Exploring Physics Students' Difficulties in Solving Symbolic Algebra Problems

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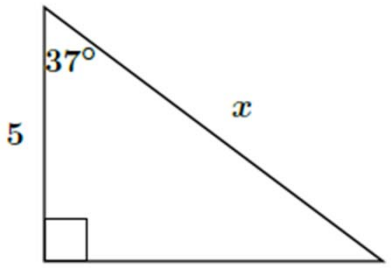
Trigonometry Questions

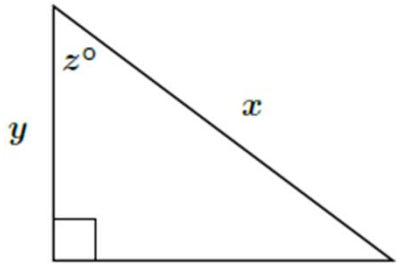
- Three basic trigonometry questions were administered on our diagnostic math quiz to a total of 1,318 students in beginning of the Spring 2018 semester.

1. What is the length of side x?



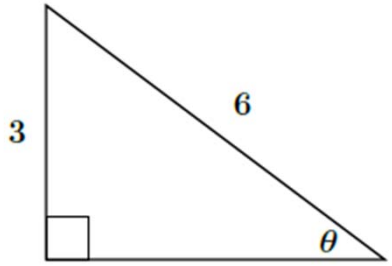
"Numeric" hypotenuse problem

2. What is the length of side x?



"Symbolic" hypotenuse problem

3. What is the value of θ?



Solving for an angle problem

Trigonometry Correct Response Rate #1-3 Combined Spring 2018

- ASU Tempe campus averages:

  PHY 121, 1st semester calculus-based course, (*N*=906): **81%**

  PHY 111, 1st semester algebra-based course, (*N*=225): **54%**

- ASU Polytechnic campus averages:

  PHY 111, 1st semester algebra-based course, (*N*=88): **41%**

  PHY 112, 2nd semester algebra-based course, (*N*=99): **40%**

  About 20%-60% of students confused on basic high school level trigonometry problems.

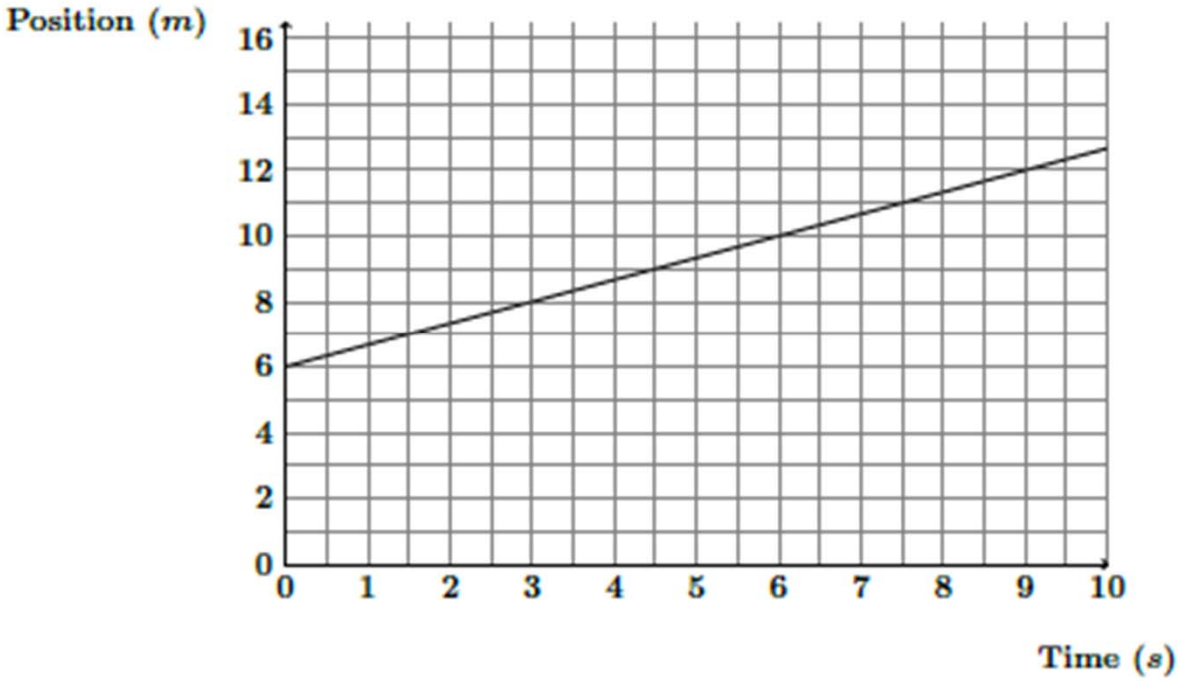
Trigonometry Errors Observed

- Use of the wrong trig function, e.g., using cosine or tangent instead of sine.
- Not understanding the definition or proper relationship for trig functions.
- Not showing all work when arriving at incorrect answer, e.g., x=ycos(z^o).

