Investigating and Addressing Physics Students' Mathematical Difficulties

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Undergraduate Student Research Assistants

- Dakota King (undergraduate and recent graduate)
- Matt Jones (now at Dartmouth U.)
- John Byrd (now at Michigan State U.)

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The Pedagogical Challenge

- Difficulties with basic math skills impact performance of introductory physics students
- The difficulties are often not resolved by students’ previous mathematical training
- Students can’t effectively grapple with physics ideas when they feel overburdened in dealing with calculational issues

Development of Students' Mathematical Thinking

- Most college physics students receive their initial mathematical preparation in middle school and high school, therefore…
- …the "mathematical landscape" of physics students' thinking must be traced back to these formative years…

Studies of Physics Students' Math Skills

- Beginning in 1918 and continuing today, investigators have probed physics students' mathematics preparation and asked whether it's adequate for college physics.
- Many mathematics diagnostic tests have been administered to high school and college physics students.

Representative Results from Diagnostic Tests

- **Hughes (1924)** argued that poor math performance by university students showed that it was *not* possible to "mathematize" high school physics to any great extent and still get satisfactory achievement.
- **Lohr (1925)** concluded that it was necessary for university physics teachers to "re-teach until [they are] sure of assimilation of the mathematics involved before attempting to give the physics using these principles."
- **Kilzer (1929)** concluded that there was a need for "maintenance drills" covering the math needed in high school physics courses.
- **Breitenberger (1992)** found that new physics *graduate* students were deficient in math skills and mathematical thinking!

Probes of Math's Impact on Physics Performance…

- **Bless (1932)** found a very high correlation between university students' physics grades and their scores on an arithmetic/algebra diagnostic test.
- **Carter (1932)** found a similarly high correlation among high school students.
  - *However*, he noted that the correlation was sharply reduced when student's "intelligence" (determined by an IQ test) was held constant
- **Kruglak and Keller (1950)** found a high correlation between math course grades and physics course grades of university students.
- **Halloun and Hestenes (1985)** found a high correlation (+0.51) between math pretest scores and physics grades, and that math scores were a factor *independent* of physics pretest scores
- **Meltzer (2002)** found that algebra pretest scores were roughly predictive of performance improvements on a *non-quantitative* physics concept test

But the Problem is More Complicated…

- Weak calculational skills are only part of the problem.
- Many early studies were flawed by conflating difficulties with *physics* concepts together with weak mathematical skills, and presuming the combination was “problems with math.”
- Up until the 1970s, there was virtually no research on which to base efforts to improve the situation.

Lillian McDermott and the Physics Education Group at the University of Washington

- Lillian McDermott and the University of Washington Physics Education Group (PEG) demonstrated that physics students' mathematical skills, physics ideas, and reasoning abilities are not easily disentangled, and must often be studied *together*, in the context of authentic physical systems.
- The PEG investigated students' abilities to work with multiple representations of physics ideas, including graphs and diagrams.

Some Examples of the PEG's Work

- Trowbridge and McDermott (1981): Probed students' thinking regarding ratios of differences, e.g. $\Delta v/\Delta t$
  - Distinguishing between a quantity (*v*), *change* in that quantity (Δv), and ratios of changes ($\Delta v/\Delta t$) is always challenging, but confusion about the distinction between velocity and acceleration introduced additional obstacles

- McDermott, Rosenquist, and Van Zee (1987): Investigated students' ideas about graphical representations of motion (position-, velocity-, and acceleration-time graphs)
  - Students' difficulties in graphical interpretation were exacerbated by misleading intuitions drawn from objects' physical trajectories.

Overview: Requirements for Successful Application of Math to Physics

1. Understanding of mathematical **concepts**
2. Technical skill with mathematical **procedures**
3. Ability to apply in **physical context**
4. Ability to apply in **problem-solving context**

Our Approach

- ***Assess*** nature and scope of difficulties using written and on-line diagnostic instruments, as well as one-on-one oral interviews.
- ***Address*** students' mathematical difficulties within the context of physics classes themselves, using in-class and out-of-class instructional materials.
  - In collaboration with Andrew Heckler's group at Ohio State University, using the "Stemfluency" online practice tool.

Data Sources

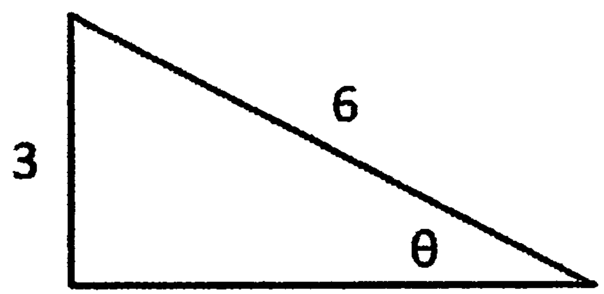
We have given diagnostic pretests covering pre-college mathematics to over 7000 introductory physics students (non-credit; calculators allowed):

- Results from five campuses at four different state universities were consistent with each other
- Results on an online version are consistent with those on the written version
- High and low scores on the diagnostic are somewhat predictive of course grades

In addition, we have carried out more than 70 one-on-one problem-solving interviews with physics students to further explore the nature of students' thinking.

Examples of Test Items

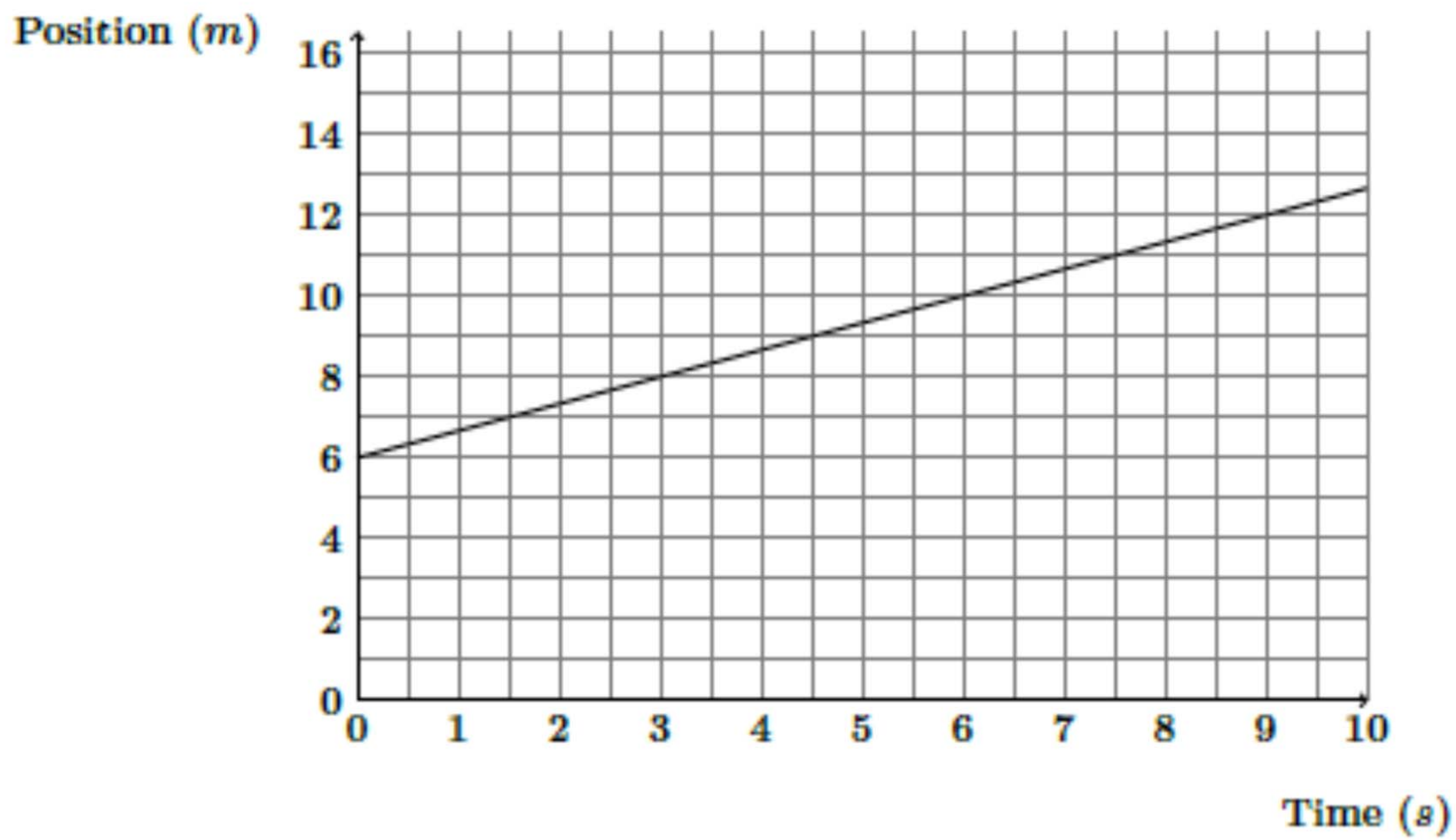
Find Unknown Angle



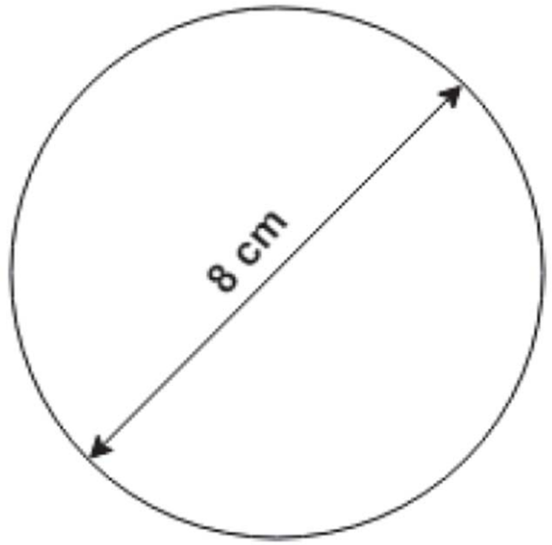
What is the value of θ?

Find Slope of Graph

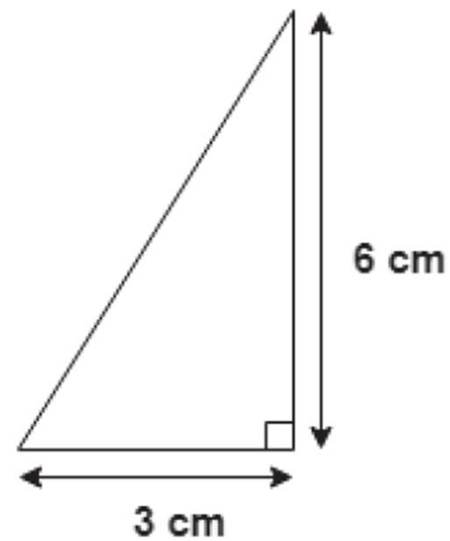
What is the slope of the graph below?



Find Area



(a) Area of the circle =



(b) Area of the triangle =

Simultaneous Equations, Symbolic Coefficients

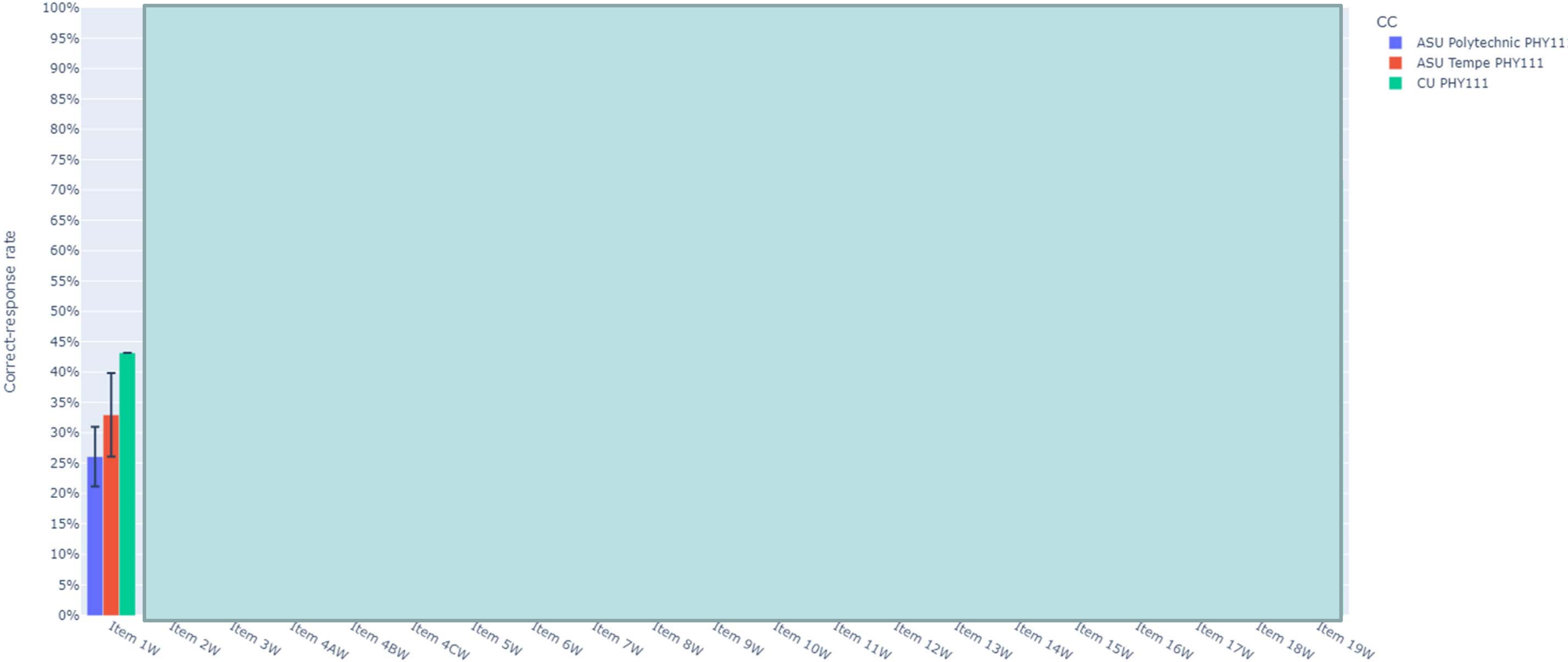
$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

High consistency of results among five campuses at four different universities (three campuses shown below) suggests findings are generalizable

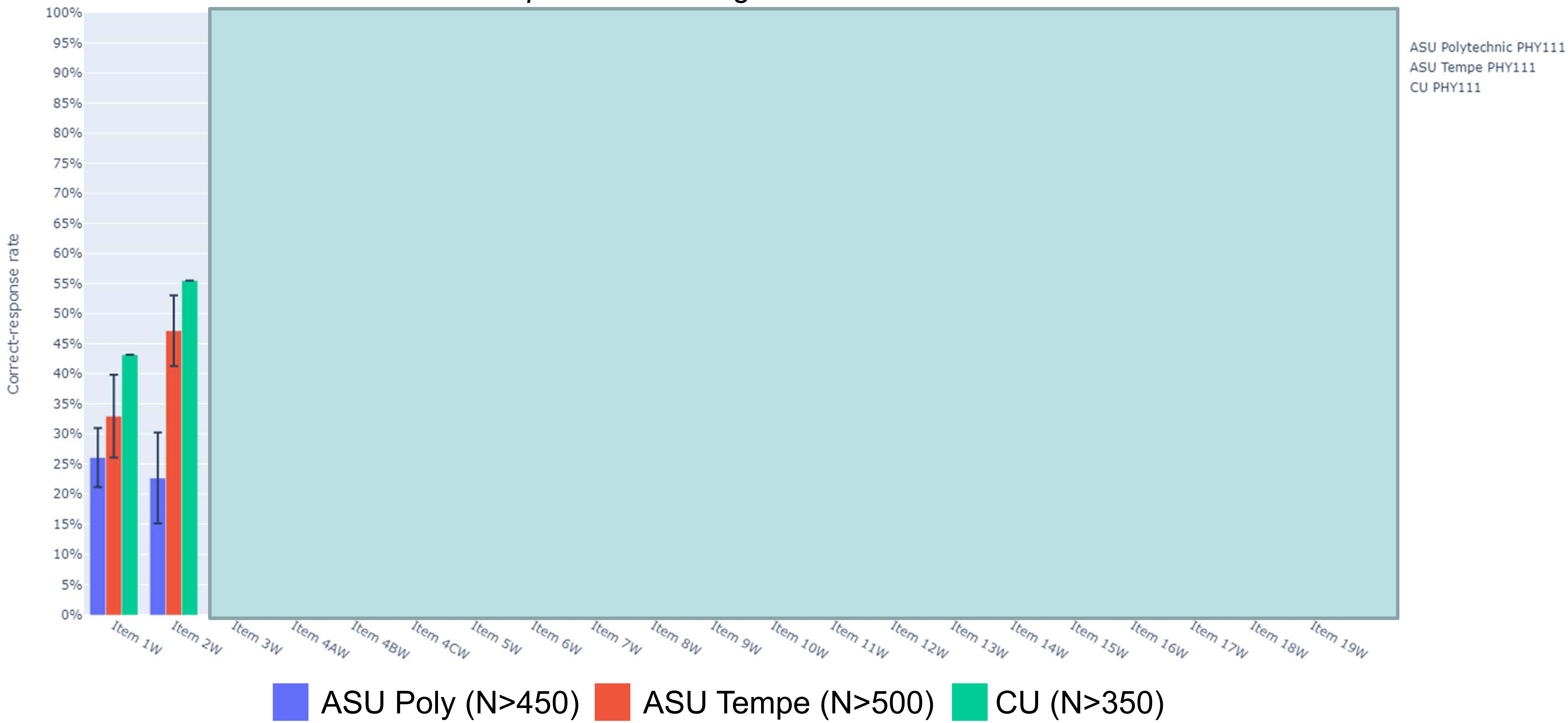
Correct-response rates: algebra-based course



ASU Poly (N>450) ASU Tempe (N>500) CU (N>350)

High consistency of results among five campuses at four different universities (three campuses shown below) suggests findings are generalizable

Correct-response rates: algebra-based course



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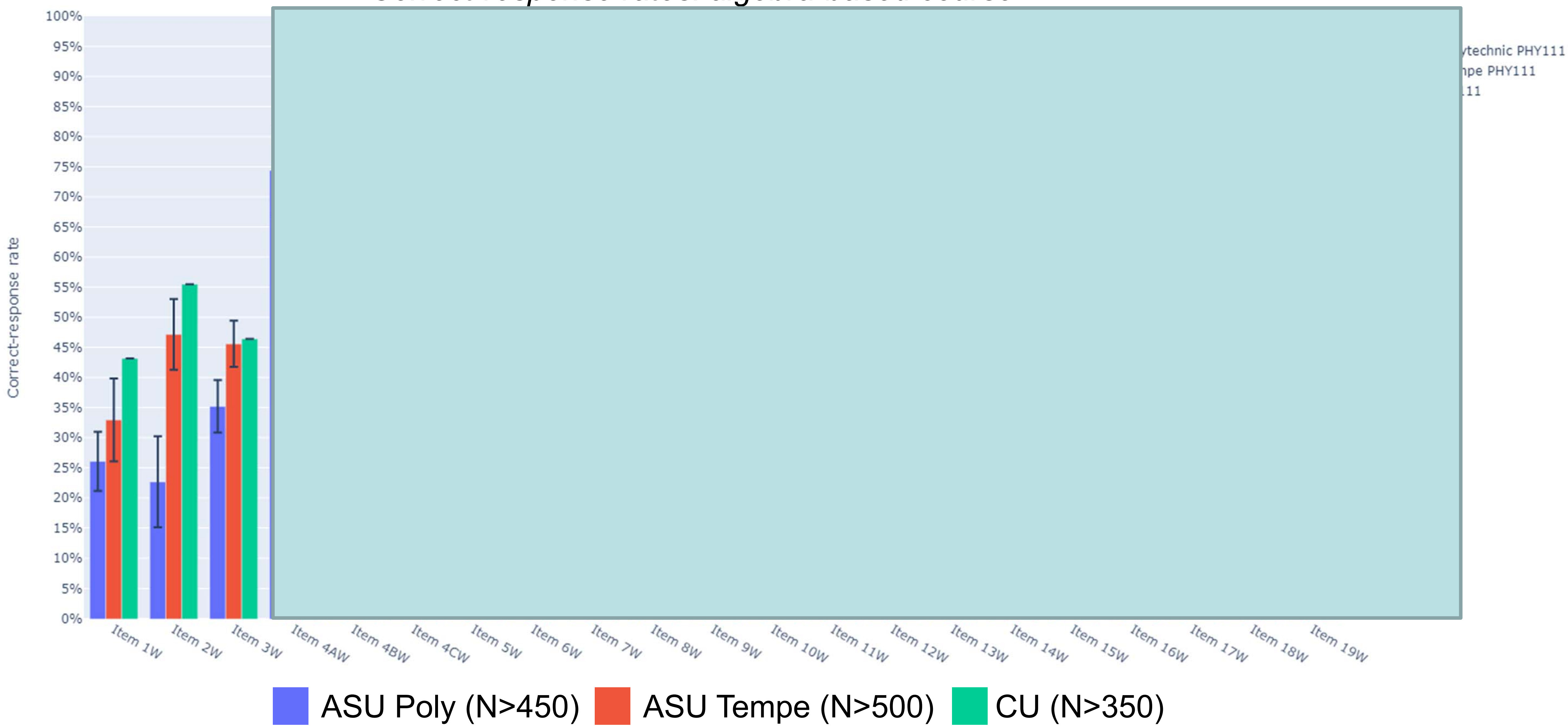

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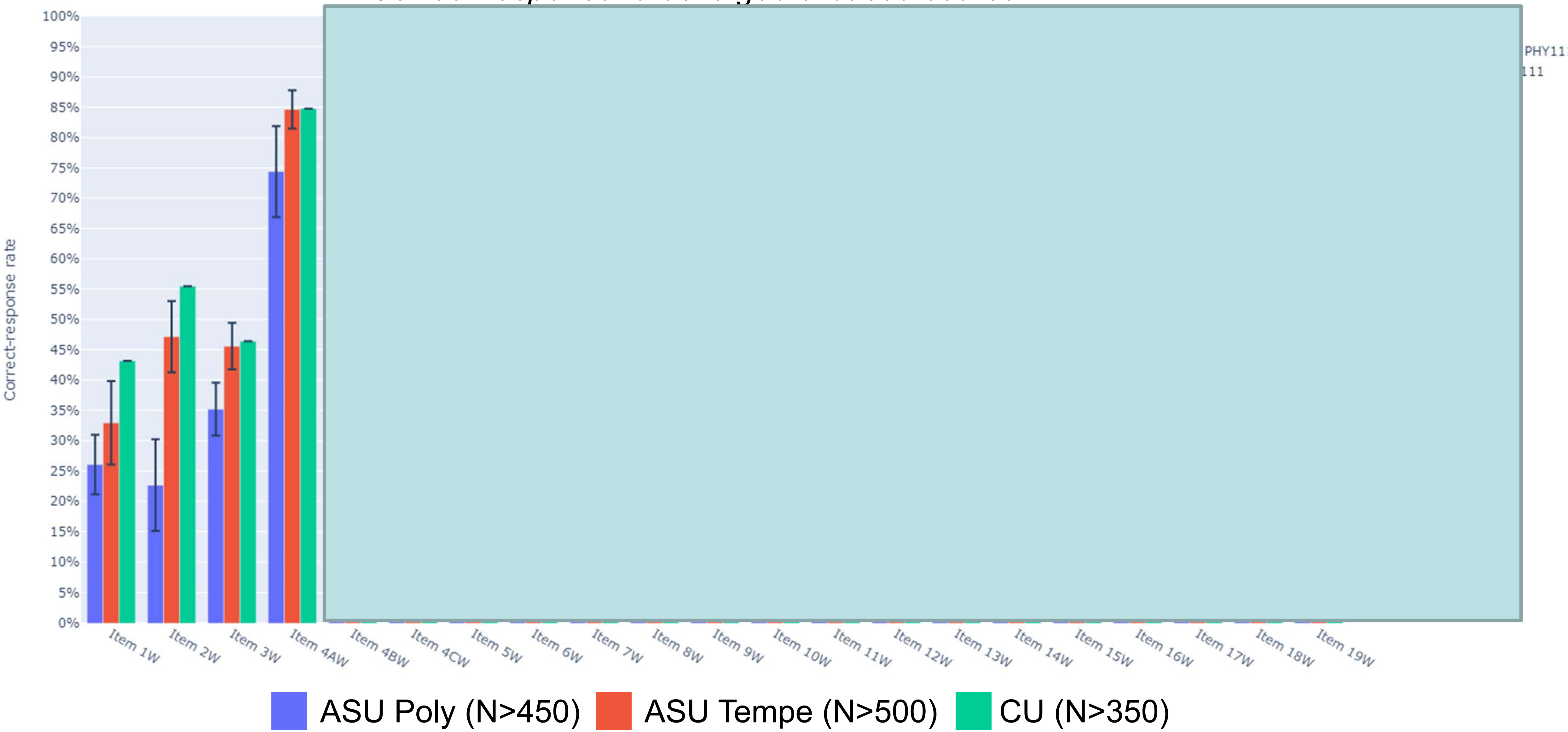

