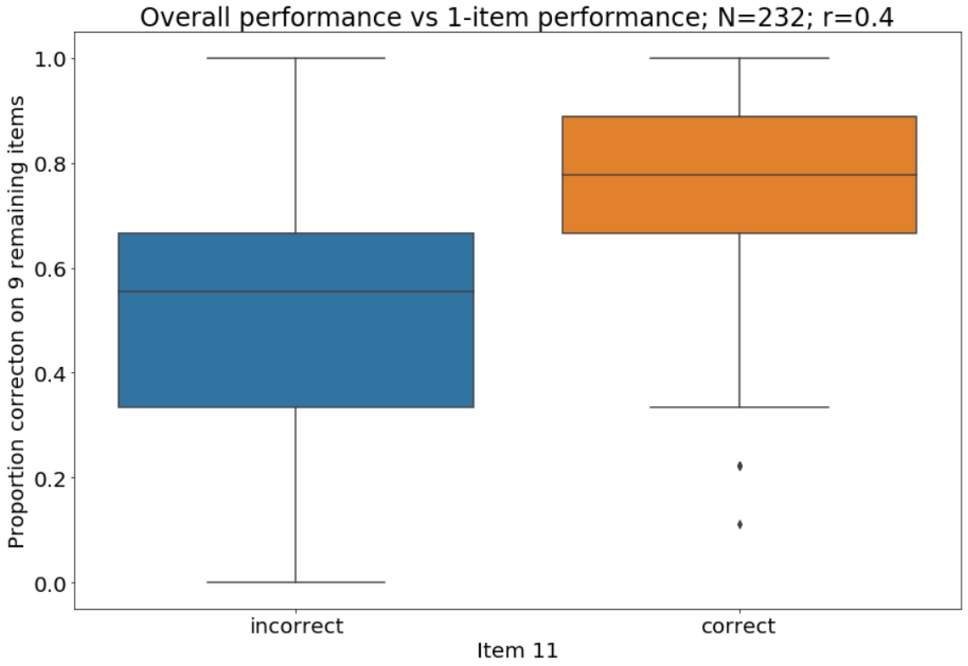
Comparing the hand-written diagnostic to the online version
Student performance correlations (math items only)

Hand-written diagnostic

Online diagnostic

Student performance on entire diagnostic (without the predictor item)





$$\frac{a/b}{c^2/d} = ?$$

A. $\frac{ac^2}{bd}$ B. $\boxed{\frac{ad}{bc^2}}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

Student performance on the predictor item

Physics portion of the online diagnostic

Four problems on Newton's 2nd and 3rd laws

Online diagnostic physics problems

Newton's second law (PHY1)

University	Campus	Course	Term	N	CRR
ASU	Poly	PHY 111	F2020	83	14%
ASU	Poly	PHY 121	F2020	36	39%
UWF	Main	PHY 2048	F2020	41	24%
UWF	Main	PHY 2049	F2020	71	32%

Correct-response rates:
14% - 39%

A rock, attached to a rope, is being pulled straight up at constant, unchanging speed. While it is being pulled, the forces acting on the rock are:

A. An upward force from the rope that is larger than the downward force of gravity

B. An upward force from the rope that is equal in magnitude to the downward force of gravity

C. An upward force from the rope that is smaller than the downward force of gravity

D. Only the downward force of gravity

E. Only an upward force from the rope

Online diagnostic physics problems

Newton's second law (PHY2)

University	Campus	Course	Term	N	CRR
ASU	Poly	PHY 111	F2020	83	24%
ASU	Poly	PHY 121	F2020	36	47%
UWF	Main	PHY 2048	F2020	41	41%
UWF	Main	PHY 2049	F2020	71	32%

Correct-response rates:
24% - 47%

A rock, attached to a rope, is being pulled straight up; it is speeding up at a steady rate (with constant acceleration) while it is being pulled. The downward force of gravity on the rock is nearly constant and unchanging. The upward force from the rope is:

A. Larger than the force of gravity, and increasing in strength

B. Larger than the force of gravity, and nearly constant in strength

C. Equal to the force of gravity

D. Smaller than the force of gravity, and decreasing in strength

E. Smaller than the force of gravity, and nearly constant in strength

Online diagnostic physics problems

Newton's third law (PHY3)