Many students seem not to be fluent with basic trigonometric functions and need additional practice.

Finding the slope of a graph.

- On our diagnostic, students were asked to find the slope of a graph:



Answer: $\frac{2}{3}$ m/s

Slope Question Correct Response Rate

- ASU Tempe campus averages:

  PHY 121, 1st semester calculus-based course, (*N*=902): **61%**

  PHY 111, 1st semester algebra-based course, (*N*=225): **53%**

- ASU Polytechnic campus averages:

  PHY 111, 1st semester algebra-based course, (*N*=88): **30%**

  PHY 112, 2nd semester algebra-based course, (*N*=99): **39%**

  About 40%-70% of students did not give a correct slope.

Slope Question Errors Observed

- 60%-76% of all incorrect responses were *slope = 1/3*.
- Students assume scaling on the x-axis is equal to that of the y-axis.
- Students are counting boxes to find the slope of the graph, which was also found to be true from the interviews.
- Almost no responses included correct units (including responses we counted as correct).

Numeric vs Symbolic Questions

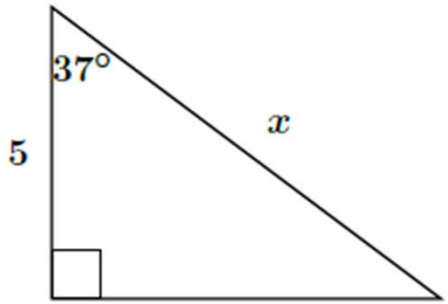
Torigoe and Gladding (2007; 2011) investigated differences in students' responses to physics problems in both numeric and symbolic form.

- "Numeric" and "symbolic" refer to the nature of the constant coefficients.
- They found that students (most of the time) had more difficulties with symbolic questions.

- We asked paired problems in both symbolic and numeric form.
  - Questions were stripped of all physics context and asked as pure mathematical problems.
- We included paired numeric vs symbolic problems from Torigoe and Gladding's (2011) study (stripped of physics context), as well as a pair of trigonometry problems.

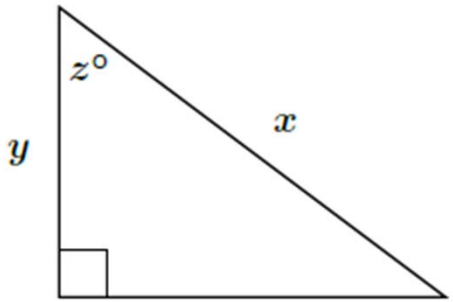
Trigonometry: Numeric vs Symbolic

1. What is the length of side x?



Correct answer: $x = \dfrac{5}{\cos(37^o)}$

2. What is the length of side x?



Correct answer: $x = \dfrac{y}{\cos(z^o)}$

Trigonometry Symbolic vs Numeric Correct Response Rates Compared Spring 2018 (*Tempe*)

- ASU Tempe campus averages:

  PHY 121, 1st semester calculus-based course, (*N*=903): **80%** **(numeric)**

  PHY 121, 1st semester calculus-based course, (*N*=903): **77%** **(symbolic)**

  PHY 111, 1st semester algebra-based course, (*N*=225): **56%** **(numeric)**

  PHY 111, 1st semester algebra-based course, (*N*=225): **52%** **(symbolic)**

Students at Tempe did 3%-4% worse on the symbolic version of the hypotenuse problem.

Trigonometry Symbolic vs Numeric Results

- For the Tempe calculus-based course, the error-rate increase in the symbolic version was found to be significant:
  - McNemar Test for Correlated Proportions Two-Tail: $p = 0.03$
- Results at Polytechnic for PHY 111/112 were similar but not significant due to the smaller sample size.

Basic Algebra Problem Symbolic vs Numeric

Numeric

$$5x + 3 = 2x + 5$$

Correct answer: $x = \frac{2}{3}$

Symbolic

$$ax + b = cx + d$$

$$x = ?$$

Correct answer: $x = \frac{d - b}{a - c}$

Basic Algebra Problem Symbolic vs Numeric Correct Response Rates Compared Spring 2018 (*Tempe*)

- ASU *Tempe* campus averages:

  PHY 121, 1st semester calculus-based course, (*N*=899): **96%** **(numeric)**

  PHY 121, 1st semester calculus-based course, (*N*=899): **87%** **(symbolic)**

  PHY 111, 1st semester algebra-based course, (*N*=222): **96%** **(numeric)**

  PHY 111, 1st semester algebra-based course, (*N*=222): **57%** **(symbolic)**

PHY 121 Tempe students did 9% worse on symbolic version and PHY 111 Tempe students did 39% worse. (PHY 111/112 for Polytechnic campus gave similar results with just a smaller *N*.) [All differences are statistically significant, $p \leq 0.001$.]

Example of not isolating x.

In the equation below, a, b, c, and d represent (unknown) numbers, for example, 3, 8, 9, 14.