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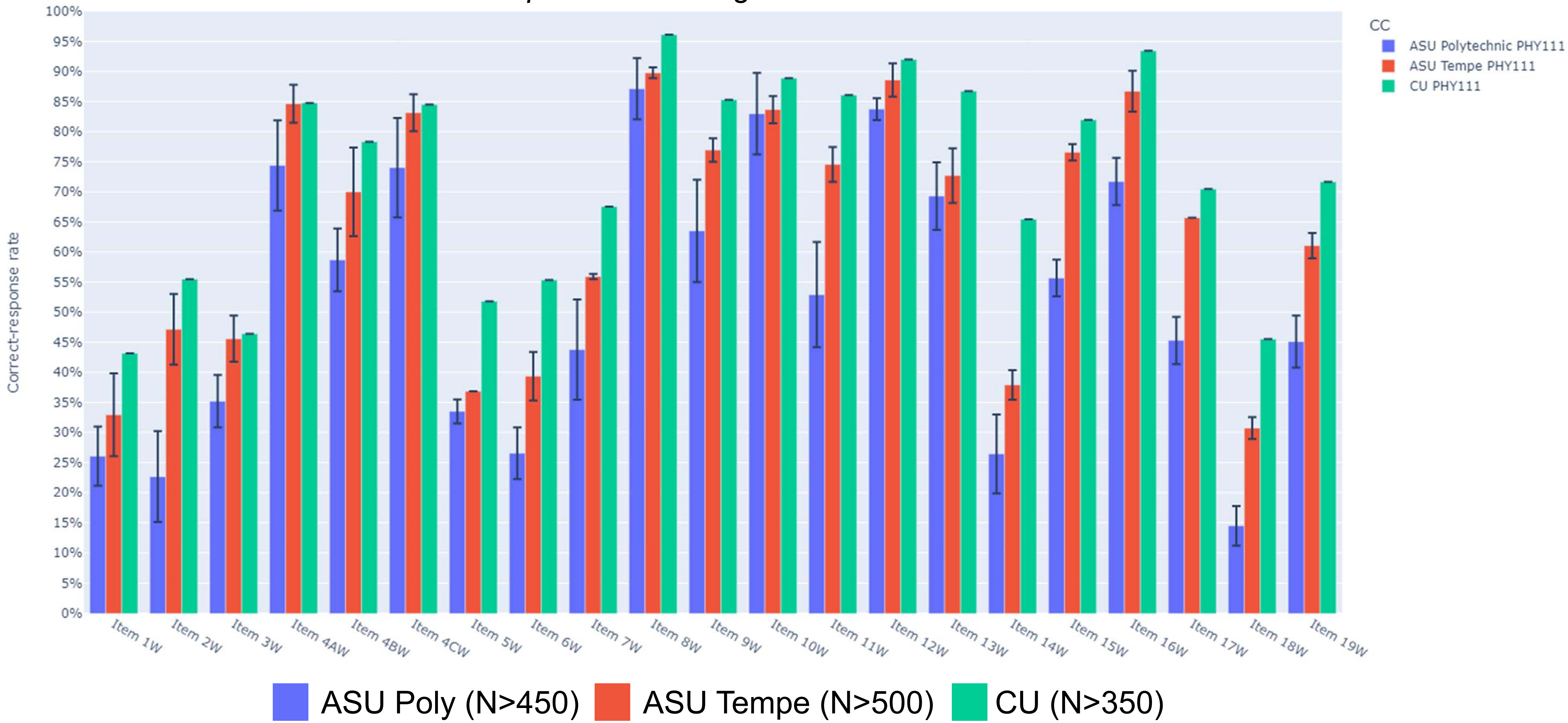
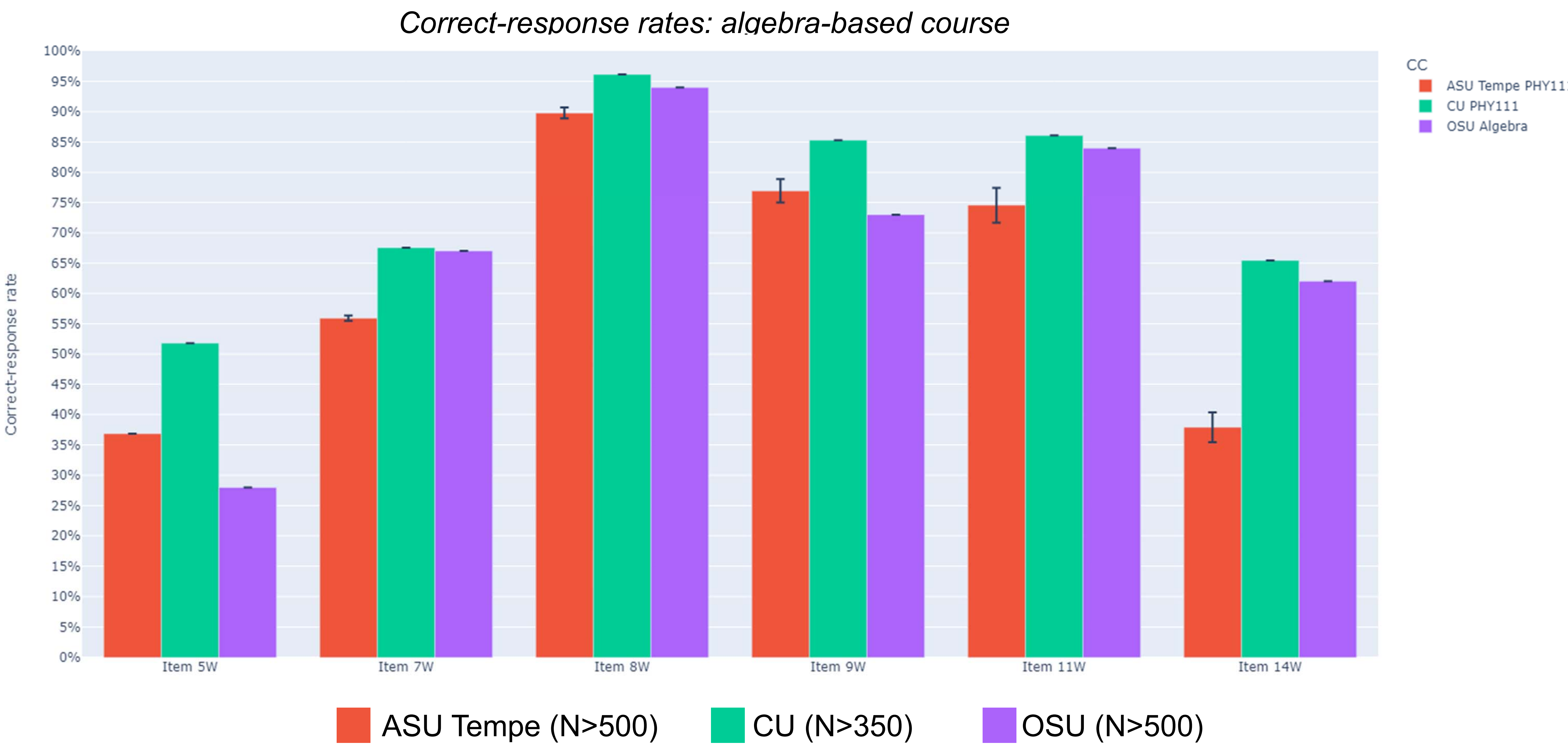
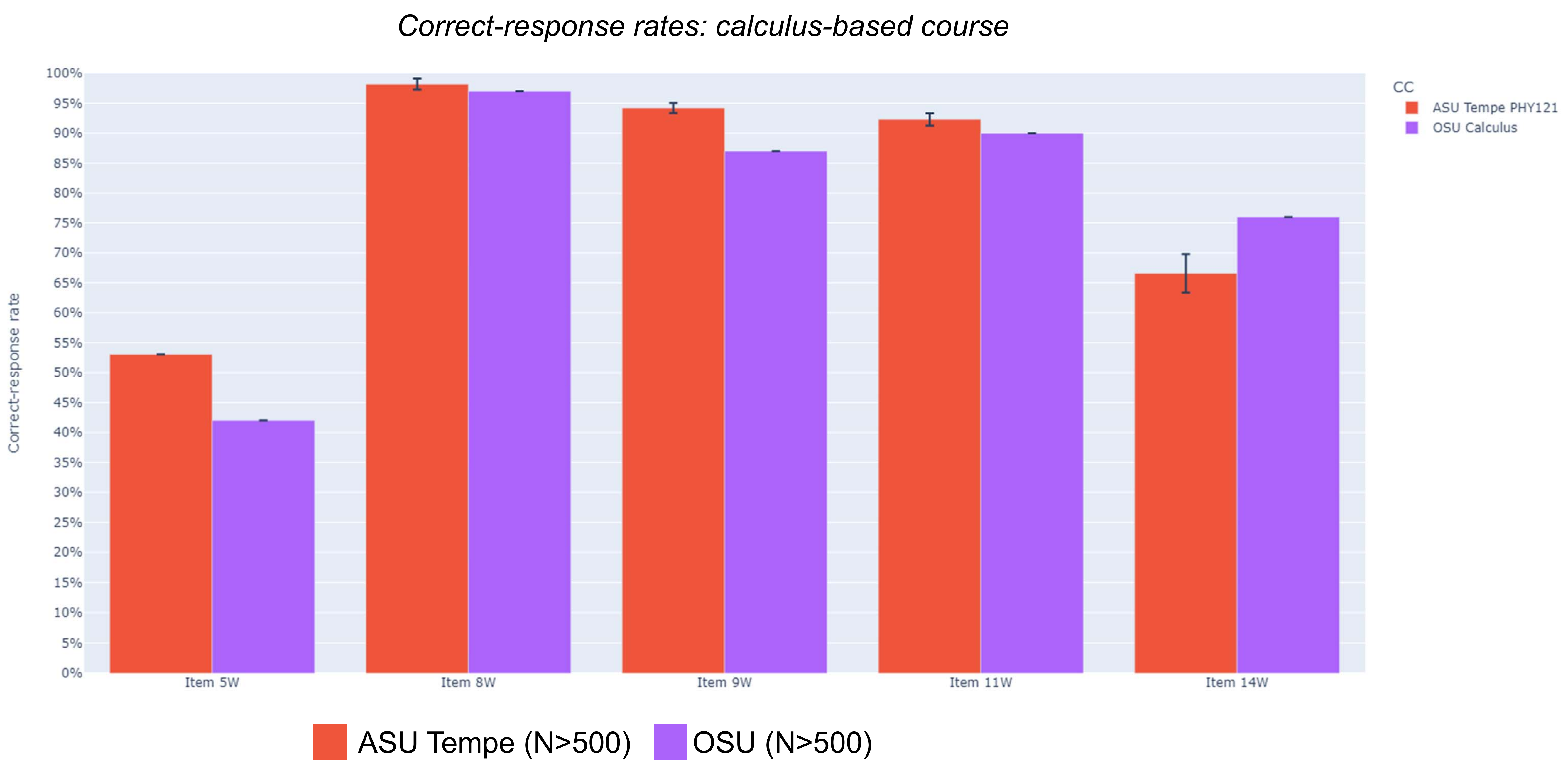
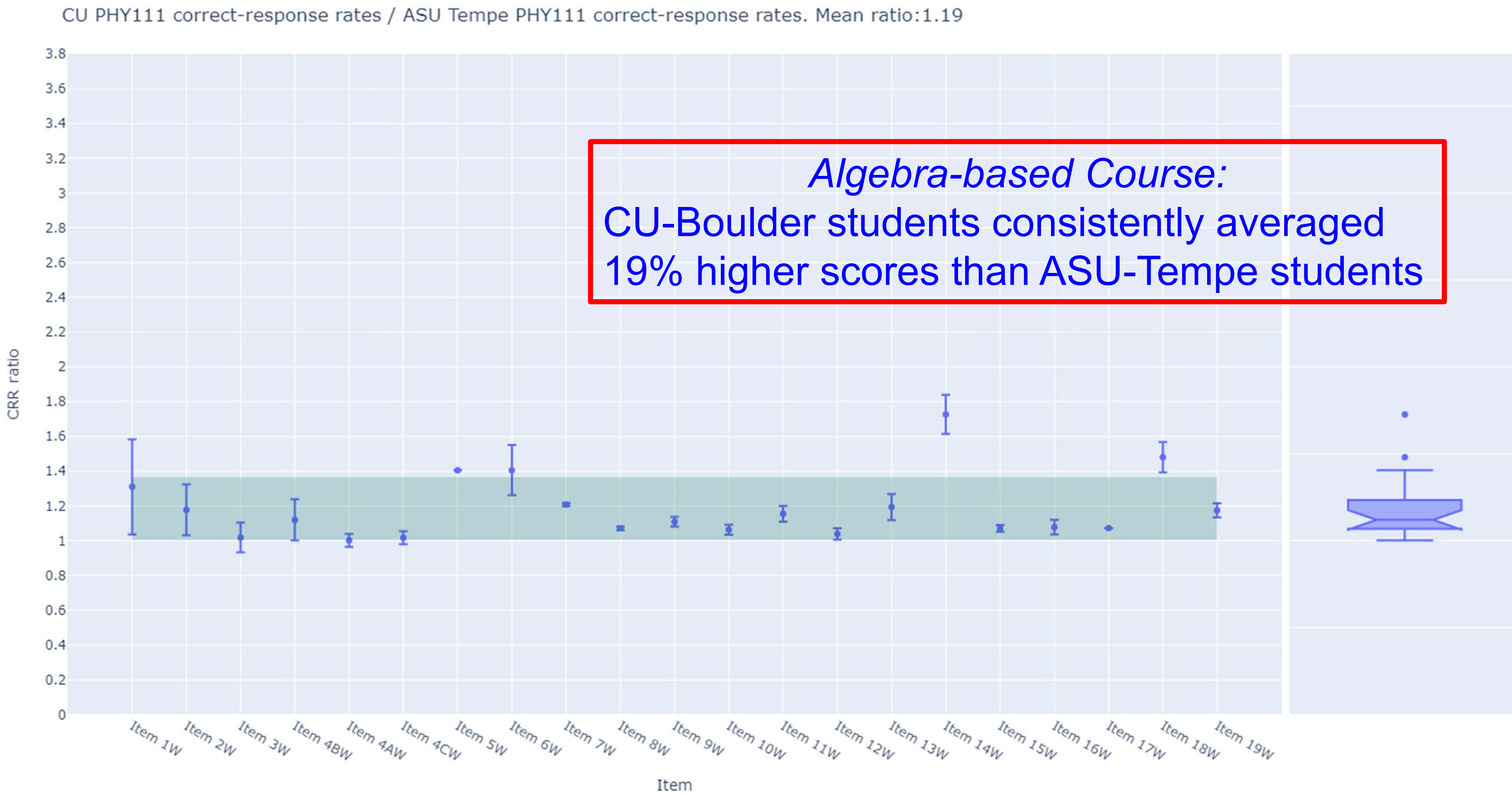


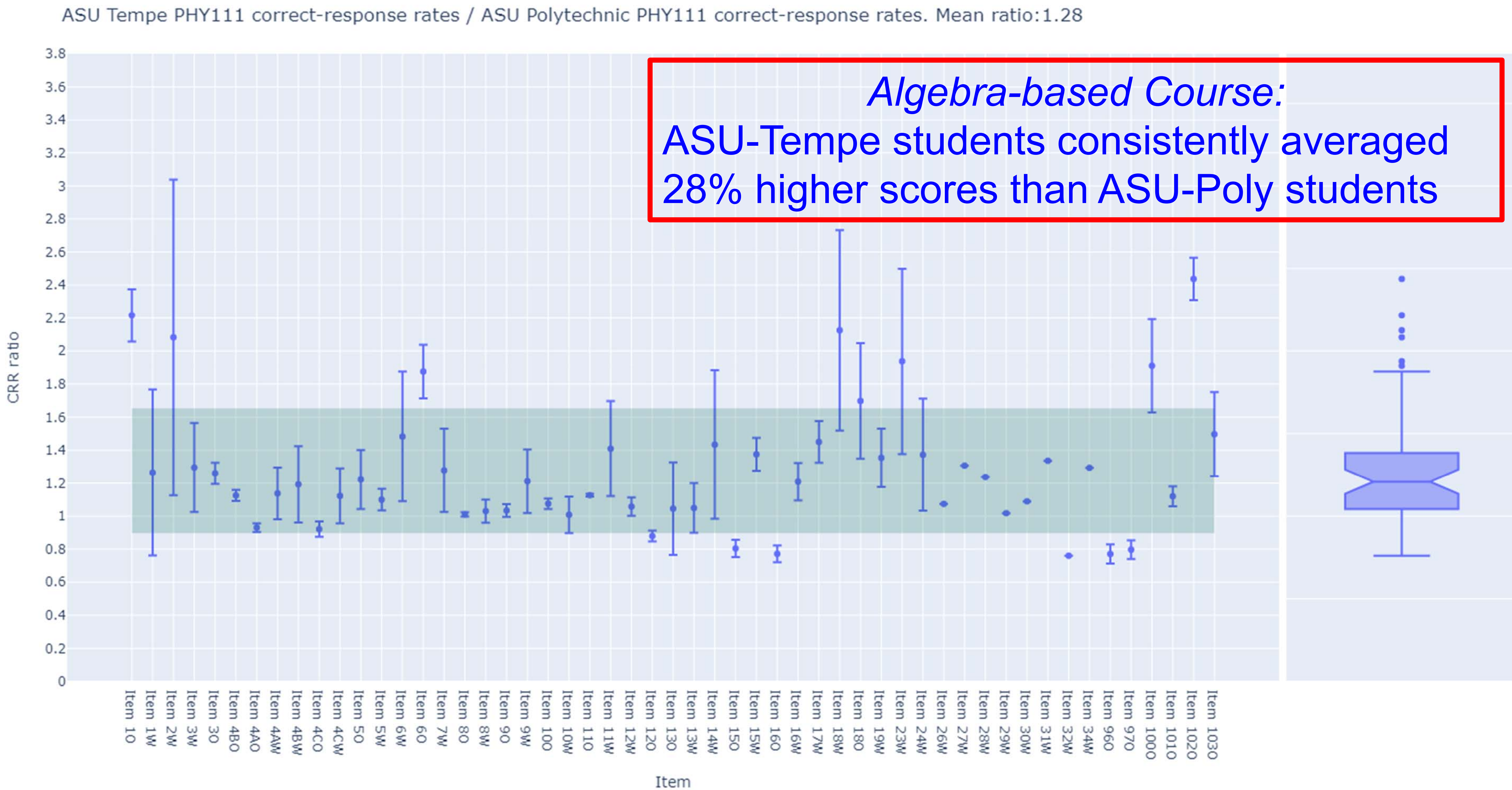


Correct-response rates: algebra-based course
Correct-response rate
100%
95%
90%
85%
80%
75%
70%
65%
60%
55%
50%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
Item 5W
Item 7W
Item 8W
Item 9W
Item 11W
Item 14W
CC
ASU Tempe PHY111
CU PHY111
OSU Algebra
ASU Tempe (N>500)
CU (N>350)
OSU (N>500)


Correct-response rates: calculus-based course
Correct-response rate
100%
95%
90%
85%
80%
75%
70%
65%
60%
55%
50%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
Item 5W
Item 8W
Item 9W
Item 11W
Item 14W
CC
ASU Tempe PHY121
OSU Calculus
ASU Tempe (N>500)
OSU (N>500)



CU PHY111 correct-response rates / ASU Tempe PHY111 correct-response rates. Mean ratio:1.19
Algebra-based Course:
CU-Boulder students consistently averaged 19% higher scores than ASU-Tempe students
CRR ratio
3.8
3.6
3.4
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Item 1W
Item 2W
Item 3W
Item 4BW
Item 4AW
Item 4CW
Item 5W
Item 6W
Item 7W
Item 8W
Item 9W
Item 10W
Item 11W
Item 12W
Item 13W
Item 14W
Item 15W
Item 16W
Item 17W
Item 18W
Item 19W
Item

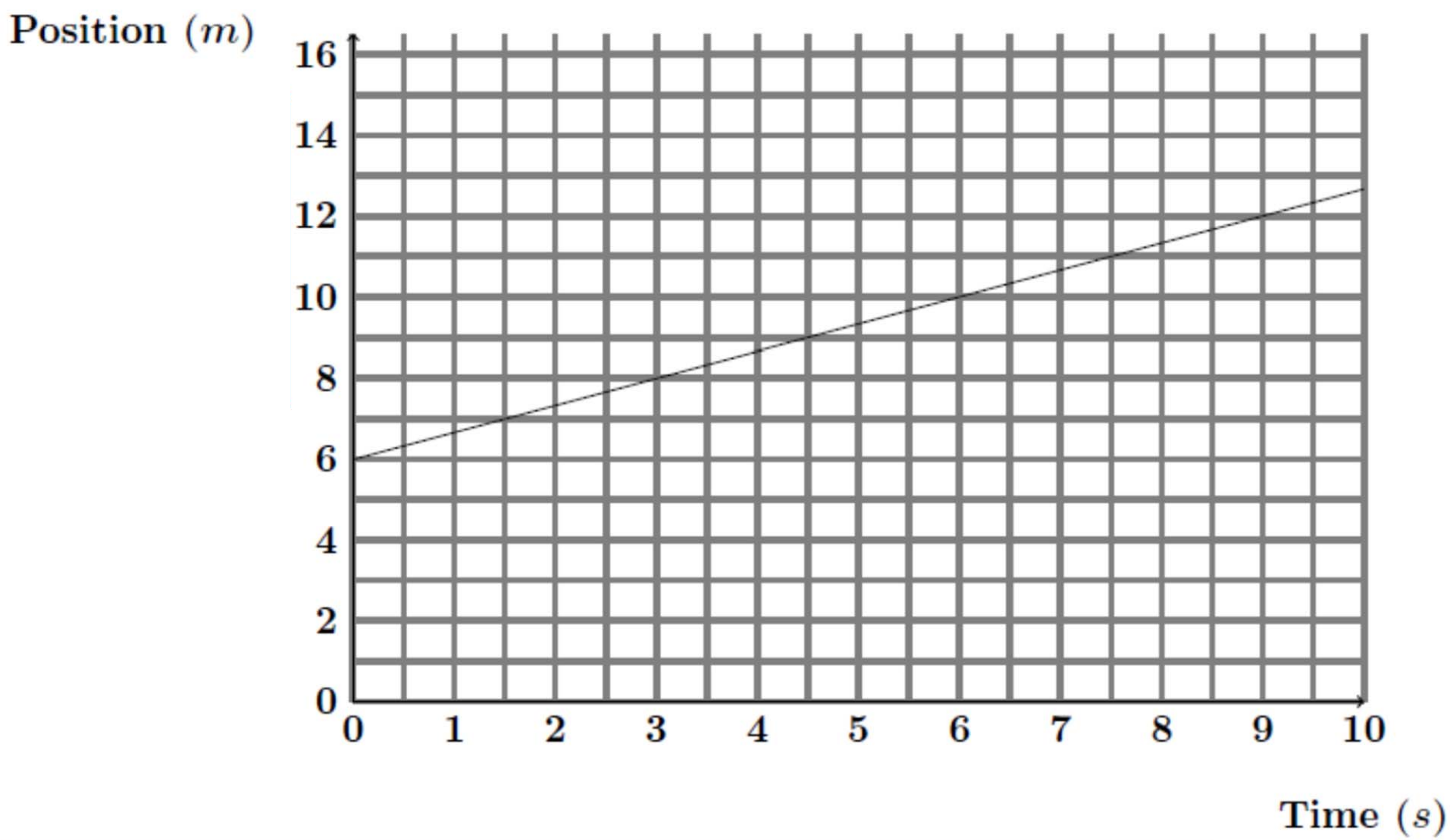

ASU Tempe PHY111 correct-response rates / ASU Polytechnic PHY111 correct-response rates. Mean ratio:1.28
CRR ratio
Item
Algebra-based Course:
ASU-Tempe students consistently averaged 28% higher scores than ASU-Poly students

Some Sample Results

Students show weakness with units and graphing

- Many students ignored graph-axis labels, and provided no or incorrect units for area and velocity.

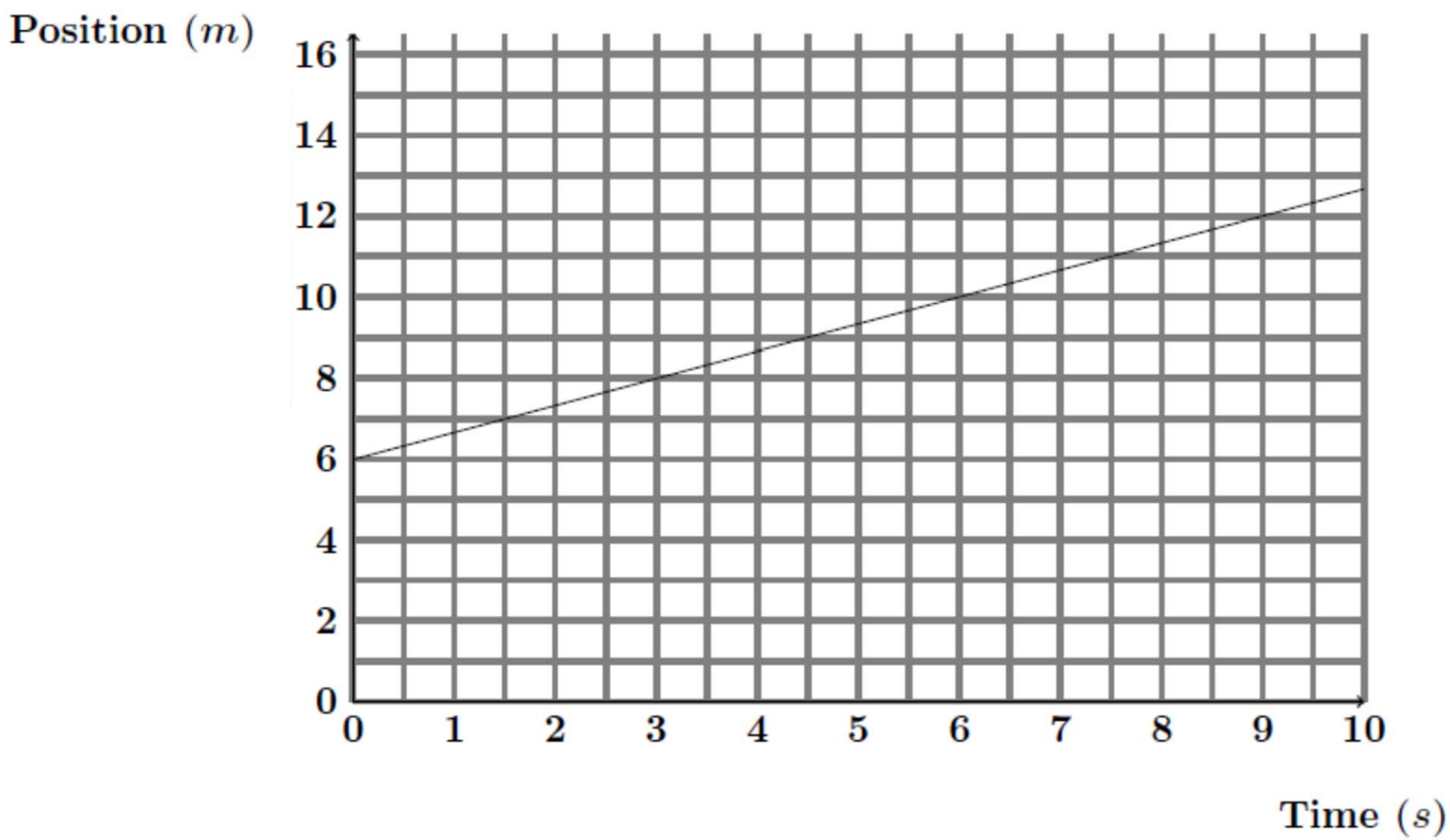
What is the slope of the graph below?



What is the slope of the graph below?