University	Campus	Course	Term	N	CRR
ASU	Poly	PHY 111	F2020	83	13%
ASU	Poly	PHY 121	F2020	36	50%
UWF	Main	PHY 2048	F2020	41	39%
UWF	Main	PHY 2049	F2020	71	41%

Correct-response rates:
13% - 50%

A small woman is pushing on a stalled car and the car is very gradually speeding up. While she is pushing on the car,

A. The force of woman on the car is larger than the force of the car on the woman

B. The force of the woman on the car is equal in magnitude to the force of the car on the woman

C. The force of the woman on the car is smaller than the force of the car on the woman

D. The woman exerts a force on the car but the car does not exert any force on the woman

E. The car exerts a force on the woman but the woman does not exert any force on the car.

Online diagnostic physics problems

Newton's second law (PHY4)

University	Campus	Course	Term	N	CRR
ASU	Poly	PHY 111	F2020	83	17%
ASU	Poly	PHY 121	F2020	36	50%
UWF	Main	PHY 2048	F2020	41	49%
UWF	Main	PHY 2049	F2020	71	41%

Correct-response rates:
17% - 50%

A child is sitting on a sled that is stranded in the middle of a frozen pond. The ice is so wet and slippery that ***friction can be assumed to be zero***. Workers in a truck on land attach a fishing line to the sled and pull on the line; a scale attached to the line ensures that it pulls with unchanging force. However, after a few moments the line breaks. Then:

A. The sled moves at constant speed while the line pulls and at a constant but slower speed after the line breaks.

B. The sled speeds up while the line pulls and slows down after the line breaks.

C. The sled speeds up while the line pulls and moves at constant speed after the line breaks.

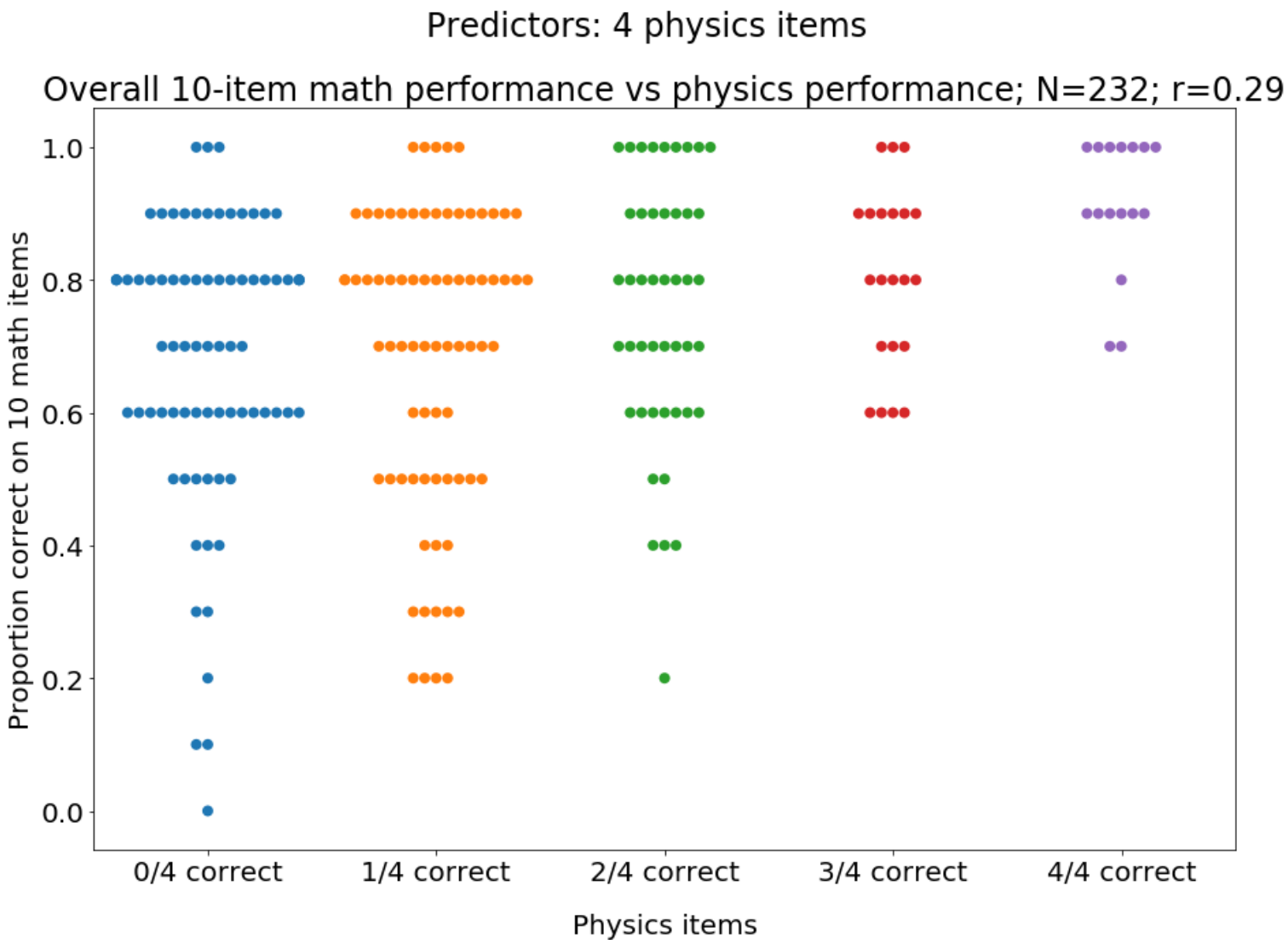
D. The sled speeds up while the line pulls and keeps gaining speed after the line breaks, but not as rapidly as before.