$ax + b = cx + d$

$ax - cx = d - b$

Student's final answer

$x =?$

(Your answer for x should have a, b, c, and d in it. Please isolate x to one side of your answer.)

Simultaneous Equations Symbolic vs Numeric

Numeric:

$$78.4 - y = 8x$$

$$0.5y = 2x$$

Correct answer: $\boldsymbol{x = \frac{78.4}{12} \approx 6.5}$

Symbolic:

$$cy = dx$$

$$a - y = bx$$

Correct answer: $\boldsymbol{x = \frac{ac}{d + bc}}$

$$x = ?$$

Simultaneous Equations Problem Symbolic vs Numeric Correct Response Rates Compared Spring 2018 (*Tempe*)

- ASU *Tempe* campus averages:

  PHY 121, 1st semester calculus-based course, (*N*=733): **87%** **(numeric)**

  PHY 121, 1st semester calculus-based course, (*N*=733): **63%** **(symbolic)**

  PHY 111, 1st semester algebra-based course, (*N*=140): **72%** **(numeric)**

  PHY 111, 1st semester algebra-based course, (*N*=140): **36%** **(symbolic)**

PHY 121 Tempe students did 24% worse on symbolic version and PHY 111 Tempe students did 36% worse. (PHY 111/112 at the Polytechnic campus gave similar results with a smaller *N*.)

Simultaneous Equations Errors Observed

- **Numeric version:** sign errors; not eliminating y.
- **Symbolic version:** not eliminating y; factoring errors; not isolating x.

- The complexity of the symbolic expressions led to many new errors that were not observed in the numeric version.

Example of not isolating x in simultaneous equations

In the two equations below, a, b, c, and d represent (unknown) numbers, for example, 3, 8, 9, 14.

$cy = dx$ $\quad x = \frac{cy}{d}$

$a - y = bx$ $\quad +y = -bx + a$

$x =?$ $\quad x = \frac{c(-bx+a)}{d}$

Leaving x in terms of x

(Your answer for x should have a, b, c, and d in it, but *not* y. Please isolate x to one side of your answer.)

Failure to isolate x was responsible for > 40% of all errors in PHY 121

Kinematic Equation Problem Numeric vs Symbolic

Numeric:

$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = 60$$

$$\Delta t = 8$$

$$v = 30$$

$$d = ?$$

Correct answer: $\mathbf{d = 60}$

Symbolic:

$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{v_1}{t_1}$$

$$v = \frac{v_1}{2}$$

$$d = ?$$

Correct answer: $\mathbf{d = \frac{v_1 t_1}{8}}$

Kinematics Equation Problem Symbolic vs Numeric Correct Response Rates Compared Spring 2018 (*Tempe*)

- ASU *Tempe* campus averages:

  PHY 121, 1st semester calculus-based course, (*N*=889): **89%** **(numeric)**

  PHY 121, 1st semester calculus-based course, (*N*=889): **72%** **(symbolic)**

  PHY 111, 1st semester algebra-based course, (*N*=215): **81%** **(numeric)**

  PHY 111, 1st semester algebra-based course, (*N*=215): **37%** **(symbolic)**

PHY 121 Tempe students did 17% worse on symbolic version and PHY 111 Tempe students did 44% worse. (PHY 111/112 at Polytechnic gave similar results with a smaller *N*.)

Kinematic Equation Problem Errors Observed

- **Numeric version:** substituting the wrong value into original equation, e.g., Δv for v.
- **Symbolic version:** incorrectly squaring and multiplying/dividing fractions.

Students seem to struggle with the additional steps and complexity in the symbolic version.

Example of common error with fractions

10. In the equations below, v_1, t_1, a, and v represent (unknown) numbers, for example, 3, 8, 9, 14.

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{v_1}{t_1}$

$v = \frac{v_1}{2}$

d = ?

$\left(\frac{v_1}{2}\right)^2 = 0^2 + 2\left(\frac{v_1}{t_1}\right)d$

$\frac{\left(\frac{v_1}{2}\right)^2}{2\left(\frac{v_1}{t_1}\right)} = d$

Correct expression

Error

$\frac{v_1^2}{4} \cdot \frac{2t_1}{2v_1} = \frac{v_1^2 t_1}{4 \cdot 2 v_1} = \frac{v_1 t_1}{4}$

(Please clearly circle your answer and show all work.)

A. $d = v_1 t_1$ B. $d = \frac{v_1 t_1}{2}$ C. $d = \frac{v_1 t_1}{4}$ D. $d = \frac{v_1 t_1}{8}$ E. $d = \frac{v_1 t_1}{16}$

Summary of Difficulties with Symbols

- We find that students have significant difficulties with most of the symbolic problems.
- Students seem not to be confused with the meaning of symbols themselves, but instead struggle with manipulations.
- Changing numbers to symbols in a simple problem (e.g., the hypotenuse problem) only produces very small differences in correct response rates.
- More complicated algebra problems result in much larger differences in response rates.
- With symbols, students are forced to deal with more obstacles which are not encountered in numeric problems.