Correct-response rate (*N* > 2000):

30-60%, nearly independent of course or campus



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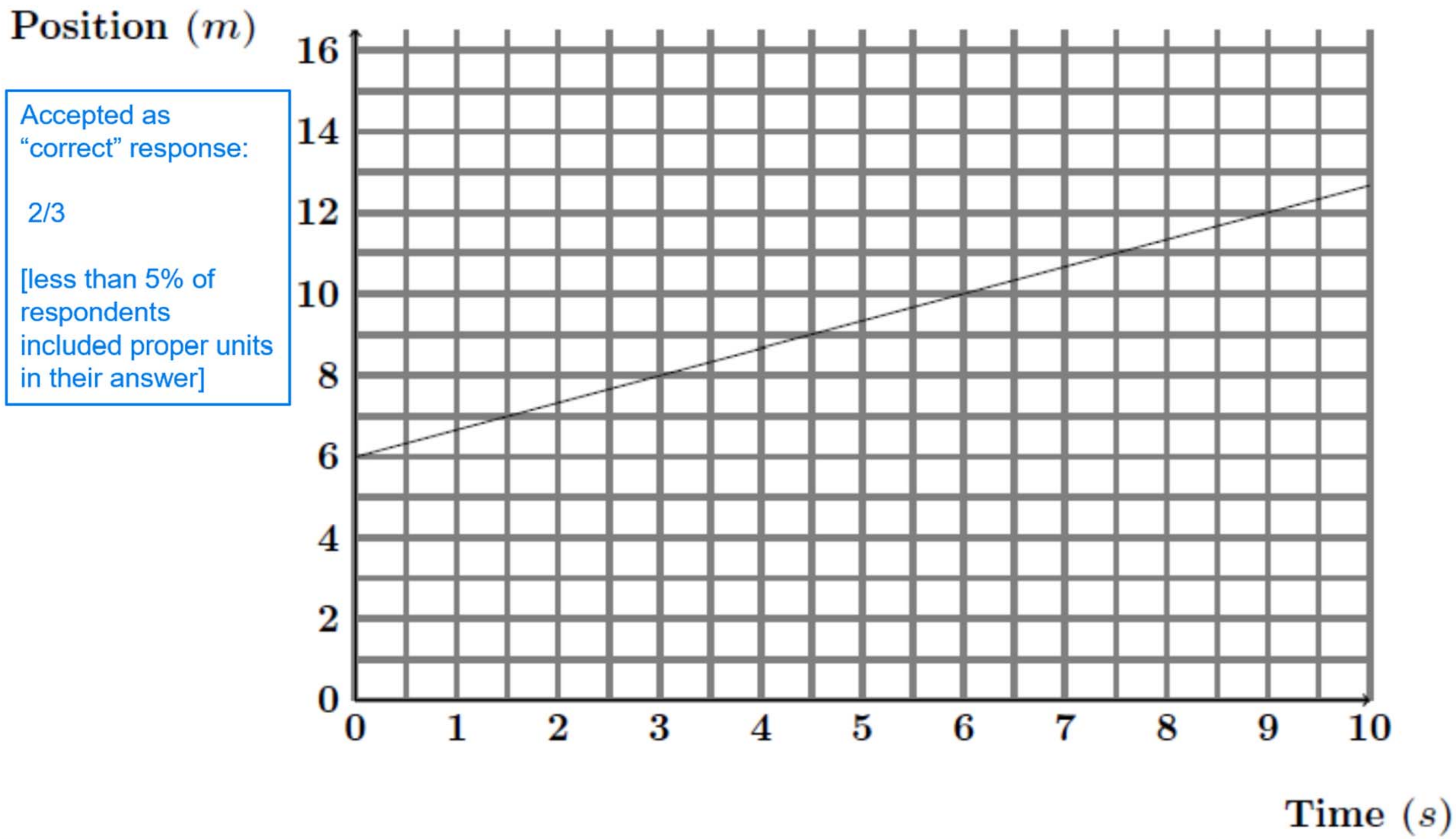
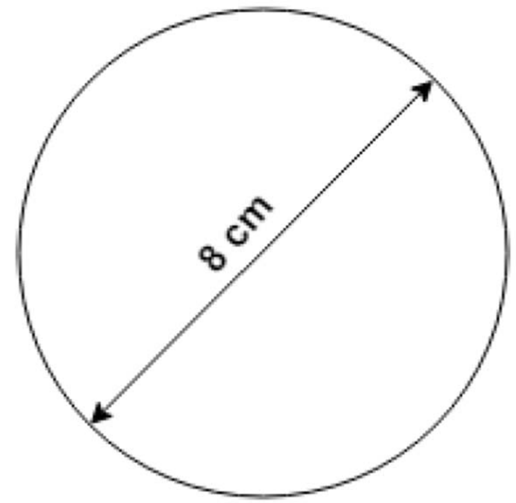


Accepted as "correct" response:

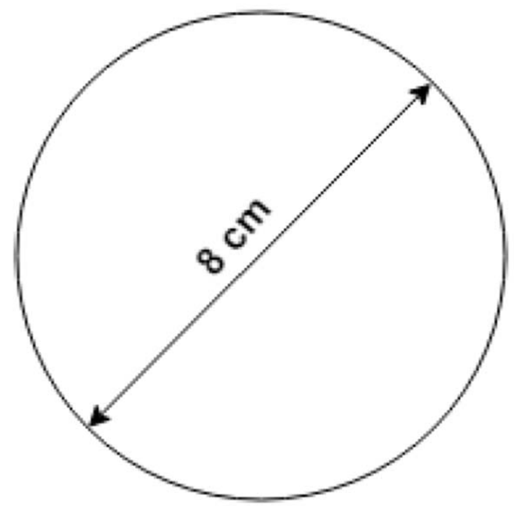
2/3

[less than 5% of respondents included proper units in their answer]

Most common error: *Counting grid squares and ignoring numbers on axes*

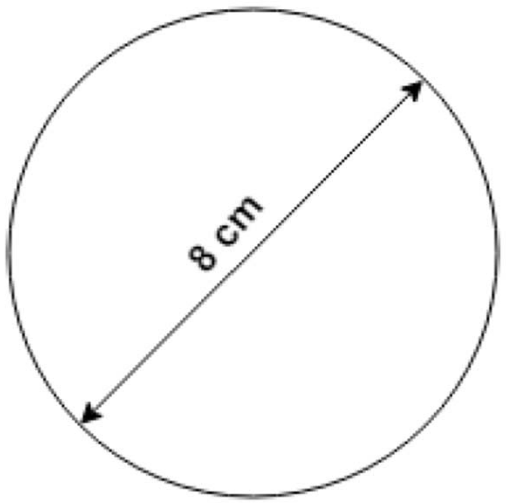


(a) **Area of the circle** =



(a) Area of the circle =

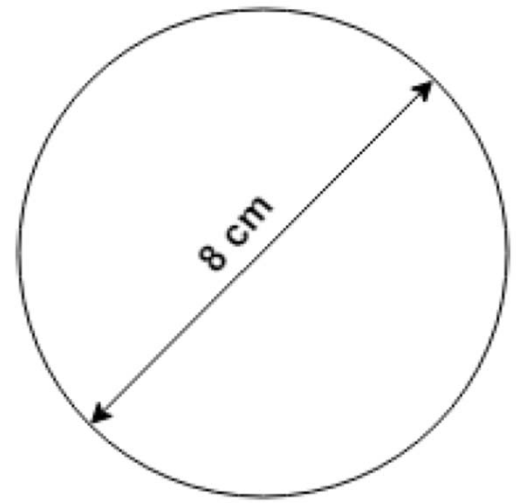
Area of Circle: Algebra- and Calculus-based courses combined, 2018



(a) Area of the circle =

Area of Circle: Algebra- and Calculus-based courses combined, 2018

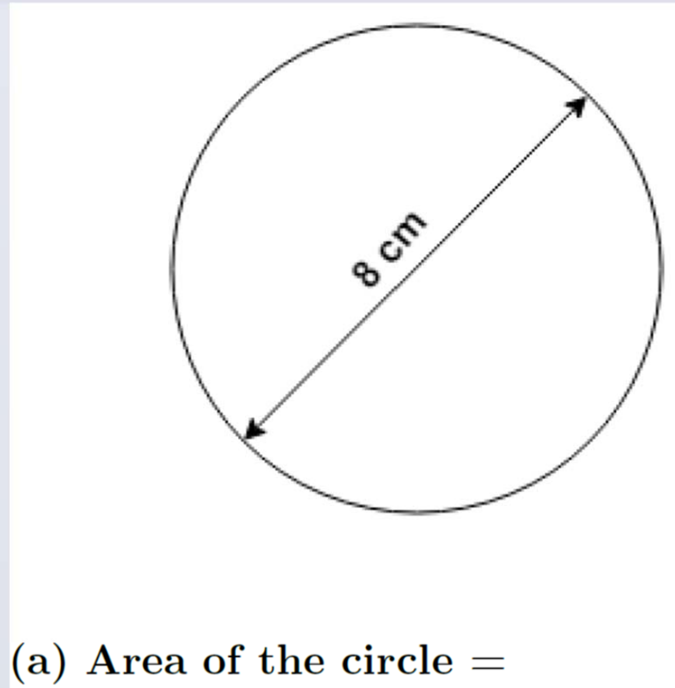
	N	Numerically correct
ASU-Polytechnic	250	57%
ASU-Tempe	1086	76%



(a) Area of the circle =

Area of Circle: Algebra- and Calculus-based courses combined, 2018

	N	Numerically correct	Correct with correct units
ASU-Polytechnic	250	57%	29%
ASU-Tempe	1086	76%	45%



Area of Circle: Algebra- and Calculus-based courses combined, 2018

	N	Numerically correct	Correct with correct units
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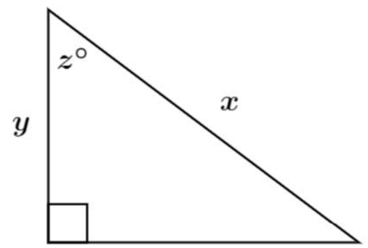
On-line Version

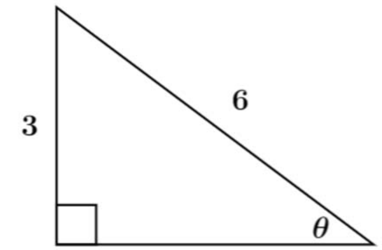
What is the length of side x?



A. $ycos(z°)$
B. $ysin(z°)$
C. $ytan(z°)$
D. $y/cos(z°)$
E. $y/sin(z°)$
F. $y/tan(z°)$
G. $cos(z°)/y$
H. $sin(z°)/y$
I. $tan(z°)/y$
J. $\sqrt{y^2 + z^2}$
K. $\sqrt{z^2 - y^2}$
L. y/z

(There may be more than one correct answer, but please select only ONE answer.)

What is the value of θ?



A. $cos(3/6)$
B. $sin(3/6)$
C. $tan(3/6)$
D. $cos^{-1}(3/6)$
E. $sin^{-1}(3/6)$
F. $tan^{-1}(3/6)$
G. 30°
H. 45°
I. 60°
J. 27°
K. 3/6
L. 0.524

(There may be more than one correct answer, but please select only ONE answer.)

$cos(0°) = ?$

A. 0 B. 1 C. undefined D. 0.707 E. 0.894

(There may be more than one correct answer, but please select only ONE answer.)

$sin(90°) = ?$

A. 0 B. 1 C. undefined D. 0.707 E. 0.894

(There may be more than one correct answer, but please select only ONE answer.)

$tan(0°) = ?$

A. 0 B. 1 C. undefined D. 0.707 E. 0.894

(There may be more than one correct answer, but please select only ONE answer.)

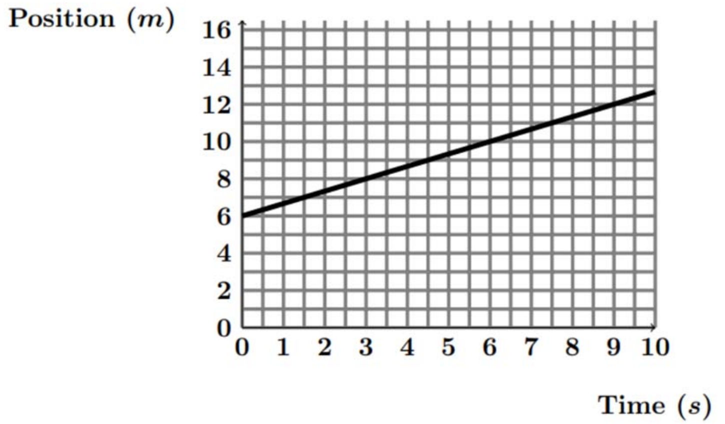
Solve for θ.

$$\gamma\theta + \eta = \lambda\theta + \omega$$

A. $\frac{\eta + \omega}{\gamma - \lambda}$
B. $\frac{\eta - \omega}{\lambda - \gamma}$
C. $\frac{\gamma - \lambda}{\omega - \eta}$
D. $\frac{\lambda - \gamma}{\eta - \omega}$
E. $\frac{\eta - \omega}{\gamma\lambda}$
F. $\frac{\omega - \eta}{\gamma\lambda}$
G. $\frac{\omega - \eta}{\gamma - \lambda}$
H. $\frac{\omega - \eta}{\gamma + \lambda}$
I. $\frac{\eta - \omega + \gamma}{\lambda}$
J. $\frac{\omega - \eta + \lambda}{\gamma}$

(There may be more than one correct answer, but please select only ONE answer.)

What is the slope of the graph below?



A. $\frac{1}{3}$ m/s because the object moves 1 meter in 3 seconds.

B. $\frac{1}{3}$ m/s because the line rises 1 box while it goes 3 boxes in the horizontal direction.

C. $\frac{2}{3}$ m/s because the object moves 2 meters in 3 seconds.

D. $\frac{2}{3}$ m/s because the line rises 2 boxes while it goes 3 boxes in the horizontal direction.

(There may be more than one correct answer, but please select only ONE answer.)

$$\left(\frac{a}{3}\right)^3 = ?$$

A. $\frac{a^3}{3}$ B. $\frac{a}{27}$ C. $\frac{a^3}{27}$

(There may be more than one correct answer, but please select only ONE answer.)

$$2\left(\frac{a}{b}\right) = ?$$

A. $\frac{2a}{b}$ B. $\frac{2a}{2b}$ C. $\frac{a}{2b}$

(There may be more than one correct answer, but please select only ONE answer.)

$$\frac{a/b}{c^2/d} = ?$$

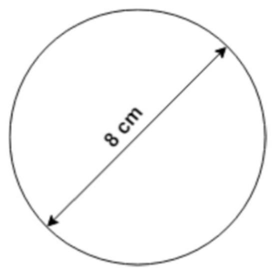
A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

(There may be more than one correct answer, but please select only ONE answer.)

$$2\left(\frac{3}{4}\right) = ?$$

A. $\frac{6}{8}$ B. $\frac{12}{8}$ C. $\frac{3}{8}$ D. $\frac{3}{2}$ E. $\frac{3}{4}$

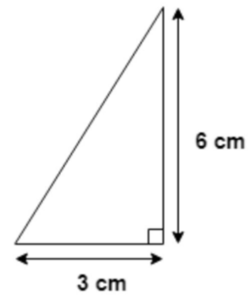
(There may be more than one correct answer, but please select only ONE answer.)



(a) Area of the circle = ?

A. 8π cm^3	F. 8π cm^2	K. 8π cm
B. 16π cm^3	G. 16π cm^2	L. 16π cm
C. 32π cm^3	H. 32π cm^2	M. 32π cm
D. 64π cm^3	I. 64π cm^2	N. 64π cm
E. 128π cm^3	J. 128π cm^2	O. 128π cm

(There may be more than one correct answer, but please select only ONE answer.)



(b) Area of the triangle = ?

A. 4.5 cm^3	F. 4.5 cm^2	K. 4.5 cm
B. 9 cm^3	G. 9 cm^2	L. 9 cm
C. 12 cm^3	H. 12 cm^2	M. 12 cm
D. 18 cm^3	I. 18 cm^2	N. 18 cm
E. 36 cm^3	J. 36 cm^2	O. 36 cm

(There may be more than one correct answer, but please select only ONE answer.)

Solve for x.

$$\frac{3}{2} = 7x$$

A. $\frac{14}{3}$ B. $\frac{3}{14}$ C. $\frac{21}{2}$ D. $\frac{21}{14}$

(There may be more than one correct answer, but please select only ONE answer.)

$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = 60$$

$$\Delta t = 8$$

$$v = 30$$

$$d = ?$$

A. $d = 30$ B. $d = 60$ C. $d = 120$ D. $d = 240$ E. $d = 480$

(There may be more than one correct answer, but please select only ONE answer.)

$$cy = dx$$

$$a - y = bx$$

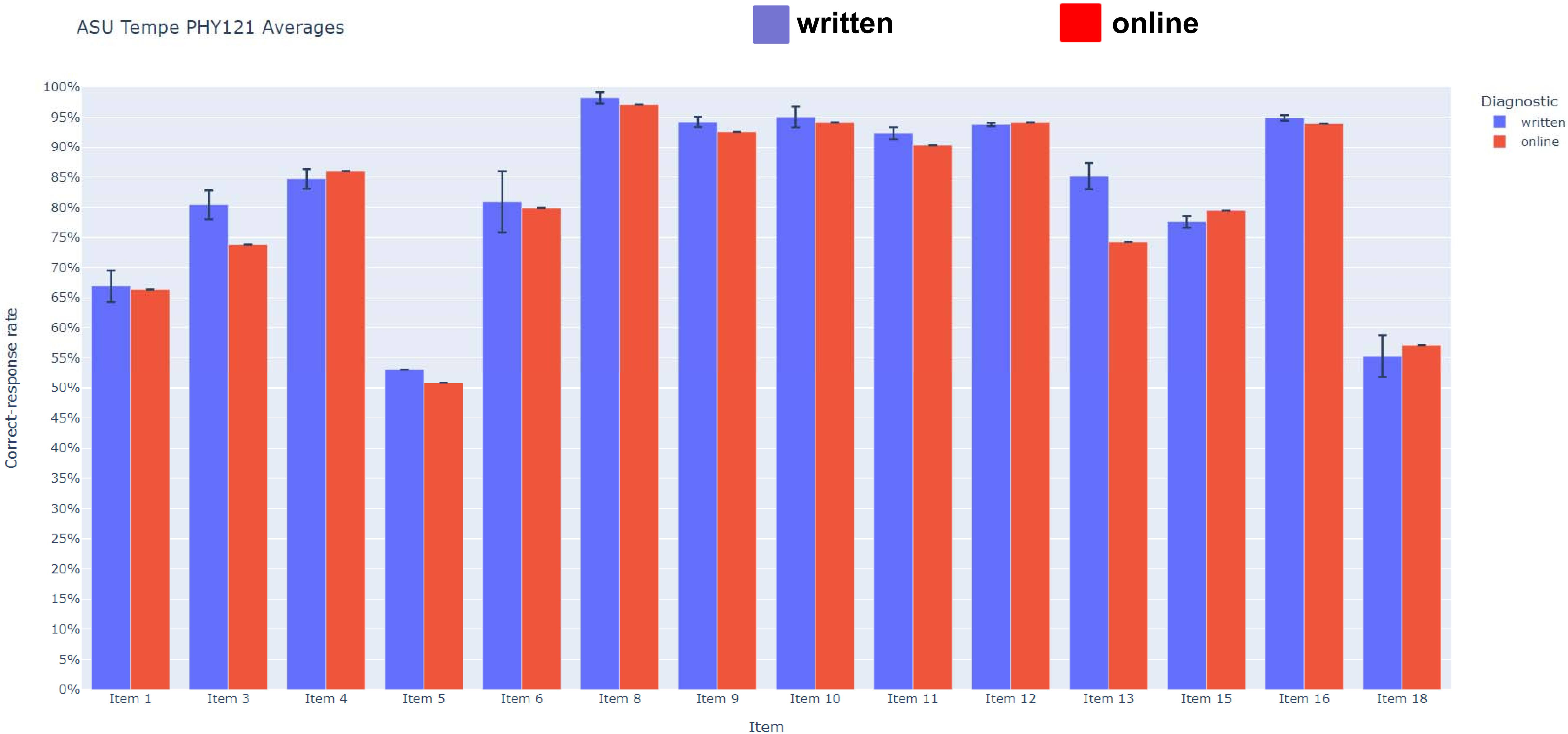
$$x = ?$$

A. $\frac{ac}{d+b}$ C. $\frac{ac}{bc-d}$ E. $\frac{ac}{db}$ G. $\frac{a}{b+\frac{d}{c}}$ I. $\frac{1}{b}\left(a - \frac{d}{c}\right)$

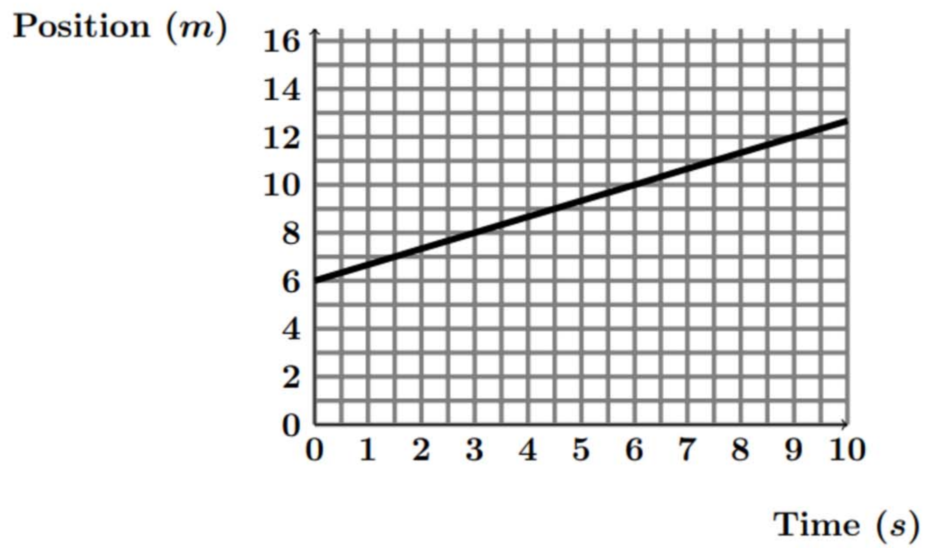
B. $\frac{ac}{d-b}$ D. $\frac{ac}{bc+d}$ F. $\frac{a}{db}$ H. $\frac{a}{b+d}$ J. $\frac{c}{d}\left(a - b\right)$

(There may be more than one correct answer, but please select only ONE answer.)

On-line and written versions yield consistent results



What is the slope of the graph below?



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C. $\frac{2}{3}$ m/s because the object moves 2 meters in 3 seconds.

D. $\frac{2}{3}$ m/s because the line rises 2 boxes while it goes 3 boxes in the horizontal direction.

***N* = 2556**

Numerically correct (C or D): 59%

***Actually* correct (C): 48%**

Consistent with results on written version

Most common error: *Counting grid squares and ignoring numbers on axes*

Findings from >70 Interviews: Students make many “careless” errors

- During interviews, students tended to self-correct approximately 60% of their initial errors with little or no prompting, suggesting that many errors are “careless.”
- These findings suggest that increased focus on improving students’ self-checking behavior might provide significant performance dividends.
  - However, studies have shown that making these improvements is quite challenging

1. Understanding of Mathematical Concepts

- Recognition of meaning and significance of mathematical operations

Example [trigonometry]: Unknown sides and angles of a right triangle may be found by applying sine, cosine, and tangent functions to known sides and angles

Example [vectors]: Direction of a vector may be defined as the angle with respect to an axis in some fixed coordinate system

1. Understanding of Mathematical Concepts

- Sherin (2001): Students' understanding of the *concepts* underlying mathematical problem solving are central to success in physics

  - *Example [wave phenomena]:* Steinberg, Wittmann, and Redish (1997) probed students' understanding of mathematical concepts related to wave propagation, and developed curricular materials to address the difficulties they observed

  - *Example [harmonic motion]:* Galle and Meredith (2014) developed tutorial worksheets to address students' confusion with meaning of, for example, $x(t) = 15 \text{ cm} \cos(2\pi f t)$

- **How to address these problems:** Have students practice *explaining the meaning* of the mathematical expressions (Galle and Meredith, 2014)

1. Understanding of Mathematical Concepts

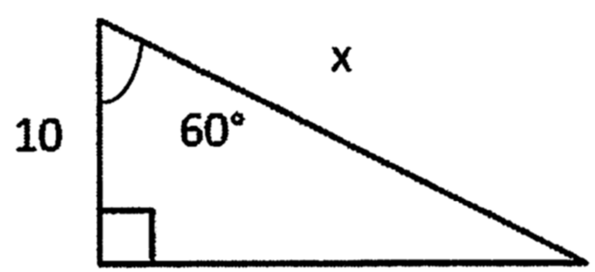
- **Trigonometry:** Many students are confused or unaware (or have forgotten) about the relationships between sides and angles in a right triangle.

- *Examples:* Questions from a diagnostic math test administered to over 7000 students, 2016-2022 (Administered as no-credit quiz during first week labs and/or recitation sections; calculators allowed)

Trigonometry Questions

with samples of correct student responses

1.



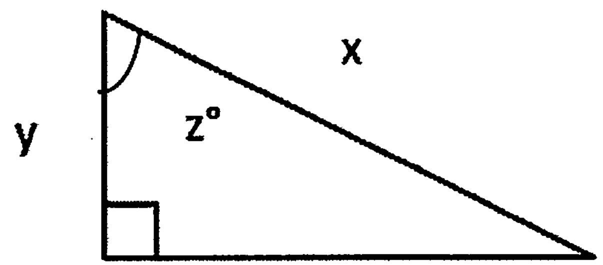
What is the value of x?

$$\cos 60 = \frac{10}{x}$$

$$x\cos 60 = 10$$

$$x = \frac{10}{\cos 60} = 20$$

2.



$\cos z = \frac{5}{x}$

What is the value of x?

A. ycos(z)

B. ycos(z)sin(z)

C. y/sin(z)

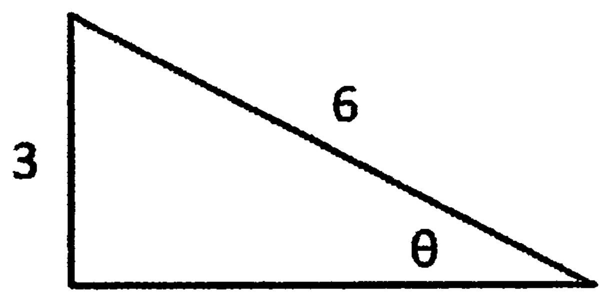
D. ysin(z)

E. ycos(z)/sin(z)

F. y/cos(z)

G. None of the above________________________

3.



What is the value of θ?

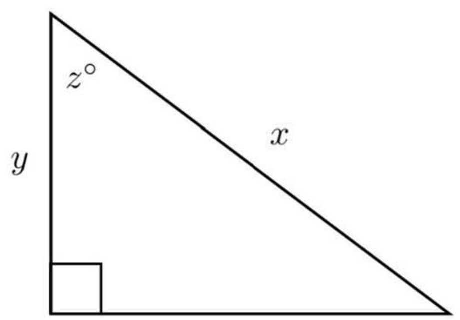
$\sin^{-1}(\theta) = \sin^{-1}\left(3/6\right)$

$\theta = 30°$

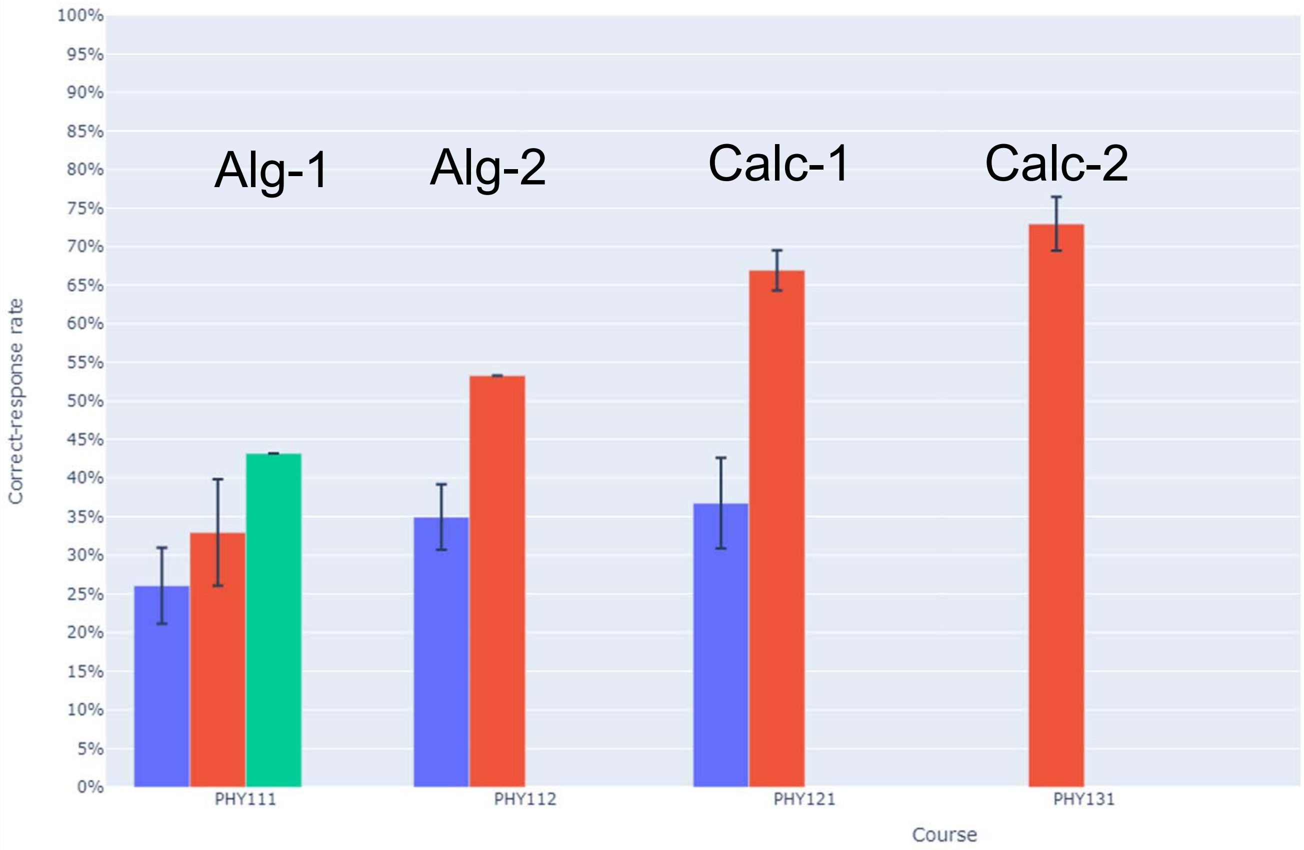
Correct-response rates

(36 classes; *N* > 3000)

What is the length of side x?