E. The sled moves at constant speed the whole time.

Comparing student performance on the physics and math portion of the online diagnostic

Online diagnostic physics problems

Math vs physics performance



Summary

- Our new online diagnostic appears comparable to our hand-written diagnostic in measuring and predicting students' mathematical preparedness
- Performance on the physics items is correlated ($r \approx 0.3$) with performance on the mathematics items
- Larger sample sizes and further analysis are needed to characterize these measures with more confidence and detail
- If you would like this diagnostic administered in your university physics course, please email me or David Meltzer at:
  - dhking1@asu.edu
  - David.Meltzer@asu.edu

Summary

- Our new online diagnostic seems to measure and predict the mathematical preparedness of students to a similar degree as our hand-written diagnostic
  - A larger sample size is desired to fully characterize the capability of making these measurements and predictions using the online diagnostic
- Performance on our conceptual physics problems (Newton's 2nd and 3rd laws) is significantly correlated with mathematics performance

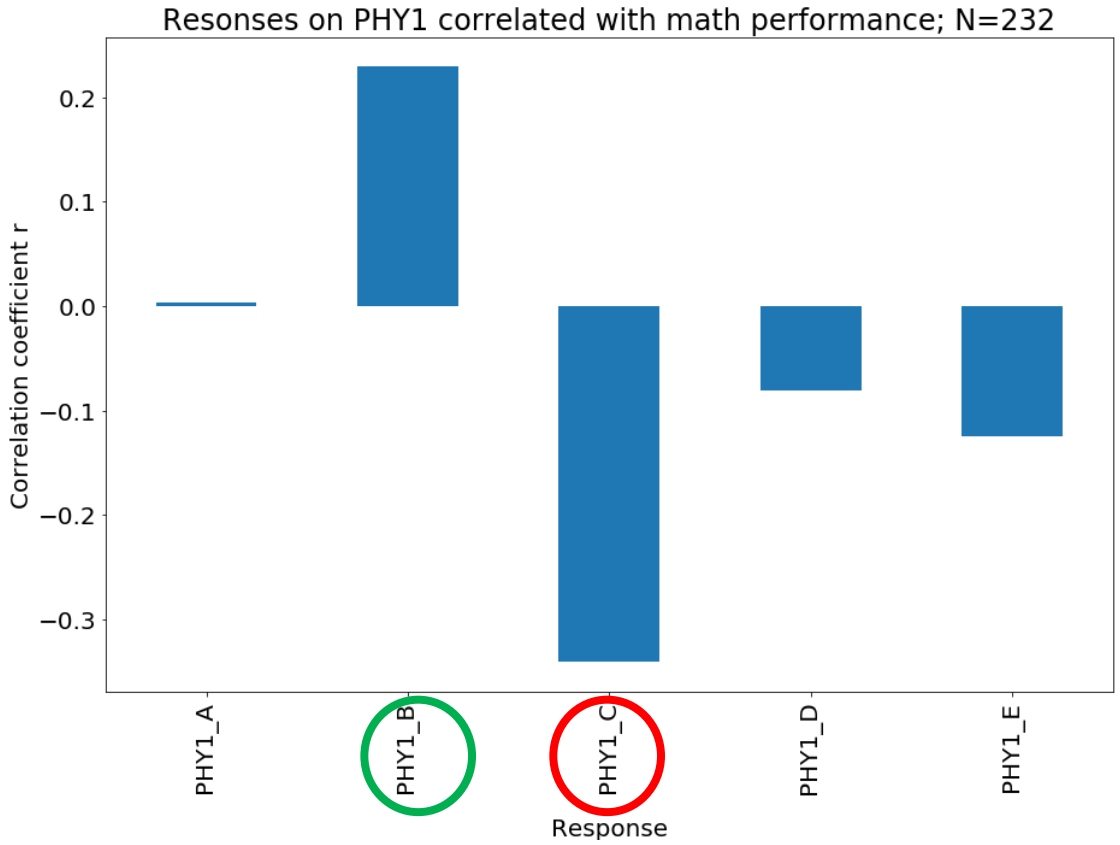
Comparing the hand-written diagnostic to the online version

Internal consistency (math items only)

Version	Hand-written (N, **KR-20**)	Online (N, **KR-20**)
A (14 items)	(1224, **0.76**)	(16, **0.85**)
B (14 items)	(1108, **0.74**)	(17, **0.85**)
C (10 items; subset of mutual items between A & B)	(2370, **0.70**)	(232, **0.68**)

Online diagnostic physics problems

Newton's second law (PHY1)



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B. An upward force from the rope that is equal in magnitude to the downward force of gravity

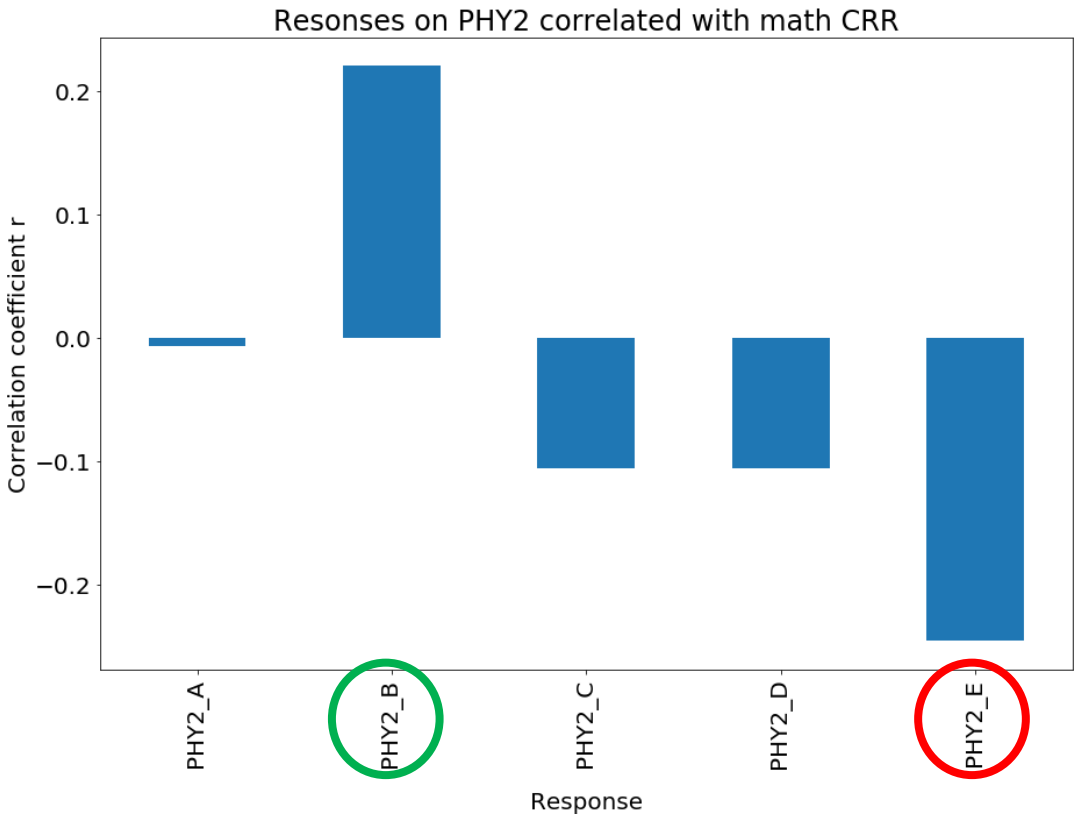
C. An upward force from the rope that is smaller than the downward force of gravity