Many results in the 30-60% range



Campus
ASU Polytechnic
ASU Tempe
CU

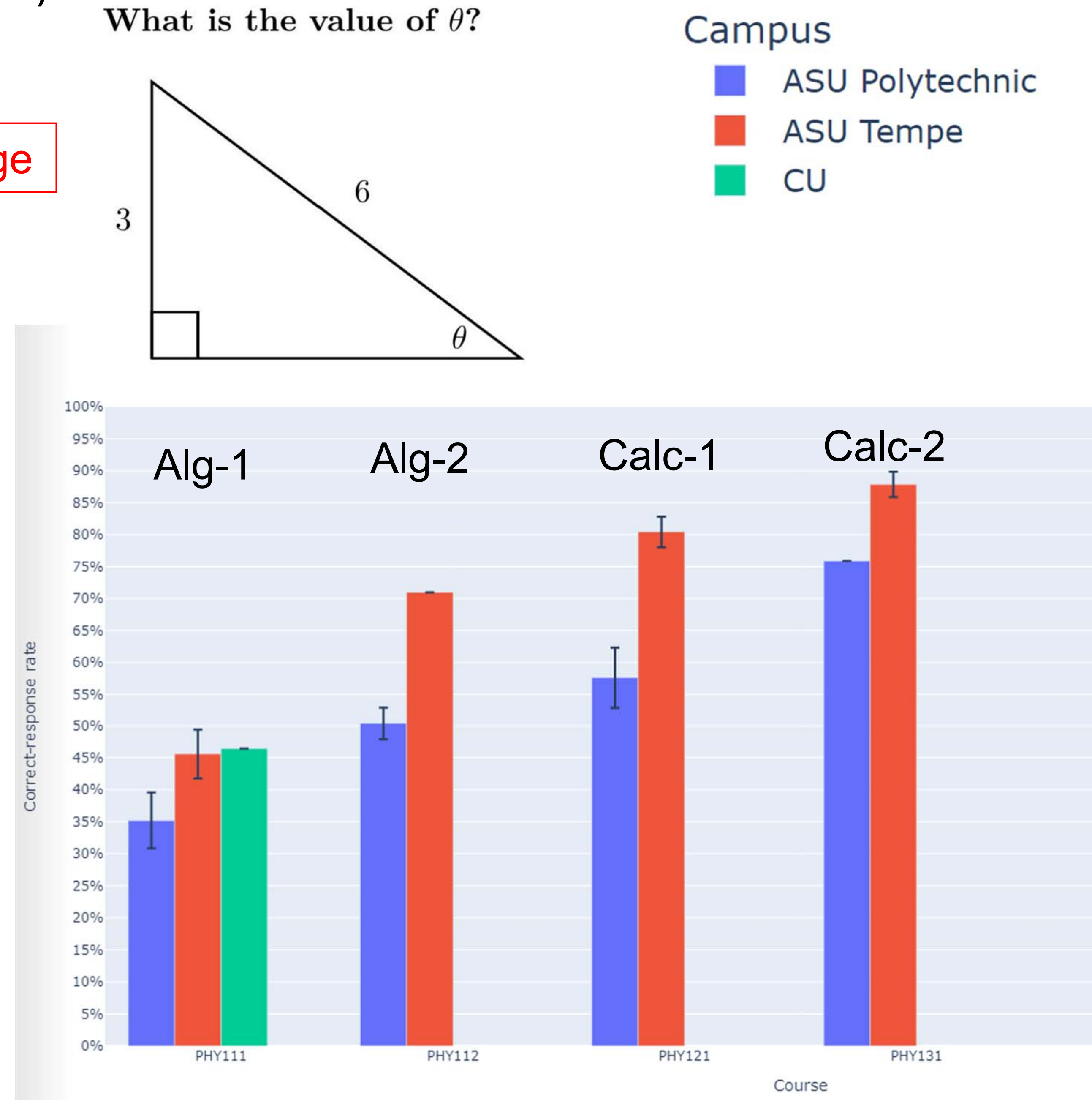
Correct-response rates

(36 classes; *N* > 3000)

What is the length of side x?

Many results in the 30-60% range

What is the value of θ?



Results on Trigonometry Questions

Errors oberved: use of incorrect trigonometric function (e.g., cosine instead of sine), calculator set on radians instead of degrees, algebra errors; *unaware (or forgot) about inverse trigonometric functions, e.g., arctan, arcsin*, *arccos [*tan^{-1}, sin^{-1}, cos^{-1}*]*

– **How to address these problems:** It seems that students require substantial additional *practice and repetition* with basic trigonometric procedures

2. Technical Skill: Vectors

- **Vectors:** Students have difficulty interpreting and manipulating vector quantities represented as arrows [Nguyen and Meltzer, 2003; Barniol and Zavala, 2014)

Example: Add (or subtract) vectors A and B to find the resultant

2. Technical Skill: Vectors

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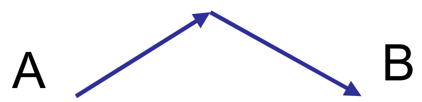

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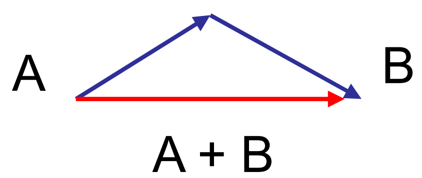


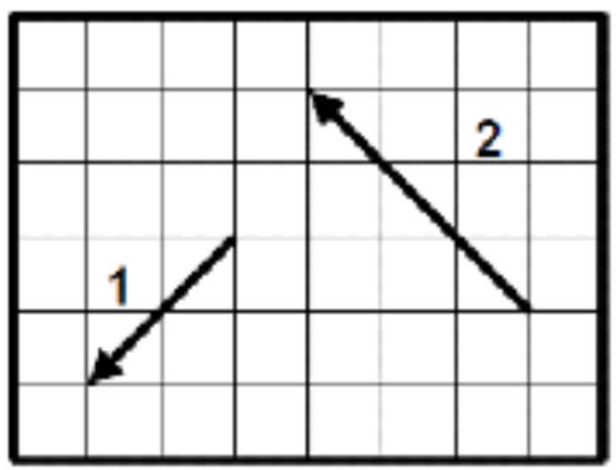
Addition of Vectors

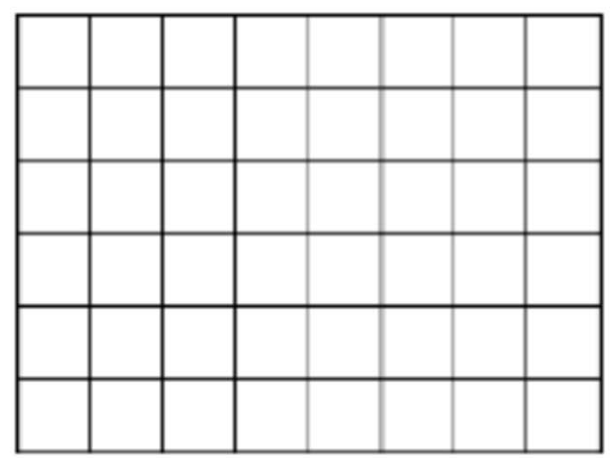
(Free Response)

6)

In the figure below there are two vectors $\overline{1}$ and $\overline{2}$. In the empty grid, draw the sum or vector addition $\overline{R}$ of the two (i.e., $\overline{R} = \overline{1} + \overline{2}$).

Note: You can draw other vectors in the empty grid, but be sure to label $\overline{R}$ clearly.





Addition of Vectors

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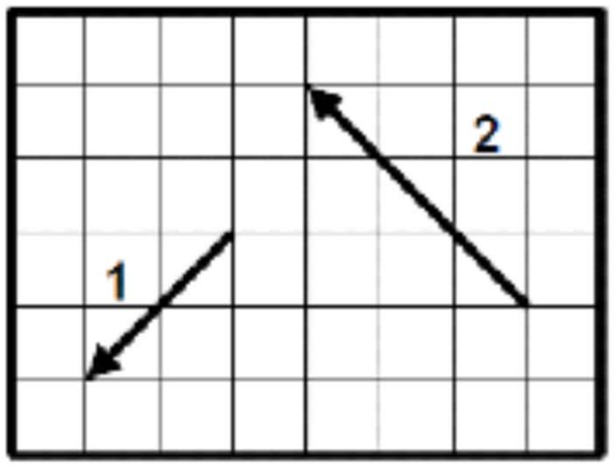
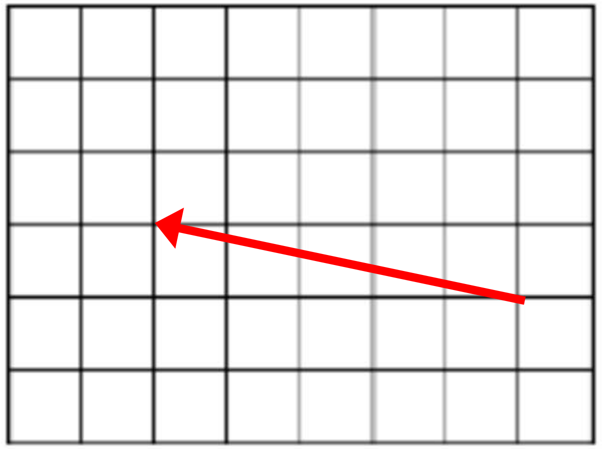




Addition of Vectors

Percent Correct Responses (Free Response)

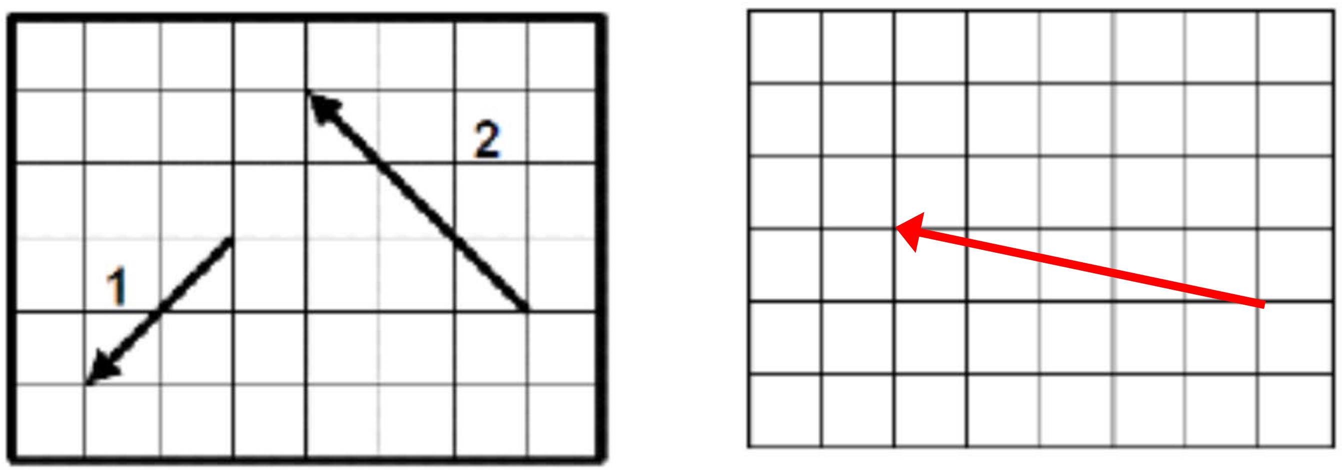
Algebra-based Course, 2nd semester (ASU-Tempe): 36% (*N* = 61)

Algebra-based Course, 2nd semester (Iowa State): 44% (*N* = 201)

6)

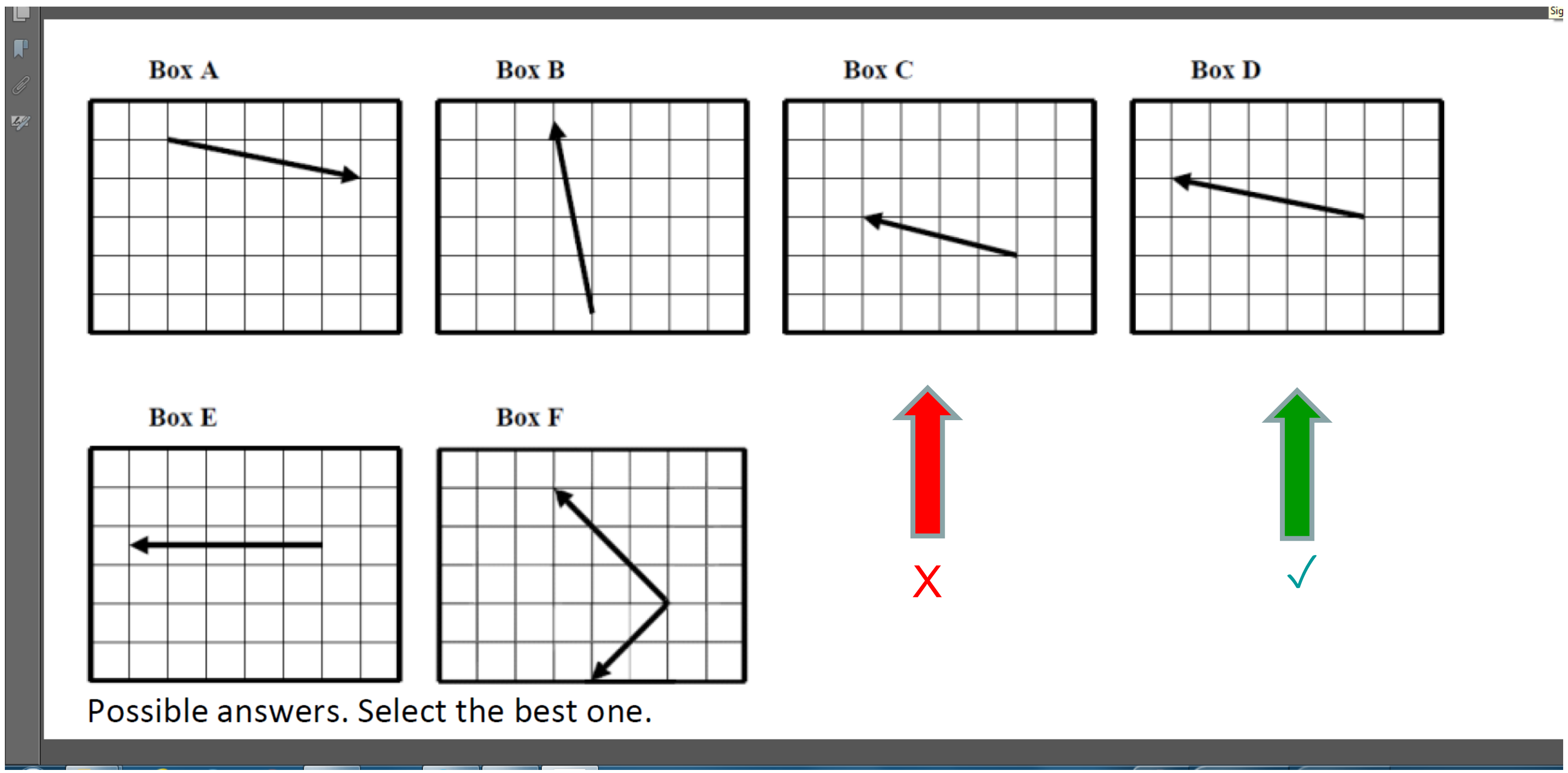
In the figure below there are two vectors $\overline{1}$ and $\overline{2}$. In the empty grid, draw the sum or vector addition $\overline{\boldsymbol{R}}$ of the two (i.e., $\overline{\boldsymbol{R}} = \overline{1} + \overline{2}$).

Note: You can draw other vectors in the empty grid, but be sure to label $\overline{\boldsymbol{R}}$ clearly.



Addition of Vectors

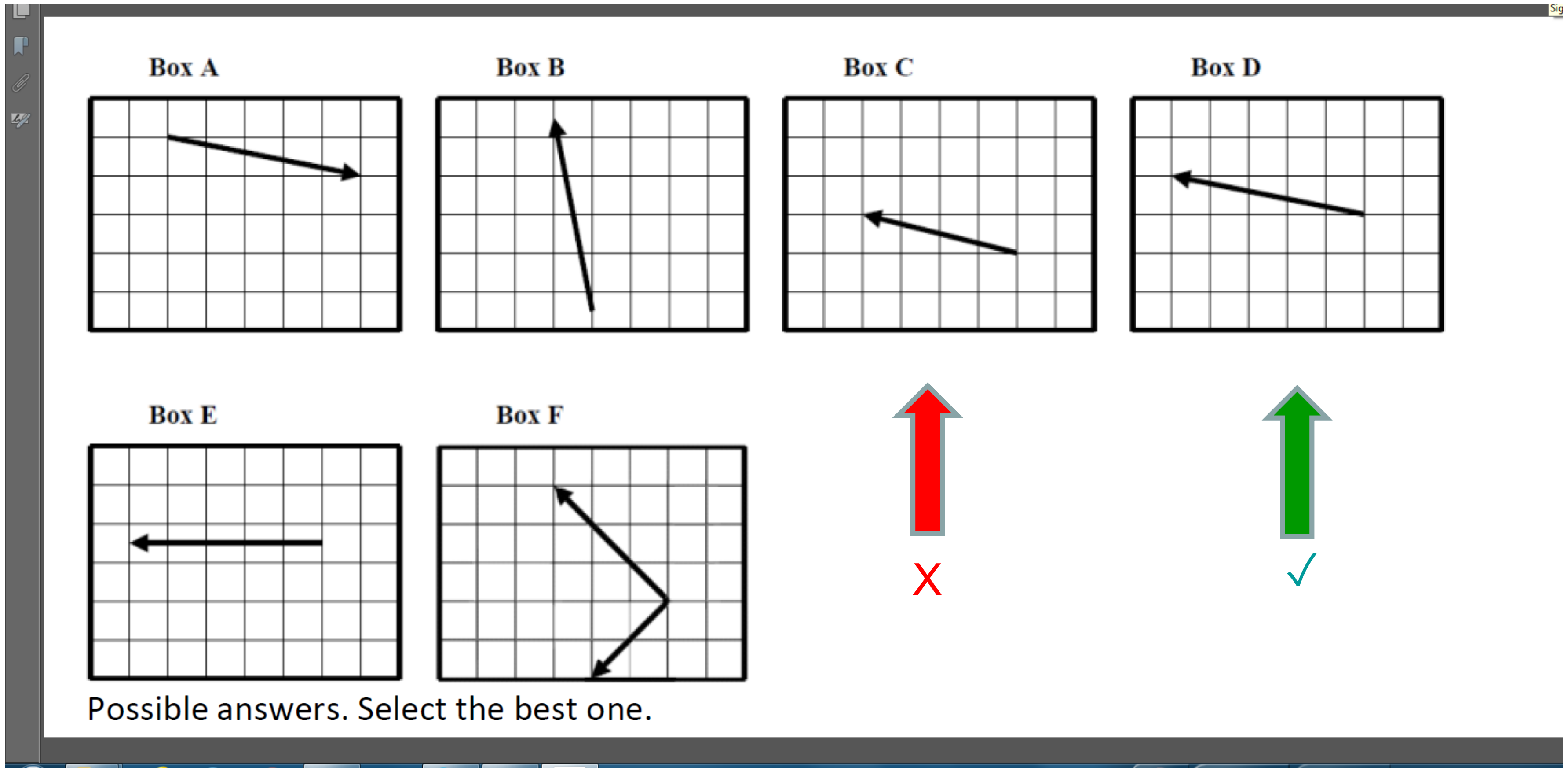
Multiple Choice



Addition of Vectors

Percent Correct Responses (Multiple Choice)

Algebra-based Course, 2nd semester (ASU-Tempe): 27% (*N* = 62)

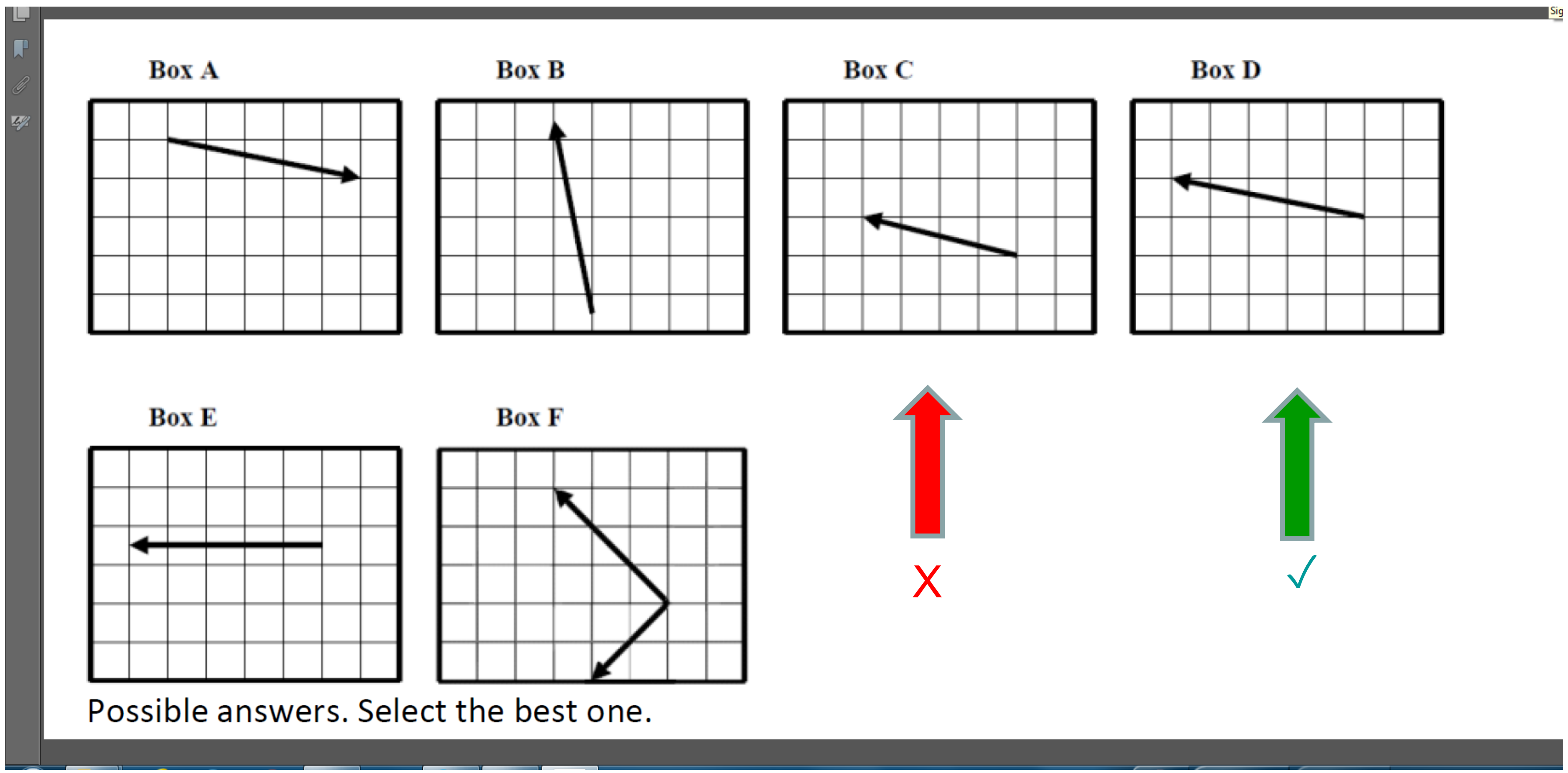


Addition of Vectors

Percent Correct Responses (Multiple Choice)

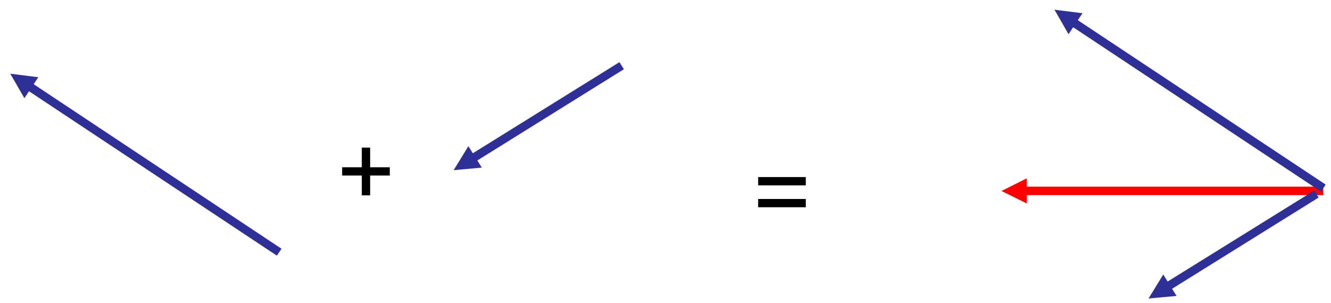
Algebra-based Course, 2nd semester (ASU-Tempe): 27% (*N* = 62)

Calculus-based Course, 1st semester (ASU-Tempe): 70% (*N* = 98)

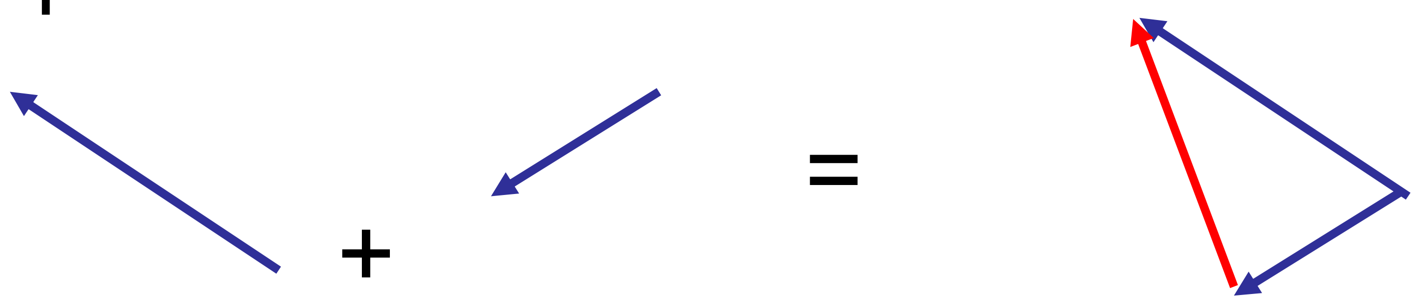


Common Student Errors With Vector Addition

- “Split the Difference” or “Bisector Vector”:



- “Tip-to-Tip”:



2. Technical Skill: Vectors

- **Vectors:** Students have difficulty interpreting and manipulating vector quantities represented as arrows [Nguyen and Meltzer, 2003; Barniol and Zavala, 2014)

How to address this problem:

- – Practice with a variety of vector orientations; introduce and use the "*ijk*" coordinate representation for vectors (Heckler and Scaife, 2015)
- – Provide extensive on-line practice and homework assignments related to frequently used vector procedures (Mikula and Heckler, 2017)
- – Design tutorial worksheet to aid students' understanding (Barniol and Zavala, 2016)

2. Technical Skill: Symbols

- “**Language mismatches”:** Students are confused by the very different symbols and techniques used in physics classes, for identical operations first seen in mathematics classes (Dray and Manogue, 1999-2004)
- **Unfamiliar symbols:** Students are often confused by new symbols or representations used in physics that are *not* used in mathematics classes, e.g., “arrow” representation of electric fields and gravitational forces; motion graphs (velocity-time, acceleration-time); “flux” [Φ] of electric field through a surface [Meltzer, 2005; Gire and Price, 2013]

2. Technical Skill: Symbols

- "**Language mismatches":** Students are confused by the very different symbols and techniques used in physics classes, for identical operations first seen in mathematics classes (Dray and Manogue, 1999-2004)
  - *Example:* The "area element" used in vector calculus to do area integrals looks very different in physics textbooks, compared to mathematics textbooks

$$\mathrm{d}S = \sqrt{[1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2]}\ \mathrm{d}x\ dy$$ *[math, general expression]*

$$\mathrm{d}S = r^2 \sin\theta\ \mathrm{d}\theta\ \mathrm{d}\phi$$ *[physics, for a sphere]*

2. Technical Skill: Symbols

- "**Language mismatches":** Students are confused by the very different symbols and techniques used in physics classes, for identical operations first seen in mathematics classes (Dray and Manogue, 1999-2004)

- **How to address this problem (Dray and Manogue, 2004):**
  - Focus on "big ideas" that provide unification, instead of memorizing many formulas and procedures; e.g., infinitesimal line element on sphere
    - » $d\mathbf{r} = dr\,\hat{r} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$
  - Improve students' geometric visualization skills, since physicists tend to think "geometrically" while math courses emphasize algebraic procedures. *Example:* manipulate vectors graphically as well as algebraically
  - Use "kinesthetic" activities to help students grasp geometrical meanings; Examples: "point fingers" in direction of vector gradient; use ruler and hoop to represent electrical flux (Gire and Price, 2012)

2. Technical Skill: Symbols

- **U**… often confused by new symbols used in physics th… sses, e.g., “arrow” representation of electric fields ar… 05); Gire and Price, 2013]

- H…

  - Ensure that students have ample practice w… ysics (e.g., diagrams, graphs, charts);
  - Include practice in “translating” between dif… “words” to “graphs”)
  - Use “kinesthetic” activities to help students … point fingers” in direction of vector gradient; use ruler and h…

-

2. Technical Skill: Symbolic Procedures

Confusion of symbolic meaning: Students perform worse on solving problems when symbols are used to represent common physical quantities in equations [Torigoe and Gladding, 2007; 2011)

Example [Multiple-choice questions; University of Illinois]:

Version #1: A car can go from 0 to 60 m/s in 8 s. At what distance *d* from the start at rest is the car traveling 30 m/s? *[93% correct]*

Version #2: A car can go from 0 to v_1 in t_1 seconds. At what distance *d* from the start at rest is the car traveling (v_1/2)? *[57% correct]*

Much worse!