D. Only the downward force of gravity

E. Only an upward force from the rope

Online diagnostic physics problems

Newton's second law (PHY2)



A rock, attached to a rope, is being pulled straight up; it is speeding up at a steady rate (with constant acceleration) while it is being pulled. The downward force of gravity on the rock is nearly constant and unchanging. The upward force from the rope is:

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B. Larger than the force of gravity, and nearly constant in strength

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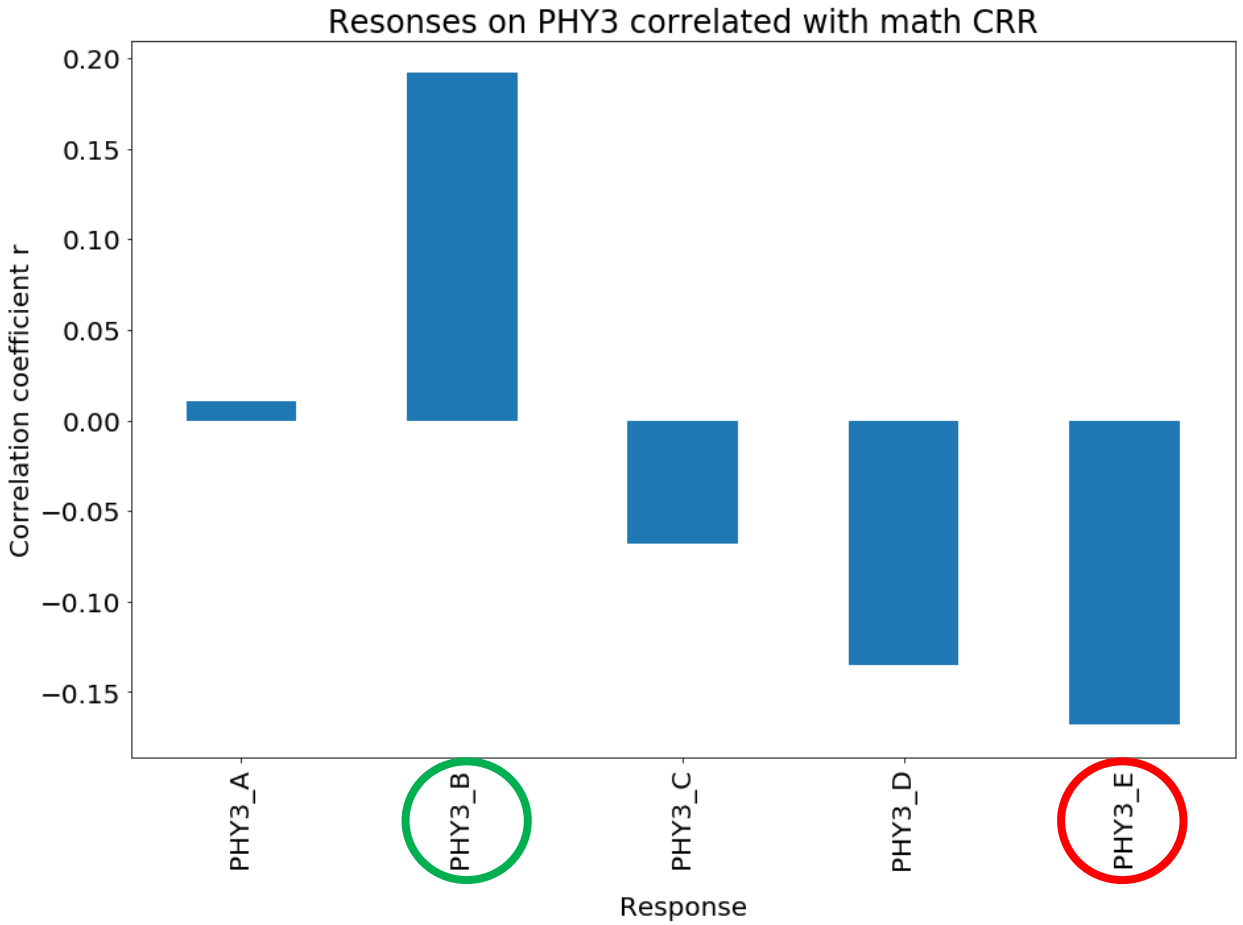
Online diagnostic physics problems

Newton's third law (PHY3)



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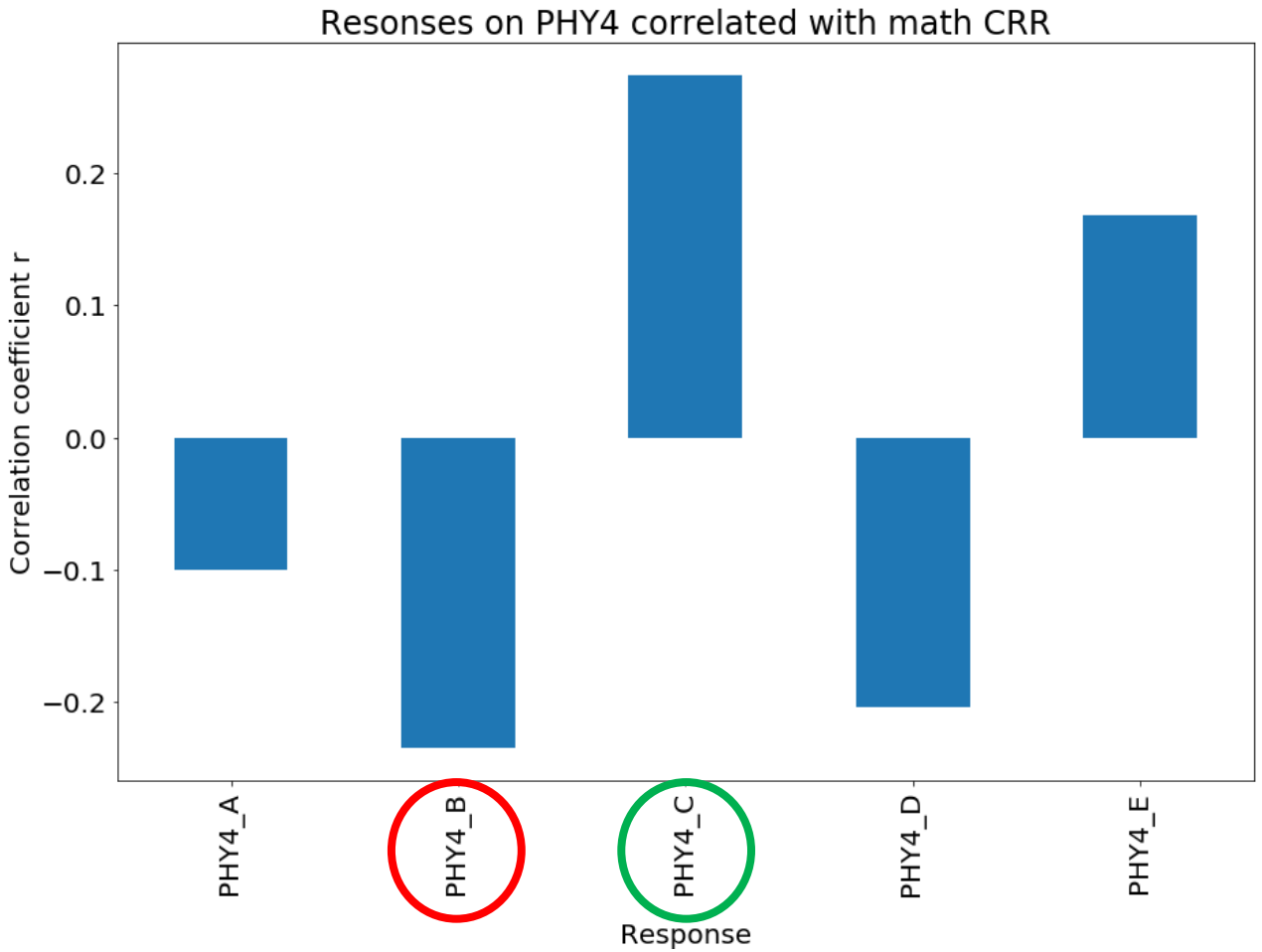
D. The woman exerts a force on the car but the car does not exert any force on the woman