- *Our results on "stripped-down" versions are analogous, although differences are smaller*

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{v_1}{t_1}$

$v = \frac{v_1}{2}$

Symbolic version

$d = ?$

A. $d = v_1 t_1$ **B.** $d = \frac{v_1 t_1}{2}$ **C.** $d = \frac{v_1 t_1}{4}$ **D.** $\boxed{d = \frac{v_1 t_1}{8}}$ **E.** $d = \frac{v_1 t_1}{16}$

Numeric version

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{\Delta v}{\Delta t}$

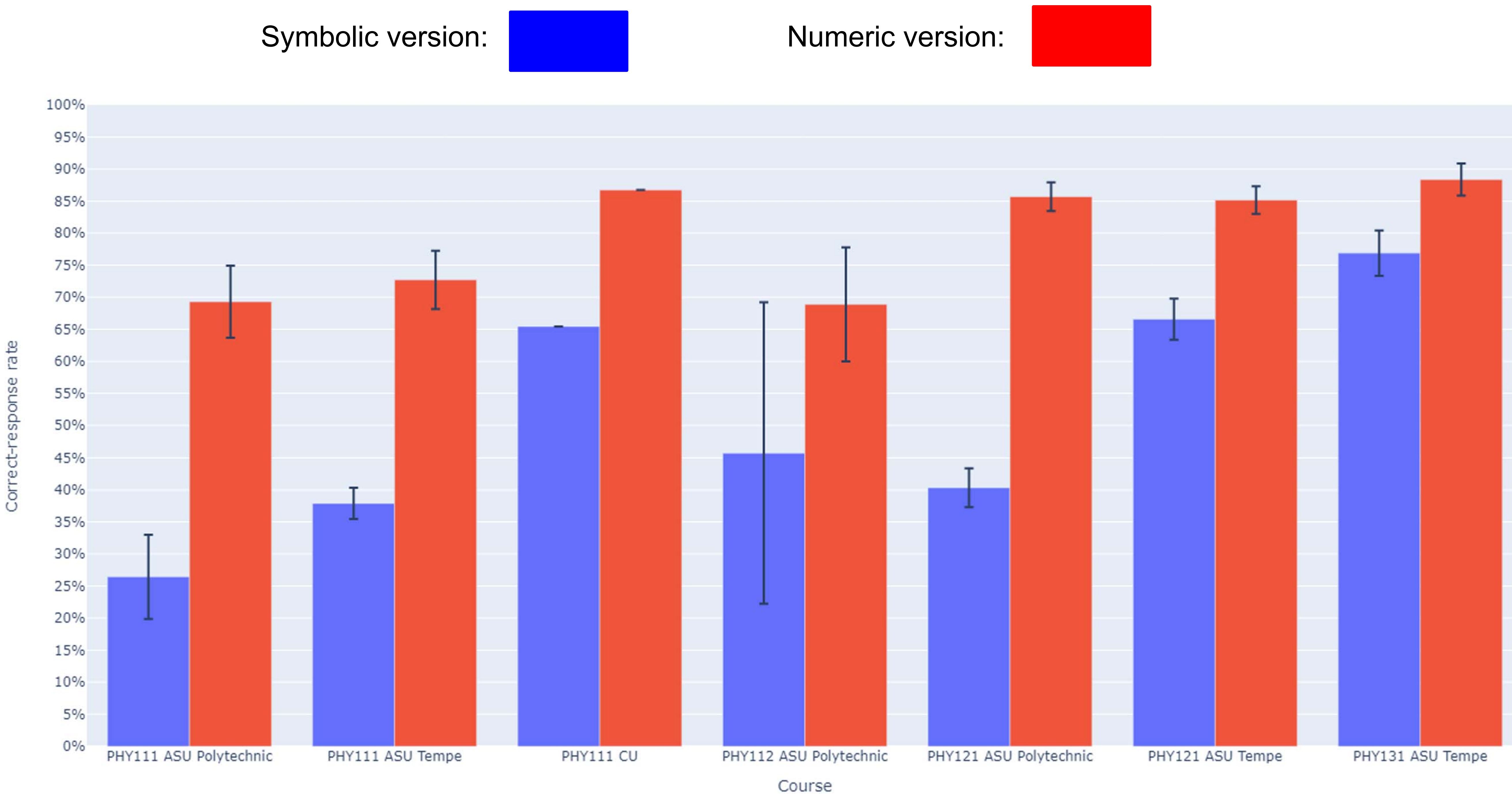
$\Delta v = 60$

$\Delta t = 8$

$v = 30$

$d = ?$

A. $d = 30$ **B.** $\boxed{d = 60}$ **C.** $d = 120$ **D.** $d = 240$ **E.** $d = 480$


Symbolic version:
Numeric version:
100%
95%
90%
85%
80%
75%
70%
65%
60%
55%
50%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
Correct-response rate
PHY111 ASU Polytechnic
PHY111 ASU Tempe
PHY111 CU
PHY112 ASU Polytechnic
PHY121 ASU Polytechnic
PHY121 ASU Tempe
PHY131 ASU Tempe
Course

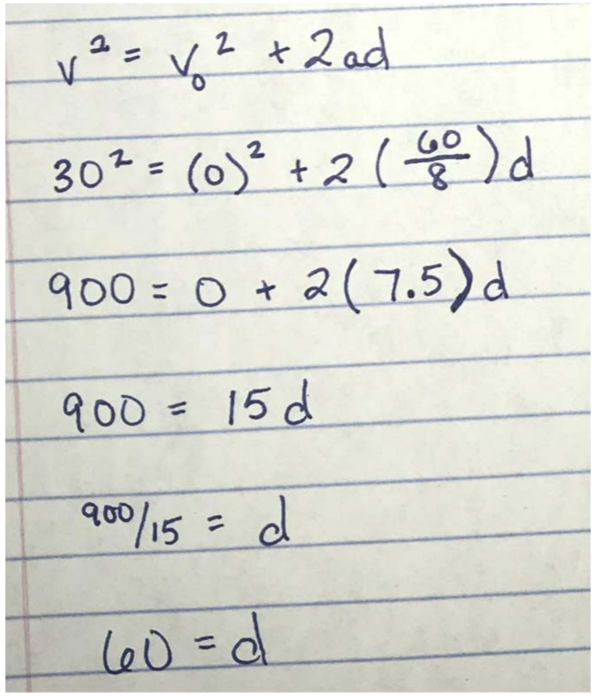
Students favor non-standard solution methods

- Introductory physics students favor semi-arithmetic methods for solving solve algebraic equations; they do not "isolate the unknown variable."

Implication: Physics instructors' habitual approach to algebraic manipulation may be confusing to their introductory students.

13. What is the numerical value of d?

53/53 students solved it this way:



$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = 60$$

$$\Delta t = 8$$

$$v = 30$$

$d = ?$ How would you solve this?

(Please clearly *circle* your answer and show all work.)

A. $d = 30$ B. $d = 60$ C. $d = 120$ D. $d = 240$ E. $d = 480$

We observed these methods used on *thousands* of students' submissions

Confusion can result from the nature of the symbols themselves

Solve for θ.

$$\gamma\theta + \eta = \lambda\theta + \omega$$

Solve for x.

$$ax + b = cx + d$$

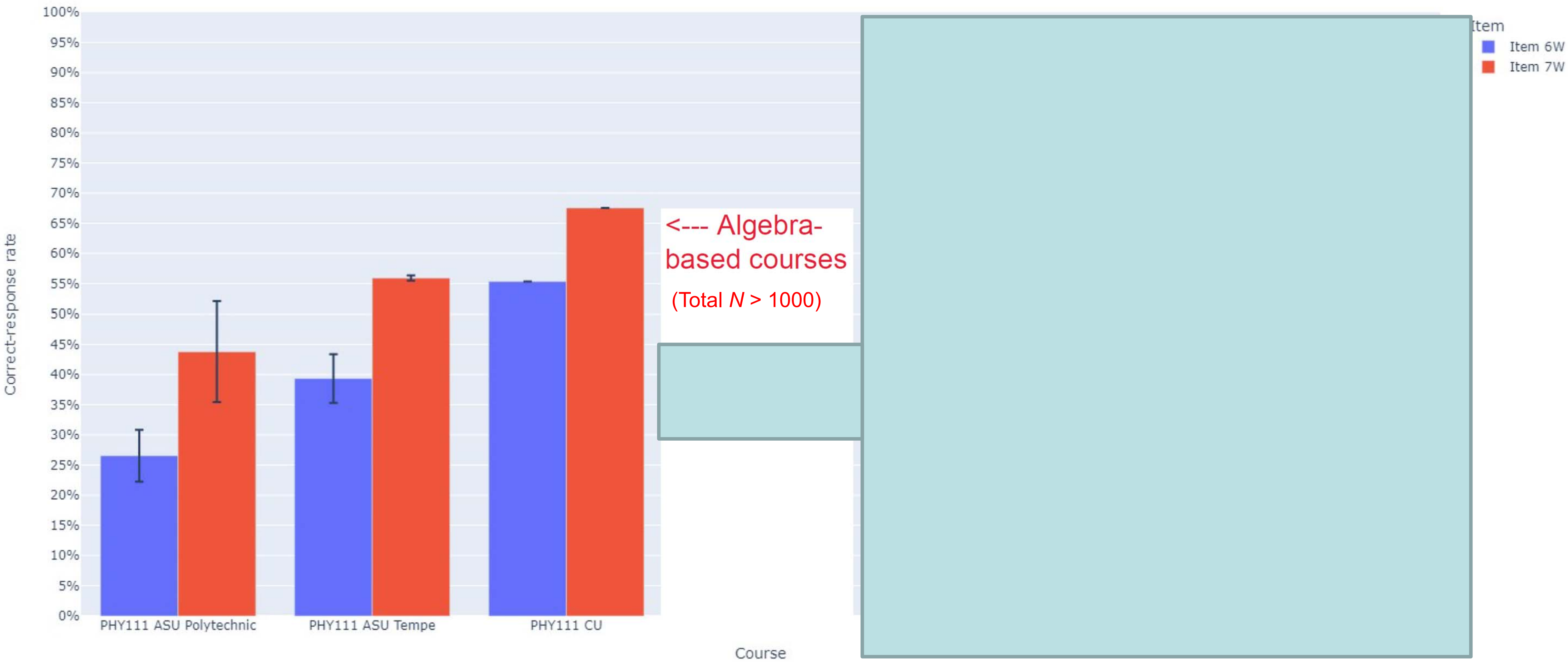
Solve for θ.

$$\gamma\theta + \eta = \lambda\theta + \omega$$

Significantly lower correct-response rates on Greek-letter version in algebra-based courses

Solve for x.

$$ax + b = cx + d$$



Solve for θ.

$$\gamma\theta + \eta = \lambda\theta + \omega$$

Significantly lower correct-response rates on Greek-letter version in algebra-based courses

Solve for x.

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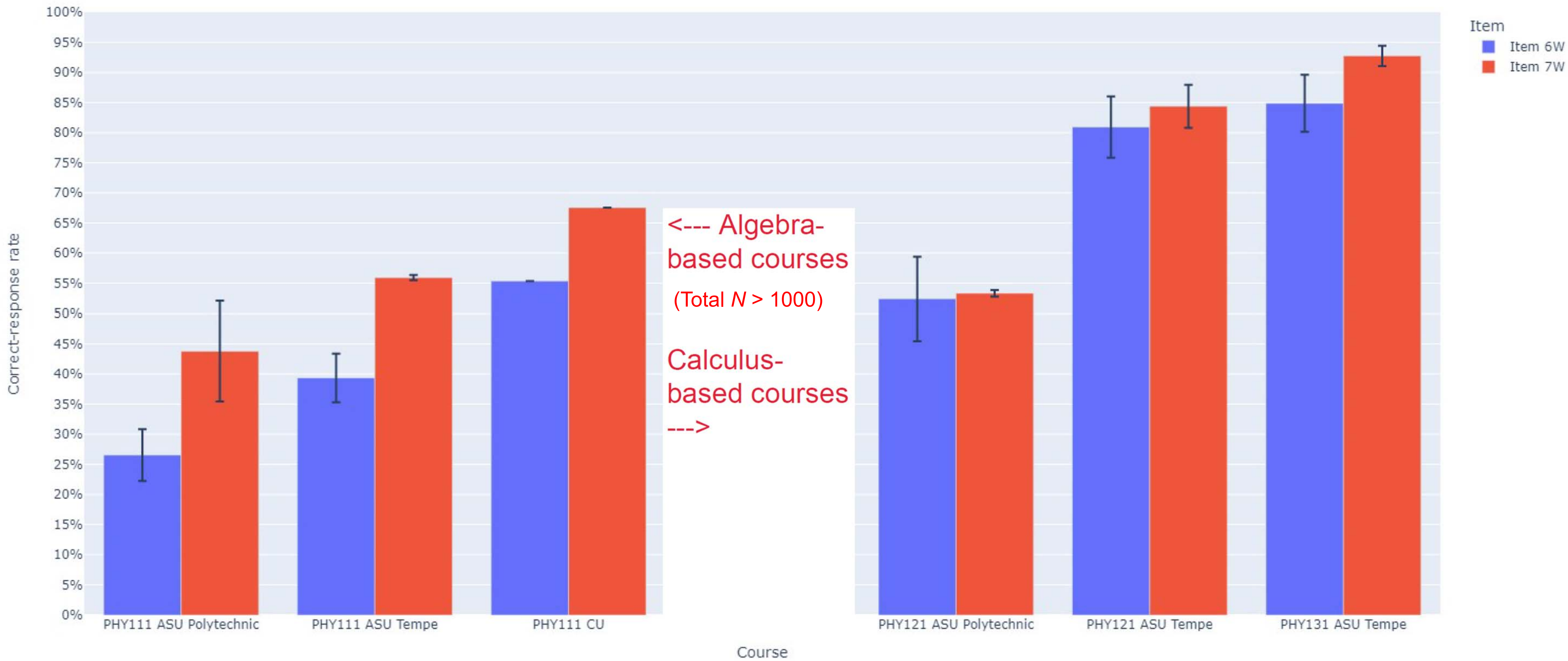


2. Technical Skill: **Symbolic Procedures**

- **Algebra (simultaneous equations):** Do differences in students' success rate between numerical and symbolic versions of same problem persist when simultaneous equations are involved? (E.g., two equations, two unknowns)

From Torigoe and Gladding (2011):

$F_{net} = ma$

$\tau_{net} = I\,\alpha$

$g = 9.8\ m/s^2$

$Mg - T = Ma$

$TR = I\alpha$

$[I = ½\ MR^2;\ \alpha = a/R]$

…→

$Mg - T = Ma$

$TR = [½\ MR^2][a/R]$

⟶ $a = 6.54\ m/s$

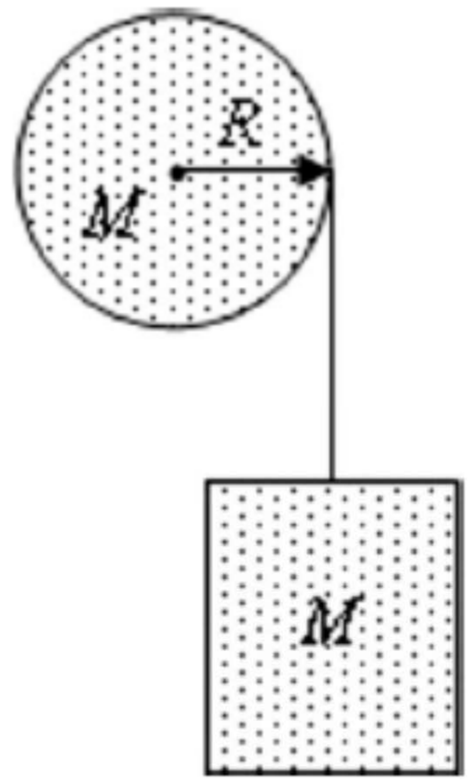


Fig. 7. Diagram for question 10.

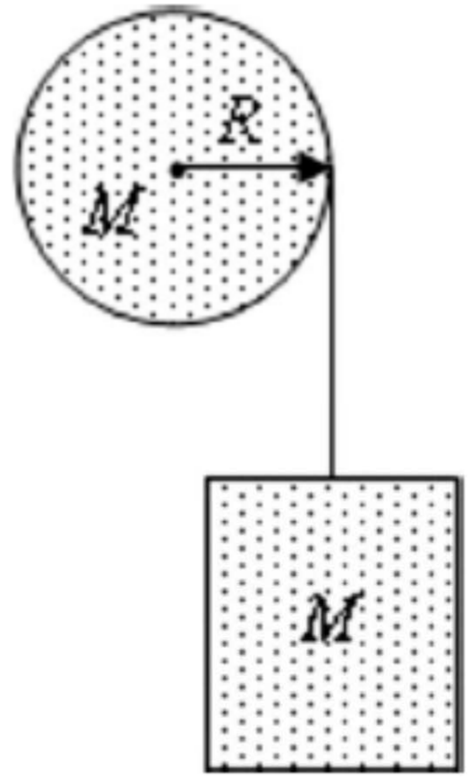
Question 10 (numeric). A uniform disk of mass $M = 8$ kg and radius $R = 0.5$ m has a string wound around its rim. The disk is free to spin about a pin through the center of the disk. A mass $M = 8$ kg (same mass as the disk) is connected to the string and is dropped from rest. What is the acceleration a of the block? (See Fig. 7.)

From Torigoe and Gladding (2011):

$$Mg - T = Ma$$
$$TR = I\alpha$$
$$[I = ½\, MR^2;\ \alpha = a/R]$$

$\ldots\rightarrow$

$$Mg - T = Ma$$
$$TR = [½\, MR^2][a/R]$$



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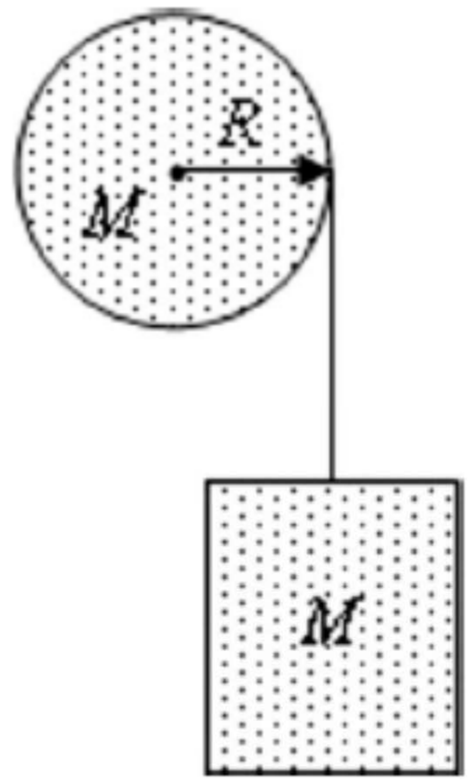


Fig. 7. Diagram for question 10.

$Mg - T = Ma$

$TR = I\alpha$

$[I = \frac{1}{2} MR^2;\ \alpha = a/R]$

$\ldots \rightarrow$

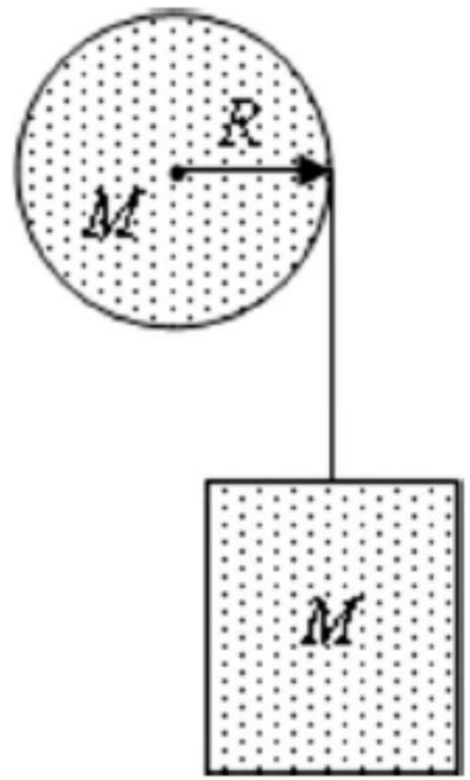
$Mg - T = Ma$

$TR = [\frac{1}{2} MR^2][a/R]$

Symbolic version

Question 10 (numeric). A uniform disk of mass M [redacted] and radius R [redacted] has a string wound around its rim. The disk is free to spin about a pin through the center of the disk. A mass M [redacted] (same mass as the disk) is connected to the string and is dropped from rest. What is the acceleration a of the block? (See Fig. 7.)

From Torigoe and Gladding (2011):



$Mg - T = Ma$

$TR = I\alpha$

$[I = \frac{1}{2} MR^2;\ \alpha = a/R]$

$\ldots \rightarrow$

$Mg - T = Ma$

$TR = [\frac{1}{2} MR^2][a/R]$

$\longrightarrow \quad a = 2/3\ g$

Symbolic version

Fig. 7. Diagram for question 10.

Question 10 (numeric). A uniform disk of mass M [redacted] and radius R [redacted] has a string wound around its rim. The disk is free to spin about a pin through the center of the disk. A mass M [redacted] (same mass as the disk) is connected to the string and is dropped from rest. What is the acceleration a of the block? (See Fig. 7.)

Results on #10

[Torigoe and Gladding, 2011]

- **Numeric version:** 49% correct ($N \approx 380$)
- **Symbolic version:** 53% correct ($N \approx 380$)

→ *No significant difference*

From Torigoe and Gladding (2011):

$Mg - T = Ma$

$TR = I\alpha$

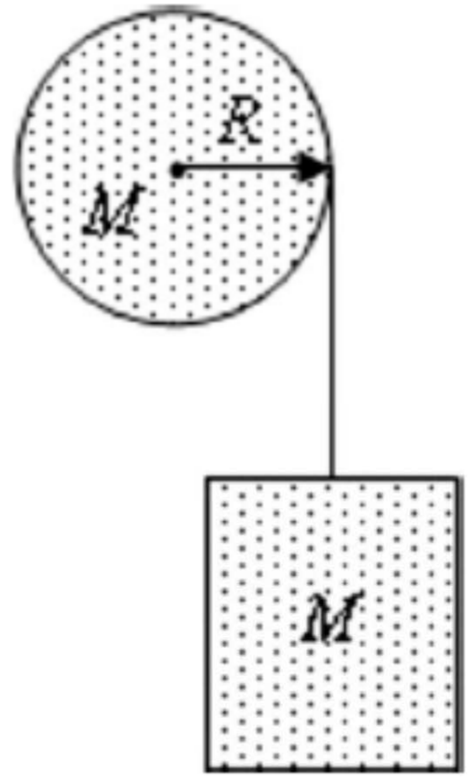
$[I = ½\, MR^2;\ \alpha = a/R]$

$\ldots \rightarrow$

$Mg - T = Ma$

$TR = [½\, MR^2][a/R]$

$a = ?$



Symbolic version

Fig. 7. Diagram for question 10.

Question 10 (numeric). A uniform disk of mass *M* [redacted] and radius *R* [redacted] has a string wound around its rim. The disk is free to spin about a pin through the center of the disk. A mass *M* [redacted] (same mass as the disk) is connected to the string and is dropped from rest. What is the acceleration *a* of the block? (See Fig. 7.)

From Torigoe and Gladding (2011):

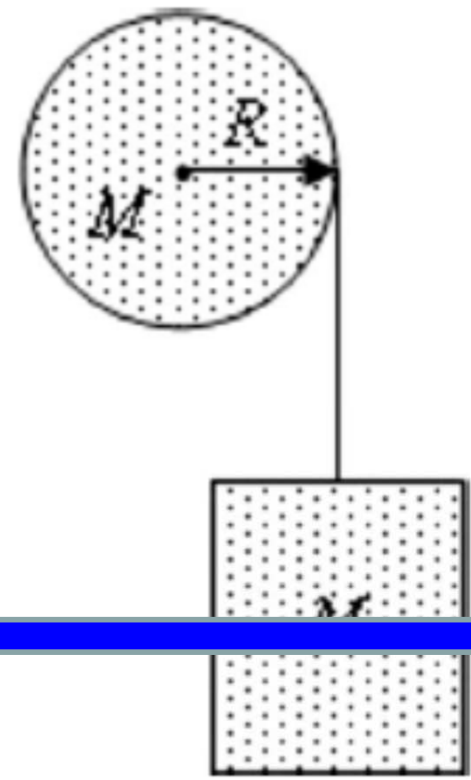
$$Mg - T = Ma$$
$$TR = I\alpha$$
$$[I = ½\ MR^2;\ \alpha = a/R]$$

$\ldots \rightarrow$

$$Mg - T = Ma$$
$$TR = [½\ MR^2][a/R]$$
$$a = ?$$

Rename to simplify:

- "Mg" → "a"
- "M" → "b"
- "R" → "c"
- "½MR" → "d"
- "T" → "y"
- "a" → "x"



$$a - y = bx$$
$$cy = dx$$
$$x = ?$$

Our Symbolic version

Fig. 7. Diagram for question 10.

Question 10 (numeric). A uniform disk of mass M [redacted] and radius R [redacted] has a string wound around its rim. The disk is free to spin about a pin through the center of the disk. A mass M [redacted] (same mass as the disk) is connected to the string and is dropped from rest. What is the acceleration a of the block? (See Fig. 7.)

From Torigoe and Gladding (2011):

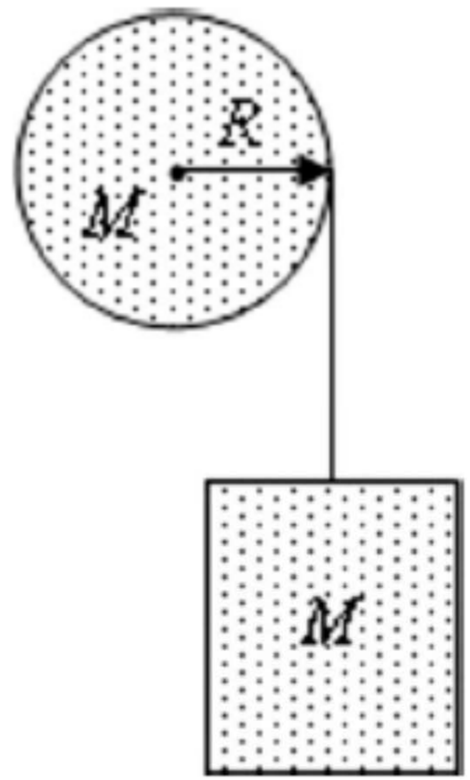


Fig. 7. Diagram for question 10.

$$Mg - T = Ma$$
$$TR = I\alpha$$
$$[I = \tfrac{1}{2} MR^2;\ \alpha = a/R]$$

…→

$$Mg - T = Ma$$
$$TR = [\tfrac{1}{2} MR^2][a/R]$$
$$a = ?$$

$$78.4 - y = 8x$$
$$0.5y = 2x$$
$$x = ?$$

Our Numeric version

Question 10 (numeric). A uniform disk of mass $M=8$ kg and radius $R=0.5$ m has a string wound around its rim. The disk is free to spin about a pin through the center of the disk. A mass $M=8$ kg (same mass as the disk) is connected to the string and is dropped from rest. What is the acceleration a of the block? (See Fig. 7.)