E. The car exerts a force on the woman but the woman does not exert any force on the car.

Online diagnostic physics problems

Newton's second law (PHY4)



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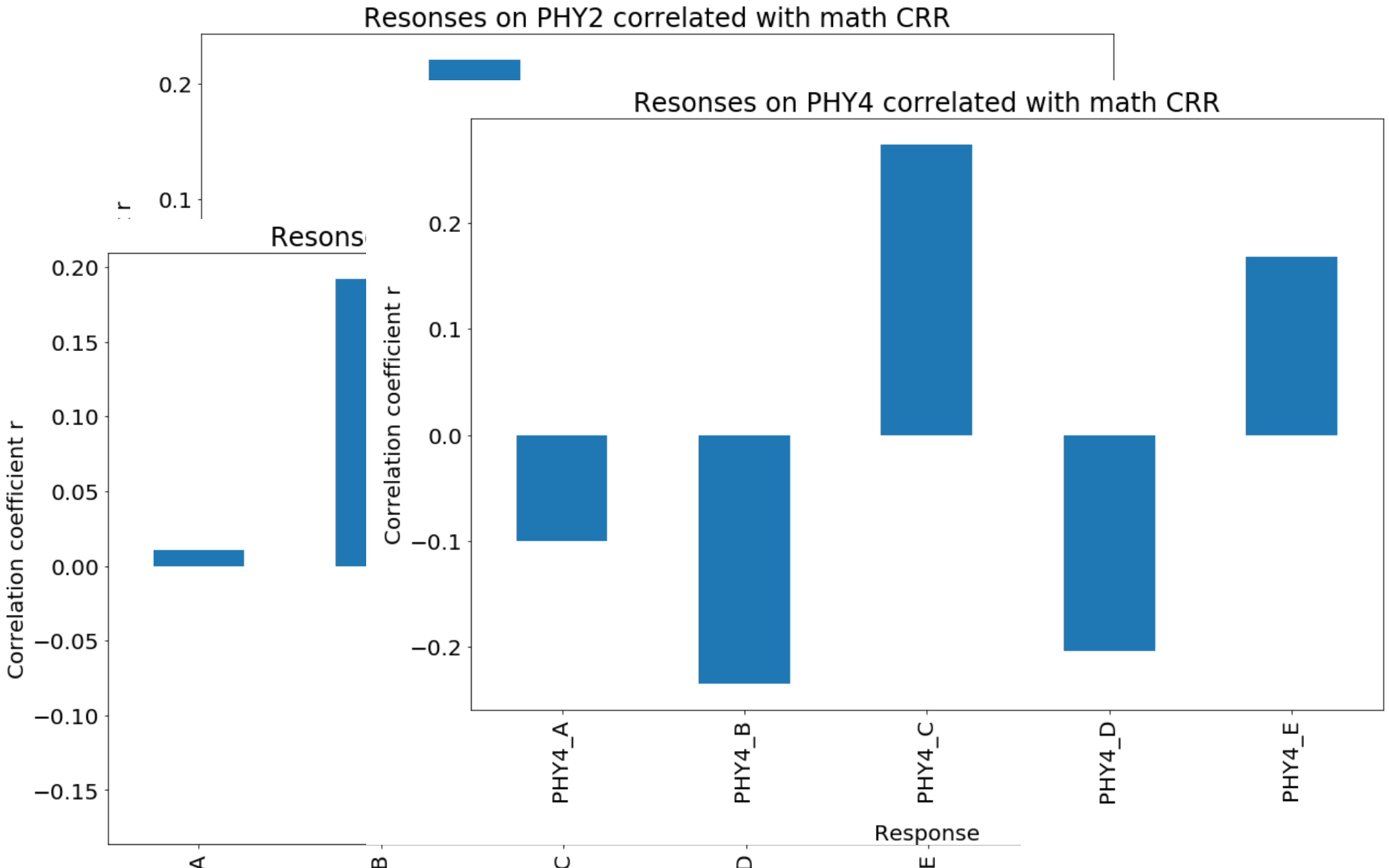
B. The sled speeds up while the line pulls and slows down after the line breaks.

C. The sled speeds up while the line pulls and moves at constant speed after the line breaks.

D. The sled speeds up while the line pulls and keeps gaining speed after the line breaks, but not as rapidly as before.

E. The sled moves at constant speed the whole time.

Online diagnostic physics problems



Online diagnostic physics problems

Math vs physics performance

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B. An upward force from the rope that is equal in magnitude to the downward force of gravity

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D. Only the downward force of gravity

E. Only an upward force from the rope

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C. The force of the woman on the car is smaller than the force of the car on the woman

D. The woman exerts a force on the car but the car does not exert any force on the woman

E. The car exerts a force on the woman but the woman does not exert any force on the car.

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B. The sled speeds up while the line pulls and slows down after the line breaks.

C. The sled speeds up while the line pulls and moves at constant speed after the line breaks.

D. The sled speeds up while the line pulls and keeps gaining speed after the line breaks, but not as rapidly as before.

E. The sled moves at constant speed the whole time.

Online diagnostic physics problems
Newton's second law (PHY1)

- Key concept:
  - The $\sum \vec{F} = 0$ when an object is moving at a constant unchanging speed
- Steps:
  1. Assess the object's motion
  2. Reason what combination of forces are causing the motion

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A. An upward force from the rope that is larger than the downward force of gravity

B. An upward force from the rope that is equal in magnitude to the downward force of gravity

C. An upward force from the rope that is smaller than the downward force of gravity

D. Only the downward force of gravity

E. Only an upward force from the rope

Online diagnostic physics problems
Newton's second law (PHY2)

- Key concept:
  - The $\sum \vec{F} = constant$ when an object is moving with a constant acceleration
- Steps:
  1. Assess the object's motion
  2. Reason what combination of forces are causing the motion

A rock, attached to a rope, is being pulled straight up; it is speeding up at a steady rate (with constant acceleration) while it is being pulled. The downward force of gravity on the rock is nearly constant and unchanging. The upward force from the rope is:

A. Larger than the force of gravity, and increasing in strength

B. Larger than the force of gravity, and nearly constant in strength

C. Equal to the force of gravity

D. Smaller than the force of gravity, and decreasing in strength

E. Smaller than the force of gravity, and nearly constant in strength

Online diagnostic physics problems
Newton's third law (PHY3)

- Key concept:
  - ➢ **Newton's third law**
- Steps:
  1. Assess the objects' motion
  2. Use Newton's third law to understand what forces are acting on each object

A small woman is pushing on a stalled car and the car is very gradually speeding up. While she is pushing on the car,

A. The force of woman on the car is larger than the force of the car on the woman

B. The force of the woman on the car is equal in magnitude to the force of the car on the woman

C. The force of the woman on the car is smaller than the force of the car on the woman

D. The woman exerts a force on the car but the car does not exert any force on the woman

E. The car exerts a force on the woman but the woman does not exert any force on the car.

Online diagnostic physics problems

Newton's second law (PHY4)

- Key concepts:
  - When $\sum \vec{F} = constant$ the object is accelerating
  - When $\sum \vec{F} = 0$ the object moves at a constant speed
- Steps:
  1. Assess the forces
  2. Reason what motion would be caused by the forces

A child is sitting on a sled that is stranded in the middle of a frozen pond. The ice is so wet and slippery that ***friction can be assumed to be zero***. Workers in a truck on land attach a fishing line to the sled and pull on the line; a scale attached to the line ensures that it pulls with unchanging force. However, after a few moments the line breaks. Then:

A. The sled moves at constant speed while the line pulls and at a constant but slower speed after the line breaks.

B. The sled speeds up while the line pulls and slows down after the line breaks.

C. The sled speeds up while the line pulls and moves at constant speed after the line breaks.

D. The sled speeds up while the line pulls and keeps gaining speed after the line breaks, but not as rapidly as before.

E. The sled moves at constant speed the whole time.