Results on Our Versions

➡ *Consistent, large, and highly significant difference*

Symbolic notation degrades student performance

- Use of symbols to replace numbers in otherwise identical algebraic equations lowered correct-response rates by ≈25%.

Algebra: Simultaneous Equations (algebra-based course, ASU-T)

$0.5y = 2x$ $78.4 - y = 8x$	[Solve for x]	Numeric Version	61% correct (N = 470)

Algebra: Simultaneous Equations (algebra-based course, ASU-T)

$0.5y = 2x$ $78.4 - y = 8x$	[Solve for x]	Numeric Version	61% correct (N = 470)
$cy = dx$ $a - y = bx$	[Solve for x]	Symbolic Version	31% correct (N = 372)

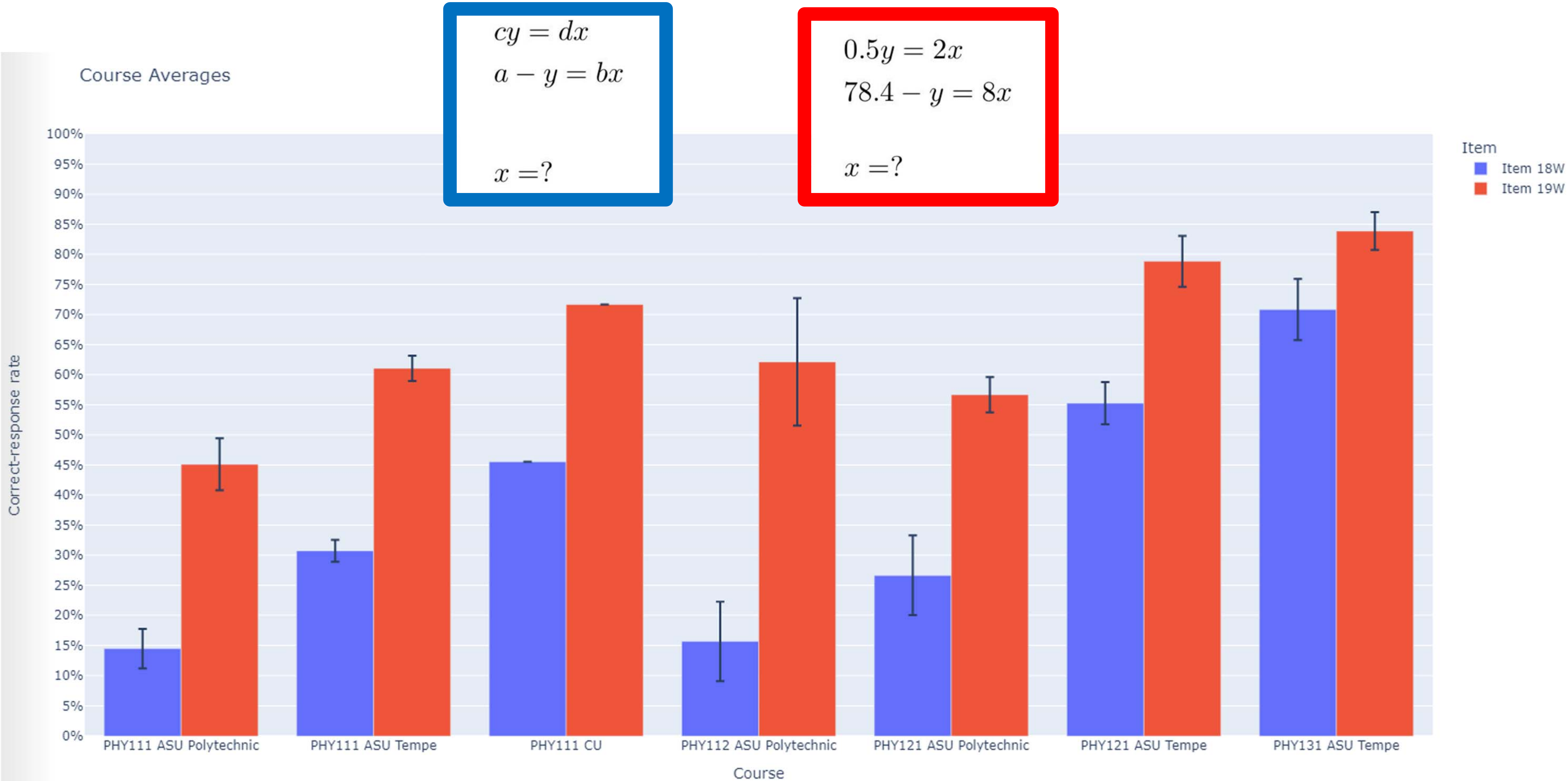
Algebra: Simultaneous Equations (calculus-based course, ASU-T)

$0.5y = 2x$ $78.4 - y = 8x$	[Solve for x]	Numeric Version	79% correct (N = 1205)

Algebra: Simultaneous Equations (calculus-based course, ASU-T)

$0.5y = 2x$ $78.4 - y = 8x$	[Solve for x]	Numeric Version	79% correct (N = 1205)

$cy = dx$ $a - y = bx$	[Solve for x]	Symbolic Version	55% correct (N = 1202)



$cy = dx$
$a - y = bx$
$x = ?$
$0.5y = 2x$
$78.4 - y = 8x$
$x = ?$
Course Averages
Correct-response rate
100%
95%
90%
85%
80%
75%
70%
65%
60%
55%
50%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
Item
Item 18W
Item 19W
PHY111 ASU Polytechnic
PHY111 ASU Tempe
PHY111 CU
PHY112 ASU Polytechnic
PHY121 ASU Polytechnic
PHY121 ASU Tempe
PHY131 ASU Tempe
Course

Sources of Difficulties

- "Carelessness"
  - Students *very frequently* self-correct errors during interviews
- Skill practice deficit: Insufficient repetitive practice with learned skills
  - e.g., dividing symbolic fractions
- Conceptual confusion
  - e.g., not realizing that *both sides* of an equation must be multiplied or divided by the same symbol

How to Address Difficulties?

- Carelessness:
  (1) review and check steps
  (2) find alternative solutions
  (3) habitual use of estimation
  (4) apply dimensional analysis (for physical problems)
- Skill deficit: Practice and repetition
- Conceptual confusion: Review and study of basic ideas

3. Ability to Apply Mathematics in a Physical Context

- Student difficulties that *appear* to be mathematical in origin may actually be due in part to application in a *physical* context [Thompson, Manogue, Roundy, and Mountcastle, 2012; Zavala and Barniol, 2013]

- *Example [calculus]:* Finding and comparing the "area under the curve" by applying the definite integral may be more challenging in a thermodynamics context (thermodynamic process represented on a Pressure-Volume diagram) [Christensen and Thompson, 2010-2012]

- *Example [vectors]:* The method used *and* the errors made by students when adding or subtracting vectors depend strongly on the specific physical context, and on whether there *is* a physical context [Shaffer and McDermott, 2005; Van Deventer and Wittmann, 2005; Barniol and Zavala, 2010]

3. Ability to Apply Mathematics in a Physical Context

- Student difficulties that *appear* to be mathematical in origin may actually be due in part to application in a *physical* context

How to address this problem:

  – Mathematics procedures must be practiced in a variety of physical contexts, and students must be made aware of possible confusion introduced by the context

4. Ability to Apply Mathematics in a Problem-Solving Context

- Students often fail to make use of specific mathematical tools that they *do* know how to use, because they don't recognize their applicability to a physics problem [Bing and Redish, 2009; Gupta and Elby, 2011]

How to address this problem:

- Vary the physical context, to provide a wide range of possible approaches
- Become aware of how students interpret problems; "exaggerate" cues regarding appropriate solution pathways (Bing and Redish, 2009)

Interim Summary:

What Options do Physics Instructors Have for Dealing with Students' Mathematics Difficulties?

- Test to assess scope of problem
- Take time to review basic math
- Assign or suggest out-of-class math review practice
  - e.g., OSU "Stemfluency" on-line practice tool
- Reduce mathematical burden of syllabus
  - more qualitative problems, fewer problems requiring algebraic manipulation
- Nothing: Leave it up to the students (??)

Caution: Difficulties with one topic implies difficulties with others as well

- Students' scores on different problem types tend to track each other closely: relatively low scores on one type imply relatively low scores on the others
- Since scores on different items are correlated with each other, scores on even a single test item can be predictive of overall score, particularly when class-average scores are considered.


Similar scores on two *very* different problems
(N > 5000)
Solve for θ.
$\gamma\theta + \eta = \lambda\theta + \omega$
What is the value of θ?
3
6
θ
100%
95%
90%
85%
80%
75%
70%
65%
60%
55%
50%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
PHY111 ASU Polytechnic
PHY111 ASU Tempe
PHY111 CU
PHY121 ASU Polytechnic
PHY121 ASU Tempe
PHY131 ASU Tempe

Even single test items are highly predictive

- Performance on **one single diagnostic item** can *accurately* predict class-average score on the full 16-item diagnostic with that item removed

Example: “Simultaneous Equations”

$$cy = dx$$

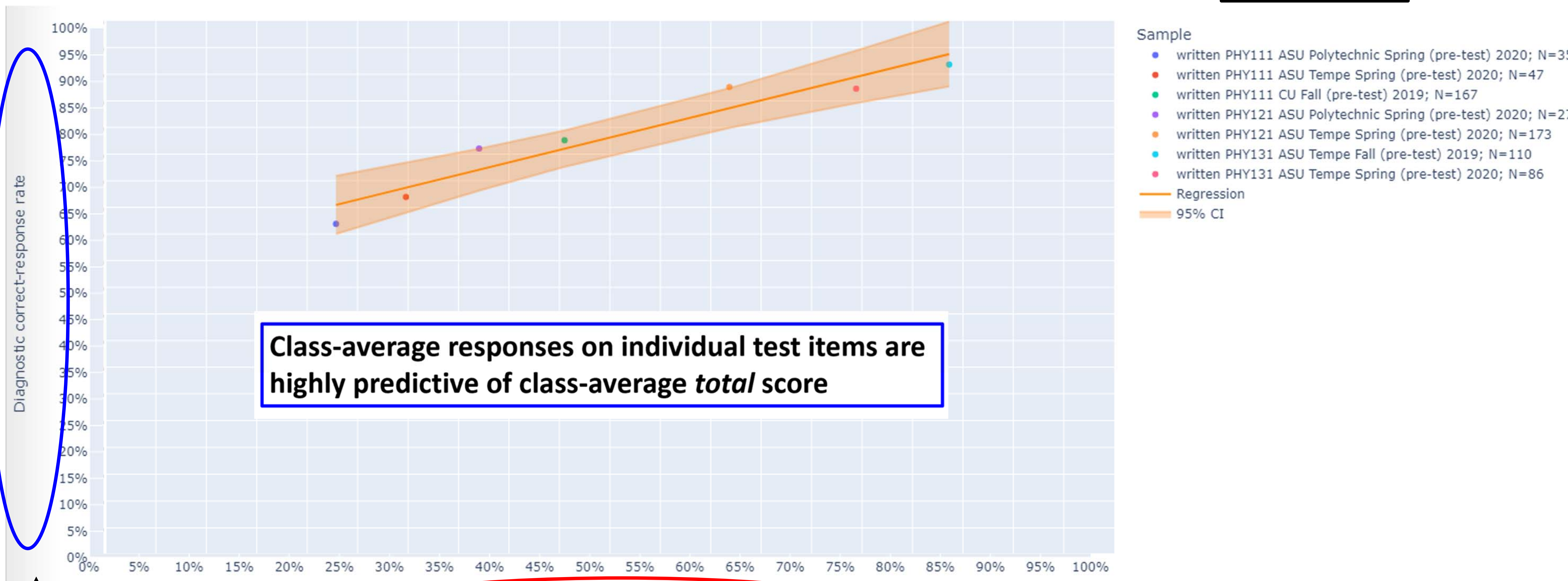
$$a - y = bx$$

$$x = ?$$

Item “Simultaneous Equations” vs. Full-Diagnostic Correct-Response Rates

$$cy = dx$$
$$a - y = bx$$
$$x = ?$$

Written Only (r = 0.96)



Class-average responses on individual test items are highly predictive of class-average *total* score

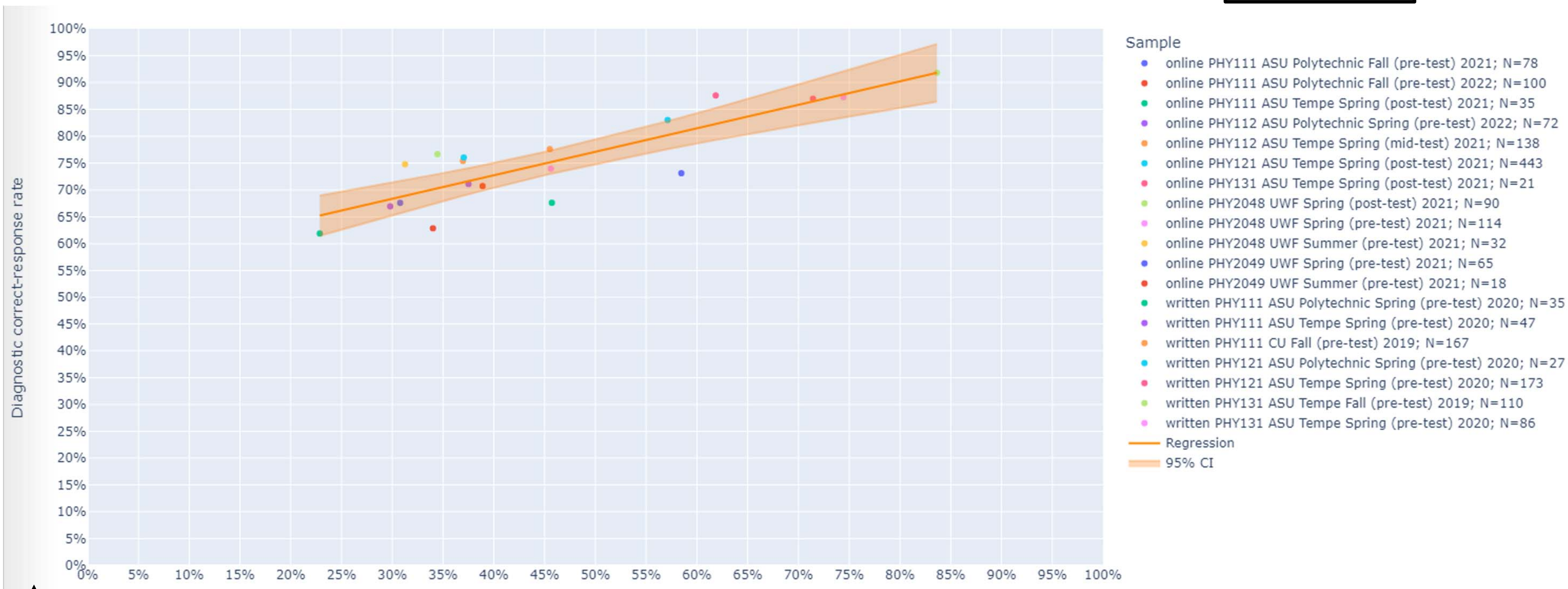
Full Diagnostic Correct-Response Rate (without this problem)

Correct-Response Rate on this Problem

Item “Simultaneous Equations” vs. Full-Diagnostic Correct-Response Rates

Written + Online (r = 0.86)

$$cy = dx$$
$$a - y = bx$$
$$x = ?$$



Full Diagnostic Correct-Response Rate (without this problem)

Correct-Response Rate on this Problem

Example: “Greek Letters”

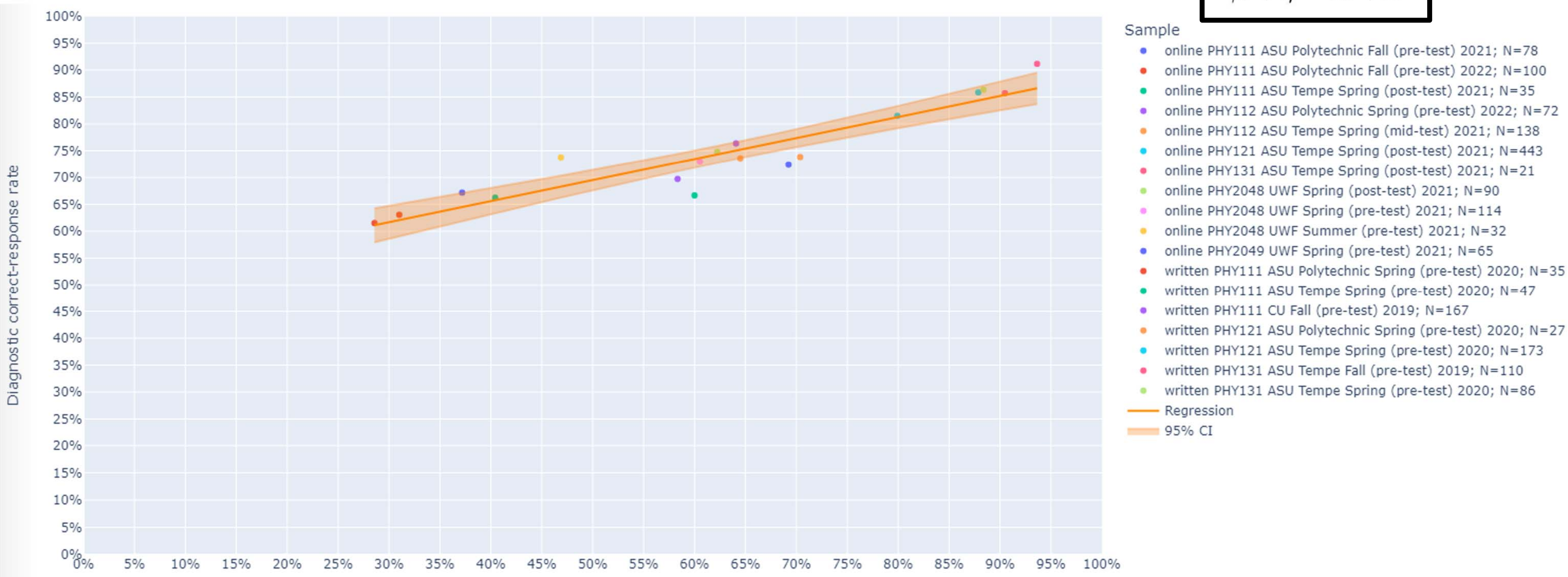
Solve for θ.

$$\gamma\theta + \eta = \lambda\theta + \omega$$

Item “Greek Letters” vs. Full-Diagnostic Correct-Response Rates
Written + Online (r = 0.93)

Solve for θ.

$$\gamma\theta + \eta = \lambda\theta + \omega$$



Full Diagnostic Correct-Response Rate (without this problem)

Correct-Response Rate on this Problem

If single items can predict total scores, what can total scores predict?

Implication: It may be possible to diagnose the level of students’ difficulties with only one or very few mathematics pretest items.

Test: 3-item subset of diagnostic items is *somewhat* predictive of students’ final grades

→ Full diagnostic offers greater predictive power

Relation Between Scores and Grades

- Performance on **full online diagnostic** can *approximately* predict final course grade

Low Course Grade vs. Full Diagnostic Score

Course	Campus	*N*	Overall % grade ≤ C+	Score ≥ 81% % grade ≤ C+	Score ≤ 57% % grade ≤ C+	Low-grade Ratio score ≤ 57% vs. score ≥ 81%

Low Course Grade vs. Full Diagnostic Score

Course	Campus	*N*	Overall % grade ≤ C+
Alg-1	ASU-P	78	26%
Alg-2	ASU-P	72	29%
Calc-1	UWF	103	39%

Alg-1: Algebra-based course, first semester
Alg-2: Algebra-based course, second semester
Calc-1: Calculus-based course, first semester
Calc-2: Calculus-based course, second semester

ASU-P: Arizona State University, Polytechnic campus
ASU-T: Arizona State University, Tempe campus
UWF: University of West Florida

Students who scored low on math diagnostic pretest had more "C" course grades than those who scored high

Low Course Grade vs. Full Diagnostic Score

Course	Campus	*N*	Overall % grade ≤ C+	Score ≥ 81% % grade ≤ C+	Score ≤ 57% % grade ≤ C+	Low-grade Ratio score ≤ 57% vs. score ≥ 81%
Alg-1	ASU-P	78	26%			
Alg-2	ASU-P	72	29%			
Calc-1	UWF	103	39%			

Alg-1: Algebra-based course, first semester
Alg-2: Algebra-based course, second semester
Calc-1: Calculus-based course, first semester
Calc-2: Calculus-based course, second semester

ASU-P: Arizona State University, Polytechnic campus
ASU-T: Arizona State University, Tempe campus
UWF: University of West Florida

Students who scored low on math diagnostic pretest had more "C" course grades than those who scored high

Low Course Grade vs. Full Diagnostic Score

Course	Campus	*N*	Overall % grade ≤ C+	Score ≥ 81% % grade ≤ C+	Score ≤ 57% % grade ≤ C+	Low-grade Ratio score ≤ 57% vs. score ≥ 81%
Alg-1	ASU-P	78	26%	19%	38%	2.1
Alg-2	ASU-P	72	29%	14%	35%	2.6
Calc-1	UWF	103	39%	26%	54%	2.1

Alg-1: Algebra-based course, first semester
Alg-2: Algebra-based course, second semester
Calc-1: Calculus-based course, first semester
Calc-2: Calculus-based course, second semester

ASU-P: Arizona State University, Polytechnic campus
ASU-T: Arizona State University, Tempe campus
UWF: University of West Florida

Students who scored low on math diagnostic pretest had more "C" course grades than those who scored high

High Course Grade vs. Full Diagnostic Score

Course	Campus	*N*	Overall % grade ≥ A-
Alg-1	ASU-P	78	35%

High Course Grade vs. Full Diagnostic Score

Course	Campus	*N*	Overall % grade ≥ A-	Score ≥ 81% % grade ≥ A-	Score ≤ 57% % grade ≥ A-	High-grade Ratio score ≥ 81% vs. score ≤ 57%
Alg-1	ASU-P	78	35%	63%	15%	4.2

High Course Grade vs. Full Diagnostic Score

Course	Campus	*N*	Overall % grade ≥ A-	Score ≥ 81% % grade ≥ A-	Score ≤ 57% % grade ≥ A-	High-grade Ratio score ≥ 81% vs. score ≤ 57%
Alg-1	ASU-P	78	35%	63%	15%	4.2
Alg-2	ASU-P	72	39%	64%	25%	2.6
Alg-2	ASU-T	129	60%	67%	55%	1.2
Calc-1	UWF	103	22%	40%	0%	"∞"
Calc-2	UWF	59	49%	61%	38%	1.6

Alg-1: Algebra-based course, first semester
Alg-2: Algebra-based course, second semester
Calc-1: Calculus-based course, first semester
Calc-2: Calculus-based course, second semester

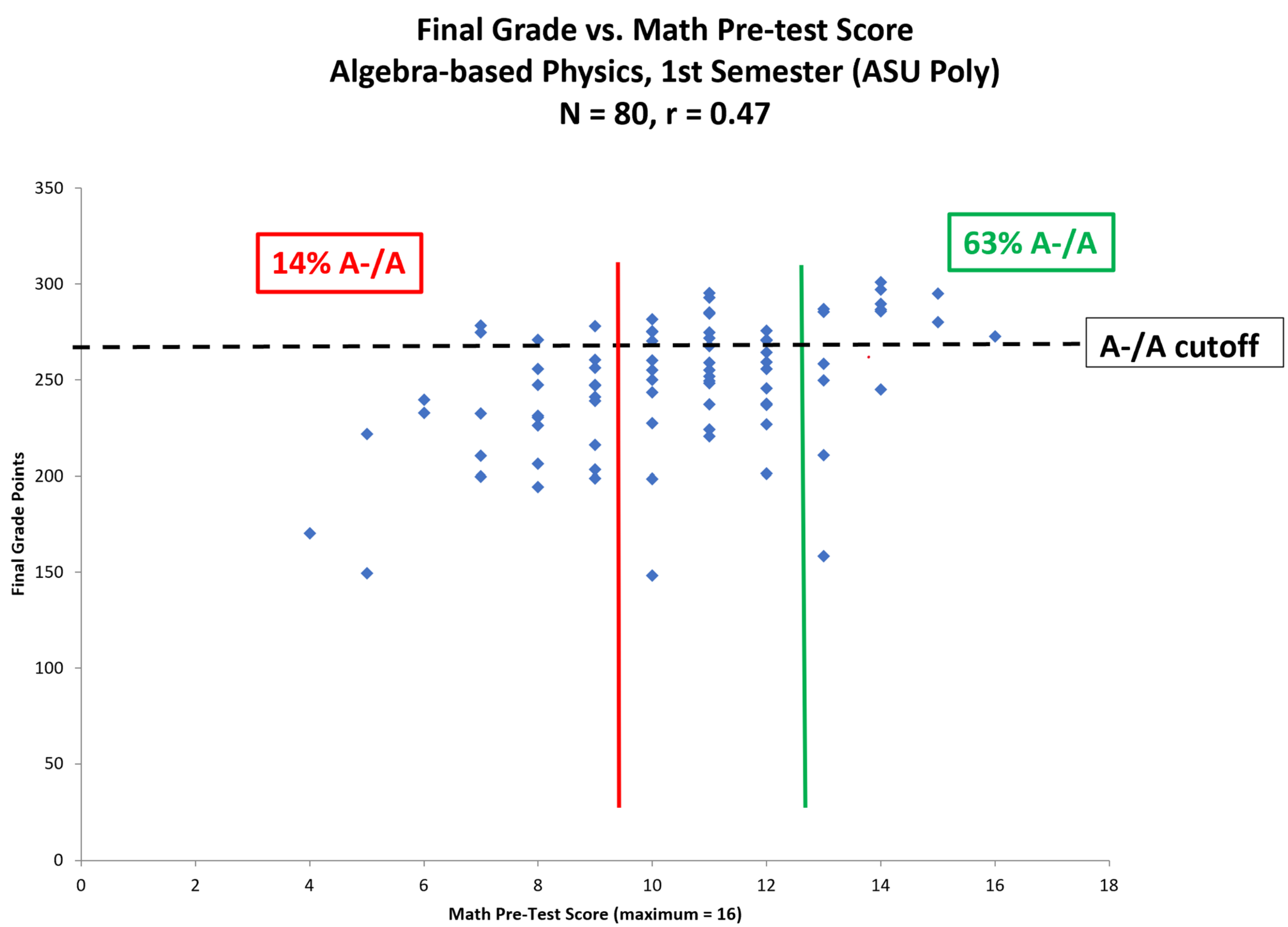
ASU-P: Arizona State University, Polytechnic campus
ASU-T: Arizona State University, Tempe campus
UWF: University of West Florida

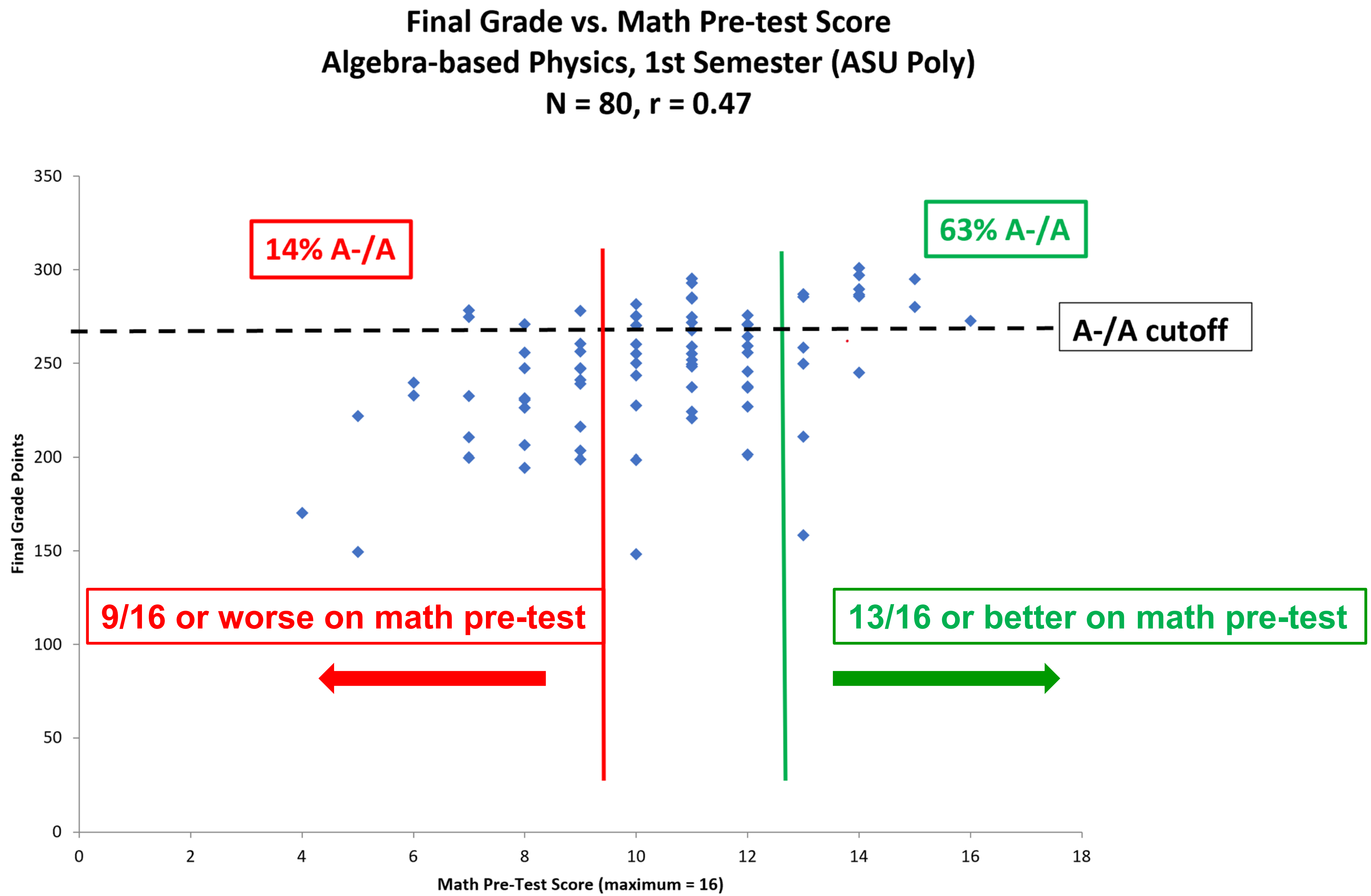
Students who scored high on math diagnostic pretest had more "A" course grades than those who scored low

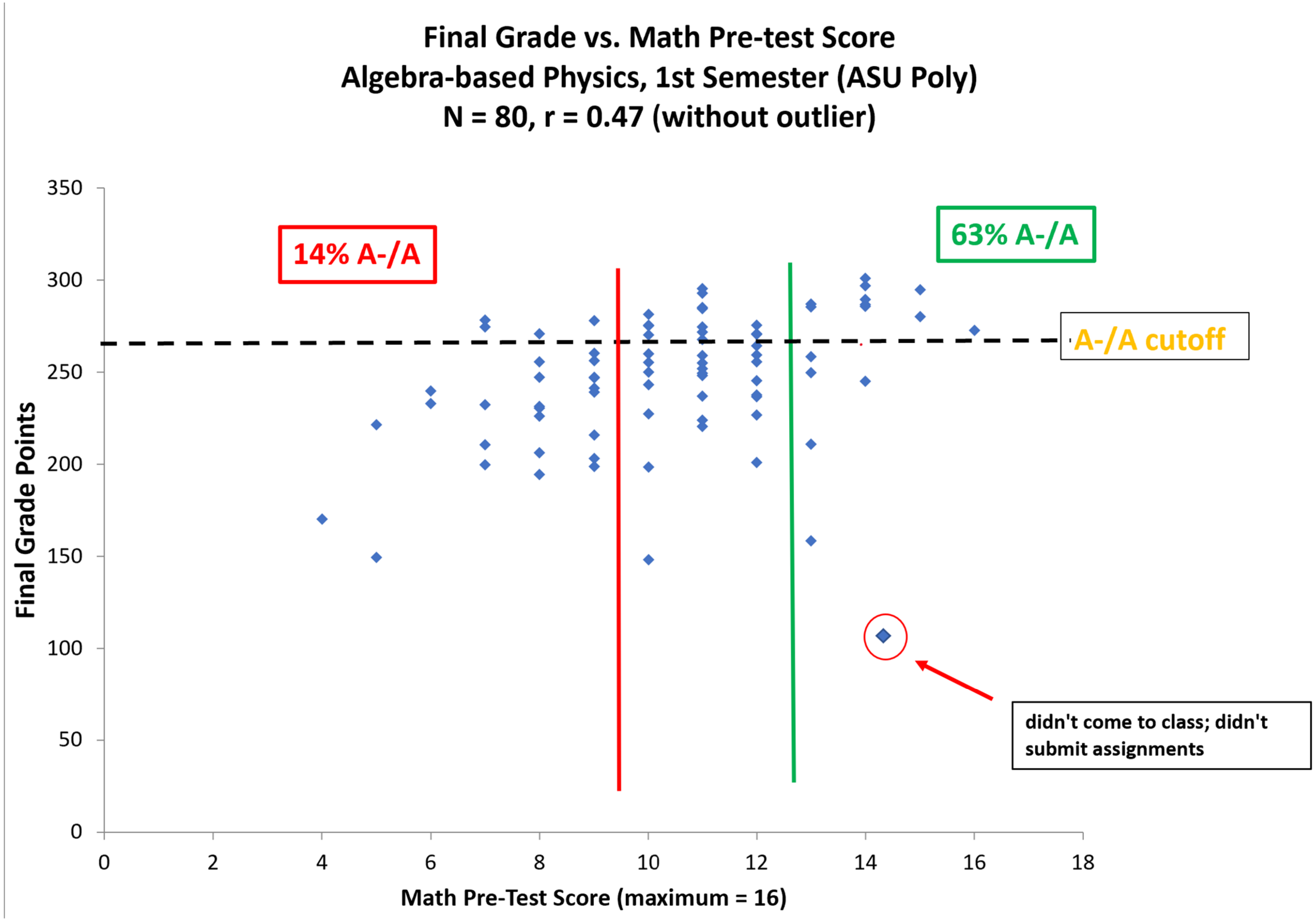
High Course Grade vs. Full Diagnostic Score

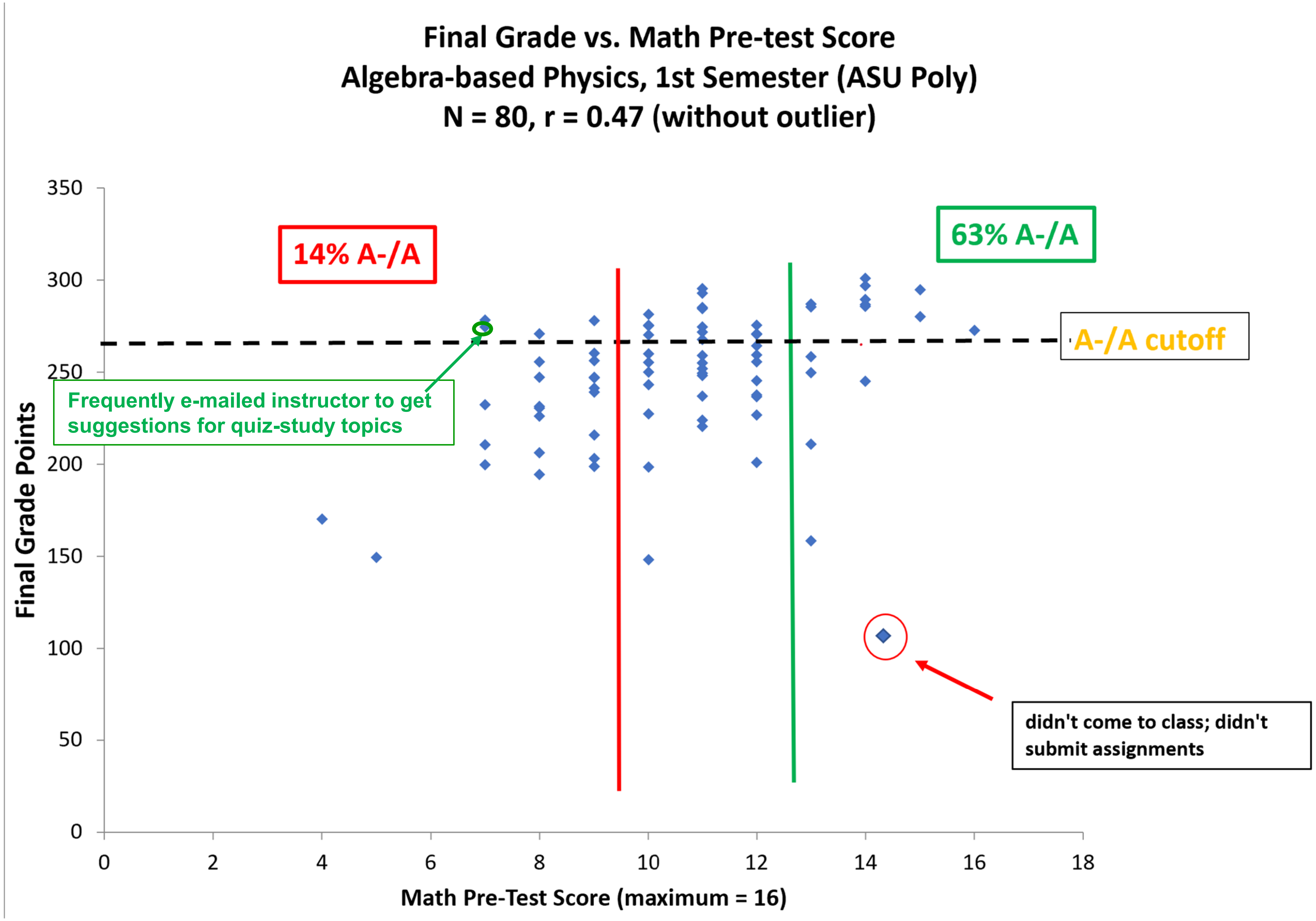
Course	Campus	*N*	Overall % grade ≥ A-	Score ≥ 81% % grade ≥ A-	Score ≤ 57% % grade ≥ A-	High-grade Ratio score ≥ 81% vs. score ≤ 57%
Alg-1	ASU-P	78	35%	63%	15%	4.2

But here, we can examine individual data points [students] in more detail…

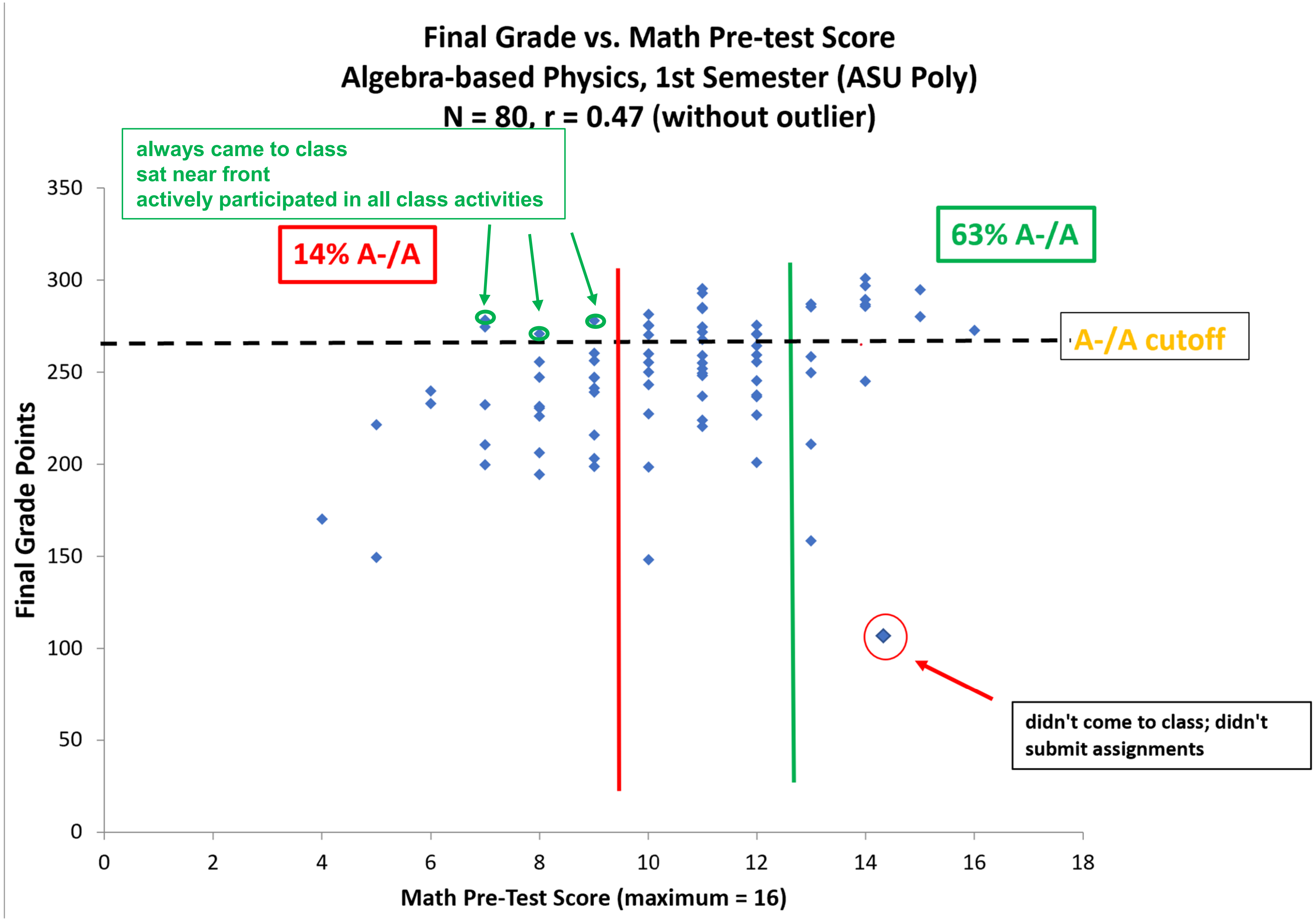

Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47
14% A-/A
63% A-/A
A-/A cutoff
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18


Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47
14% A-/A
63% A-/A
A-/A cutoff
9/16 or worse on math pre-test
13/16 or better on math pre-test
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18

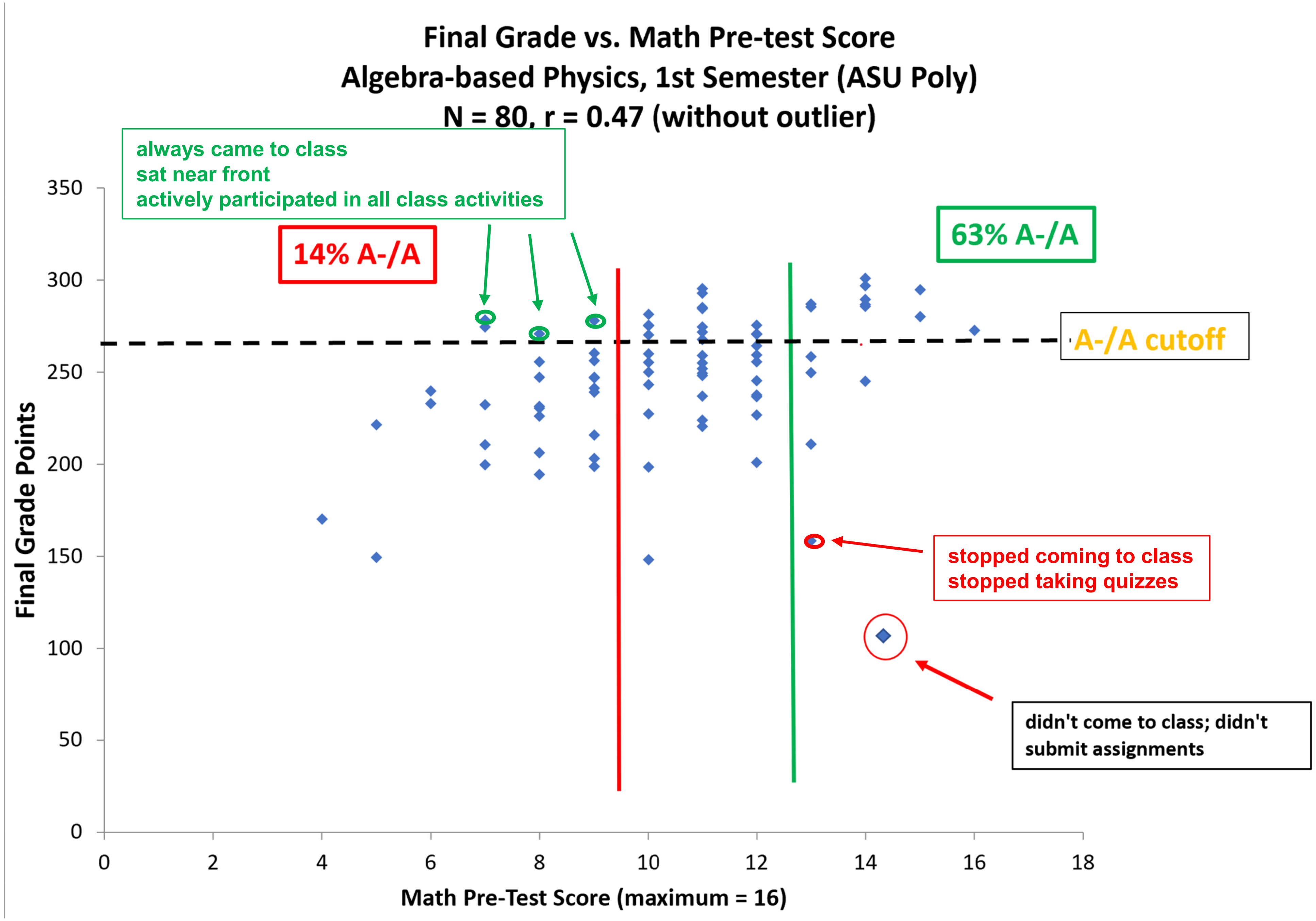

Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
14% A-/A
63% A-/A
A-/A cutoff
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
350
300
250
200
150
100
50
0
0
2
4
6
8
10
12
14
16
18



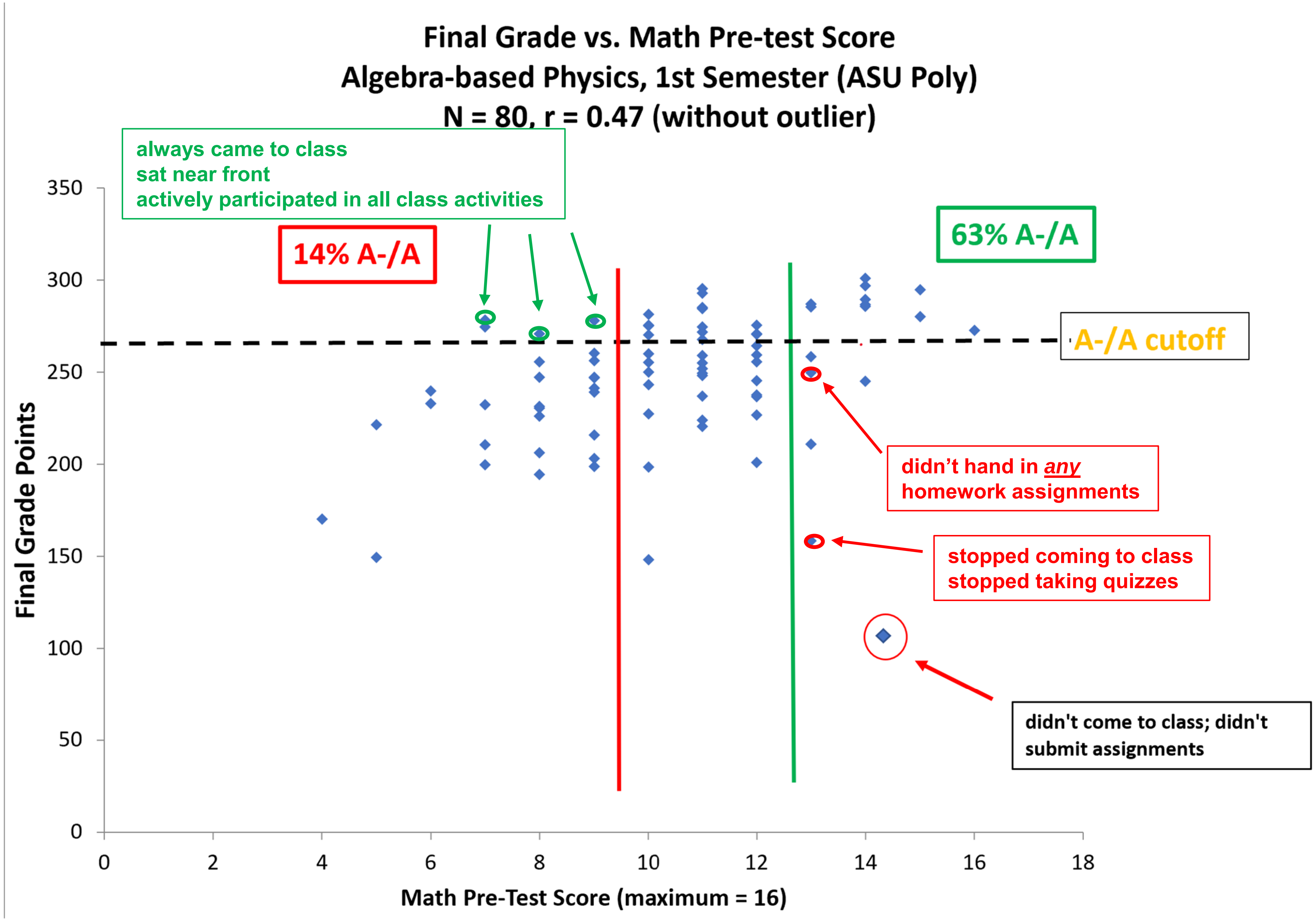
Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
14% A-/A
63% A-/A
A-/A cutoff
Frequently e-mailed instructor to get suggestions for quiz-study topics
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
350
300
250
200
150
100
50
0
0
2
4
6
8
10
12
14
16
18

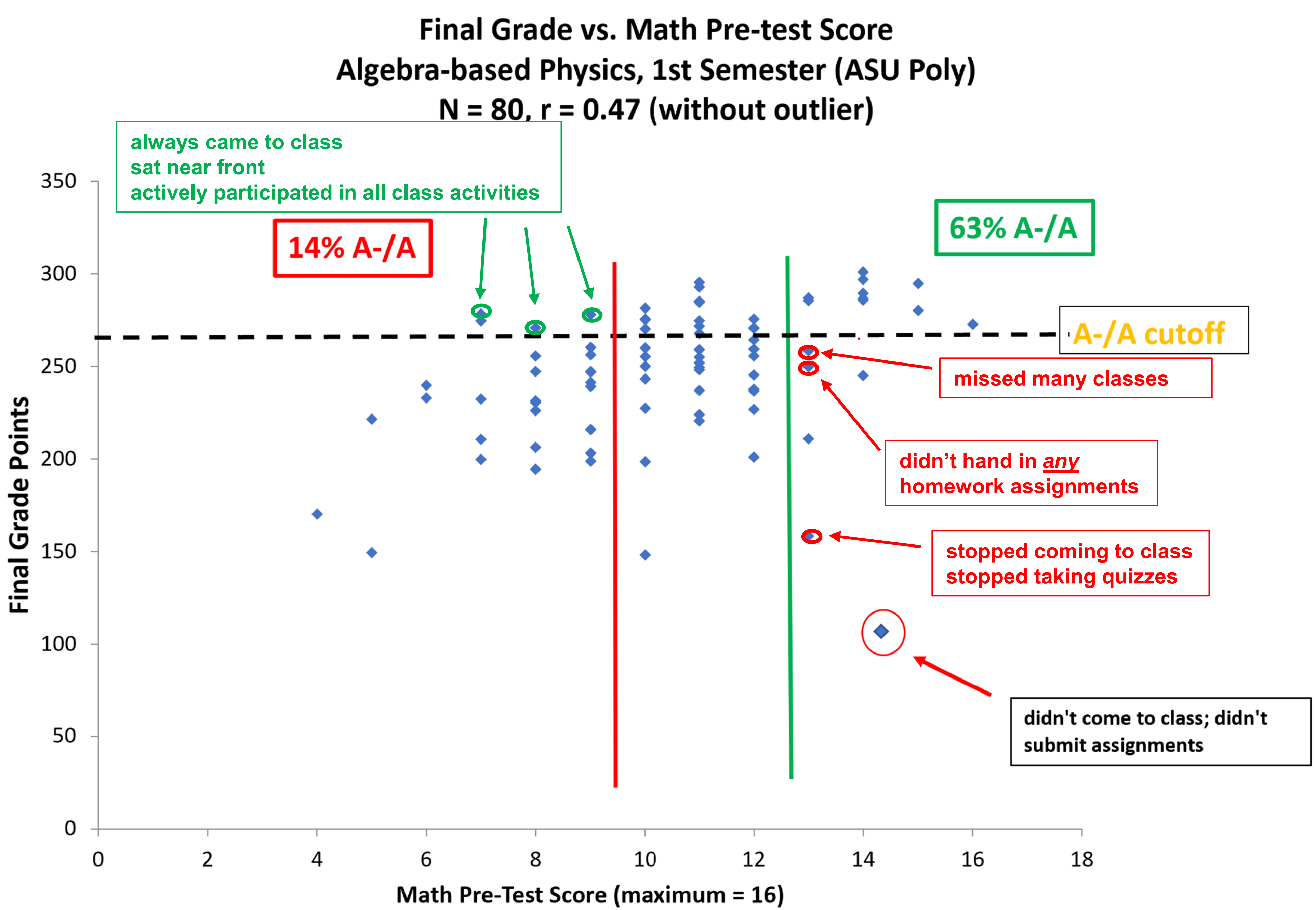


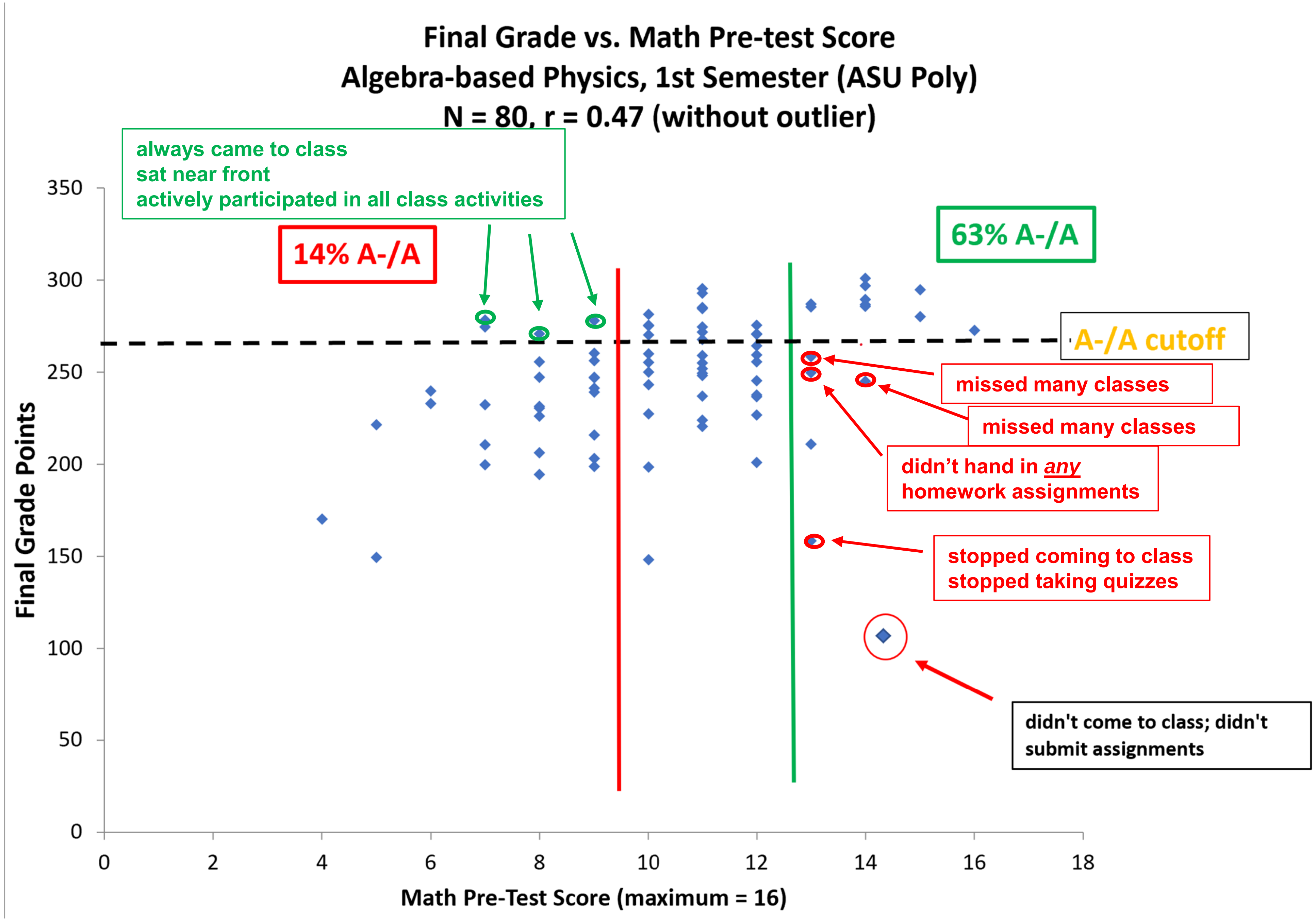
Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
didn't come to class; didn't submit assignments
Final Grade Points
350
300
250
200
150
100
50
0
0
2
4
6
8
10
12
14
16
18
Math Pre-Test Score (maximum = 16)

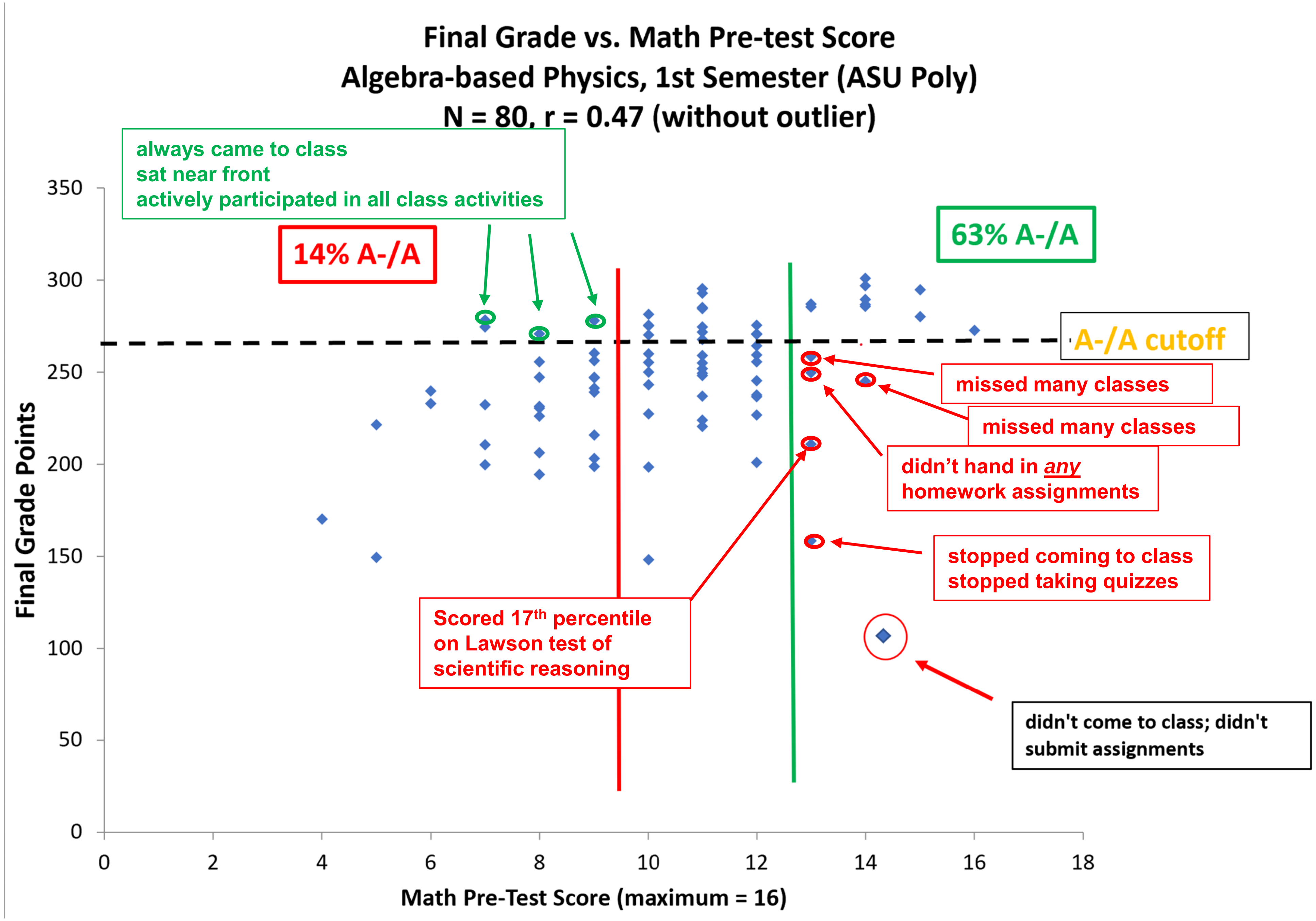


Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
stopped coming to class
stopped taking quizzes
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
350
300
250
200
150
100
50
0
0
2
4
6
8
10
12
14
16
18

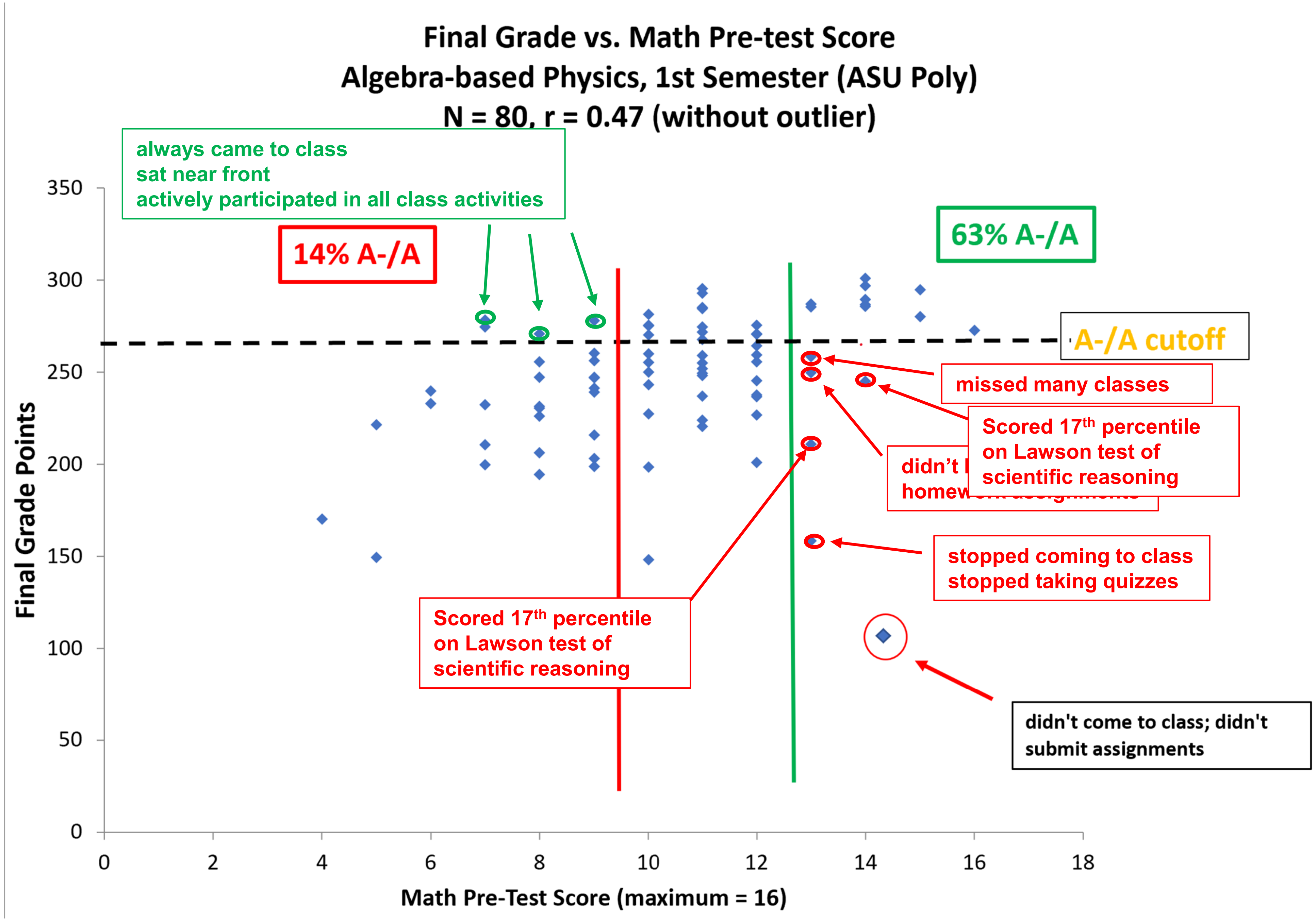

Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
didn’t hand in any homework assignments
stopped coming to class
stopped taking quizzes
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18


Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
missed many classes
didn't hand in *any* homework assignments
stopped coming to class
stopped taking quizzes
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18

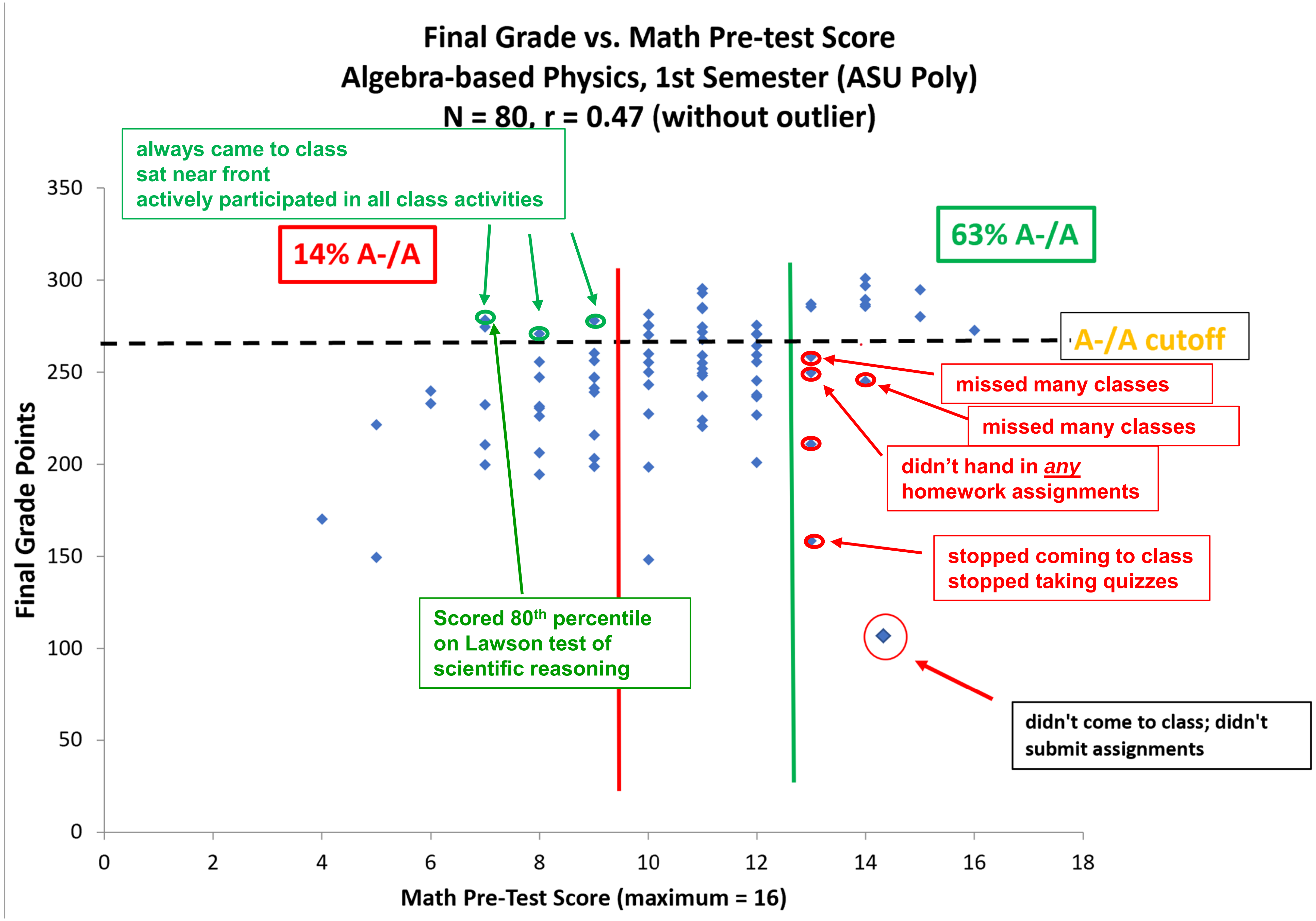

Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
missed many classes
missed many classes
didn't hand in any homework assignments
stopped coming to class
stopped taking quizzes
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18



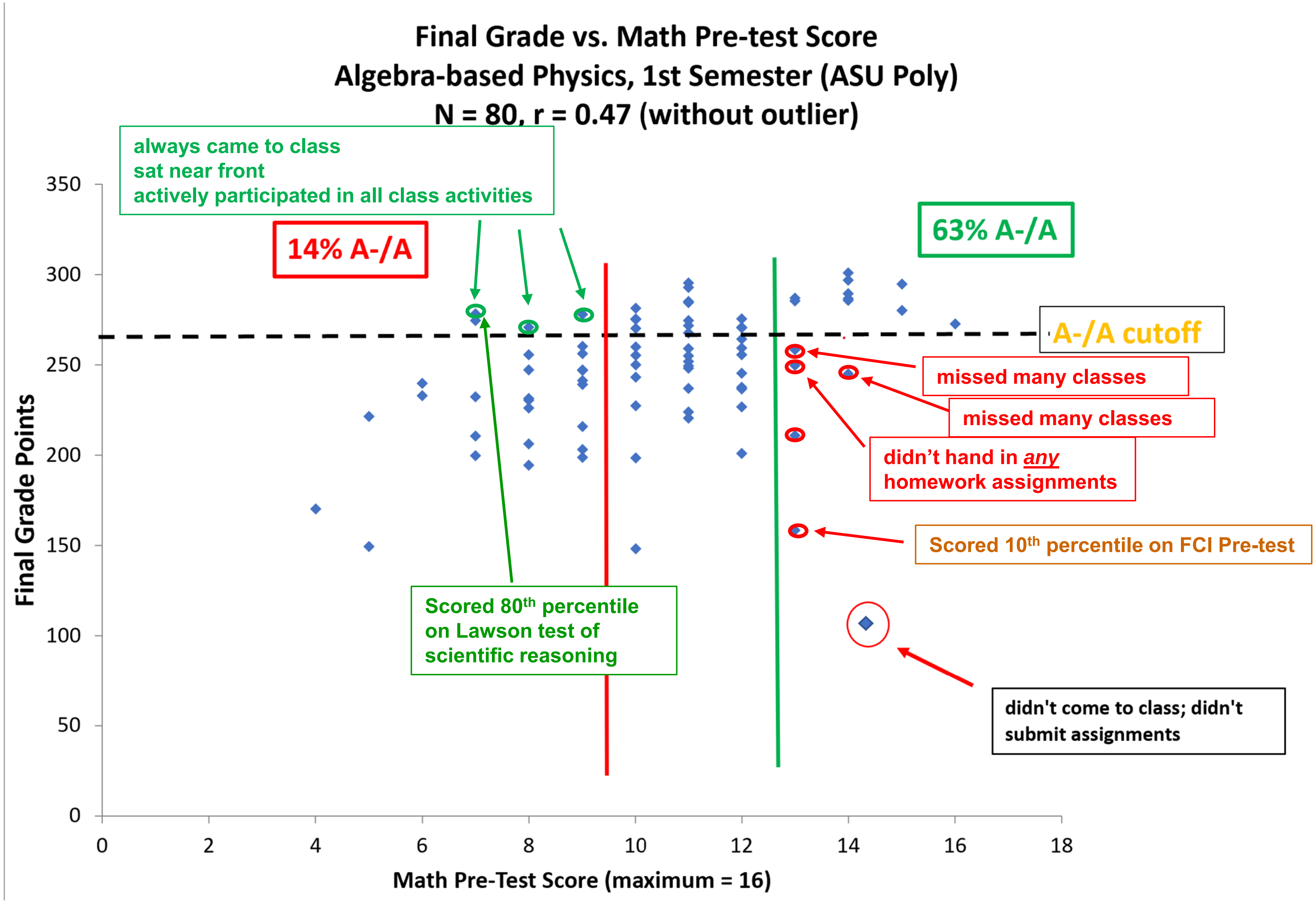
Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
missed many classes
missed many classes
didn't hand in any homework assignments
stopped coming to class
stopped taking quizzes
Scored 17th percentile on Lawson test of scientific reasoning
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18

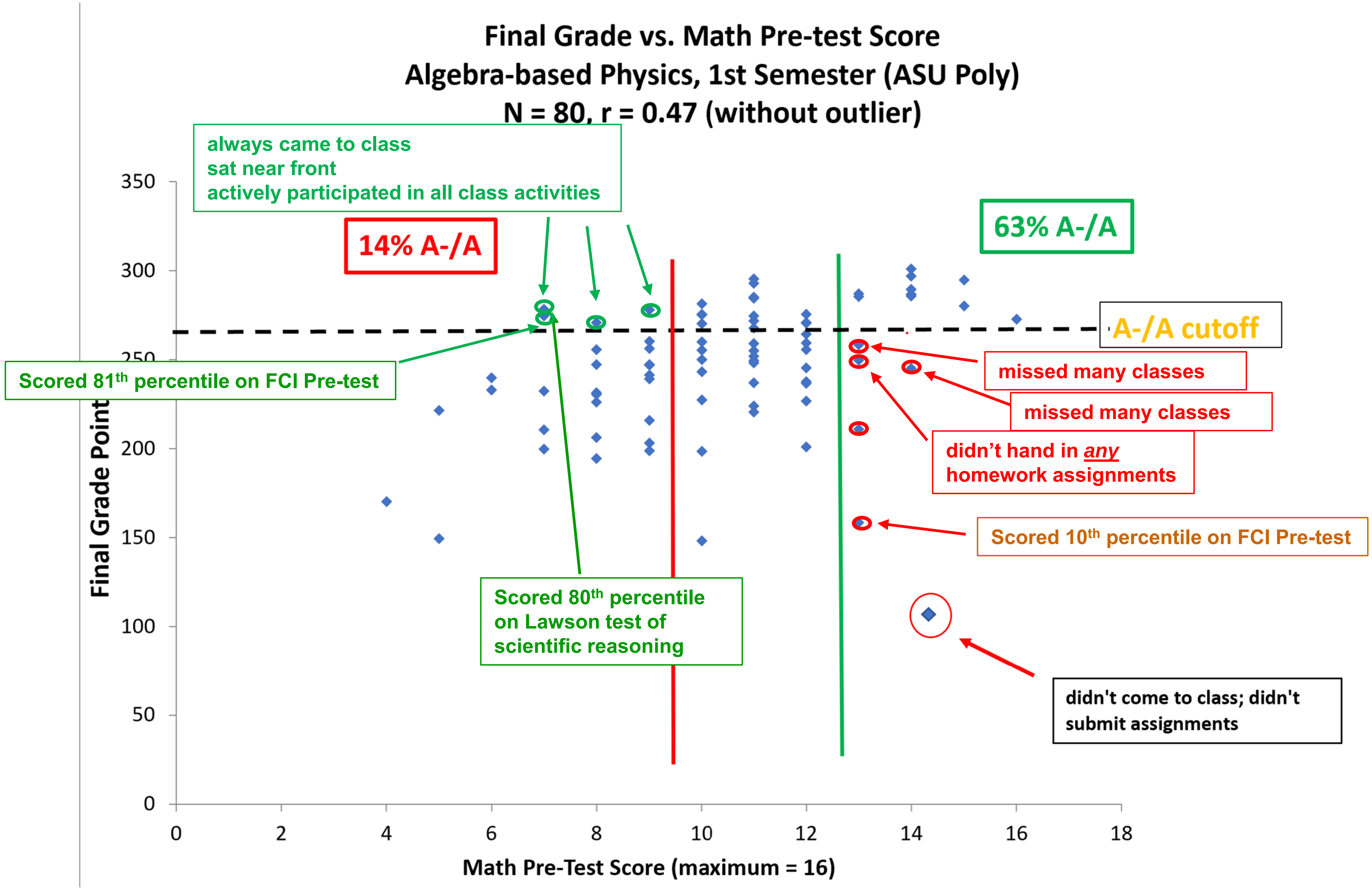


Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
missed many classes
Scored 17th percentile on Lawson test of scientific reasoning
stopped coming to class
stopped taking quizzes
Scored 17th percentile on Lawson test of scientific reasoning
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18



Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
missed many classes
missed many classes
didn't hand in any homework assignments
stopped coming to class
stopped taking quizzes
Scored 80th percentile on Lawson test of scientific reasoning
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18


Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
missed many classes
missed many classes
didn't hand in *any* homework assignments
Scored 10th percentile on FCI Pre-test
Scored 80th percentile on Lawson test of scientific reasoning
didn't come to class; didn't submit assignments
Final Grade Points
350
300
250
200
150
100
50
0
0
2
4
6
8
10
12
14
16
18
Math Pre-Test Score (maximum = 16)


Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
Scored 81th percentile on FCI Pre-test
missed many classes
missed many classes
didn't hand in any homework assignments
Scored 10th percentile on FCI Pre-test
Scored 80th percentile on Lawson test of scientific reasoning
didn't come to class; didn't submit assignments
Final Grade Point
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
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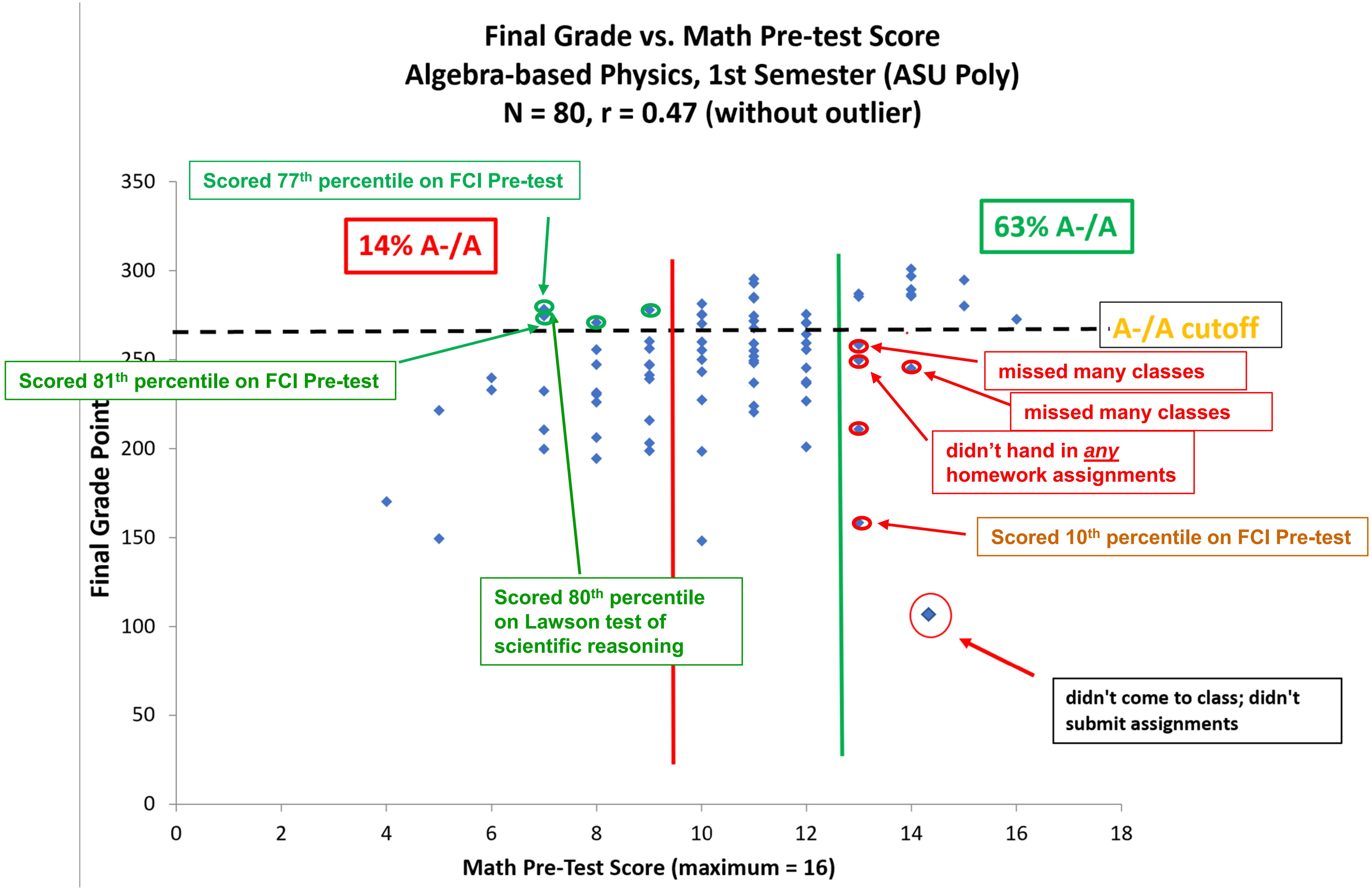
16
18



Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
Scored 77th percentile on FCI Pre-test
14% A-/A
63% A-/A
A-/A cutoff
Scored 81th percentile on FCI Pre-test
missed many classes
missed many classes
didn't hand in any homework assignments
Scored 10th percentile on FCI Pre-test
Scored 80th percentile on Lawson test of scientific reasoning
didn't come to class; didn't submit assignments
Final Grade Point
Math Pre-Test Score (maximum = 16)
350
300
250
200
150
100
50
0
0
2
4
6
8
10
12
14
16
18

Recommendations Summary

- Instructors should be wary of assumptions about students’ mathematics preparation before making assessments
  - Pre-instruction performance on a brief mathematics diagnostic may provide indications of students’ difficulties and of students at risk
- Instructors may wish to modify their instructional practices to take account of students’ mathematical difficulties and behaviors
  - e.g., constrained use of symbolic manipulations, addition of math practice
- Recognize that deep-lying difficulties may be hard to address, but motivational factors can provide some compensation