Physics Students’ Familiarity with Mathematical Facts and Procedures

David E. Meltzer and Dakota H. King
Arizona State University

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The Problem

- College physics instructors often report deficiencies in introductory students' mathematical skills
- Weak skills with basic pre-college mathematics can severely impact physics students' course performance
- We have explored the nature and prevalence of physics students' difficulties with elementary mathematics, using "stripped-down" problems with little or no physics context

Work to Date

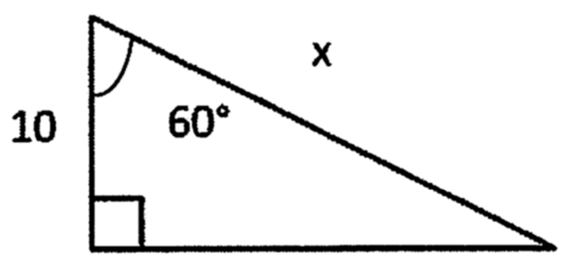
- Administer (and analyze) written diagnostic quiz, given to > 4000 students in ≈ 30 algebra- and calculus-based physics classes over seven semesters at Arizona State University during 2016-2019; calculators *are* allowed

- Carry out individual interviews with 75 students enrolled in those or similar courses during same period (Primary interviewer: Matt Jones)

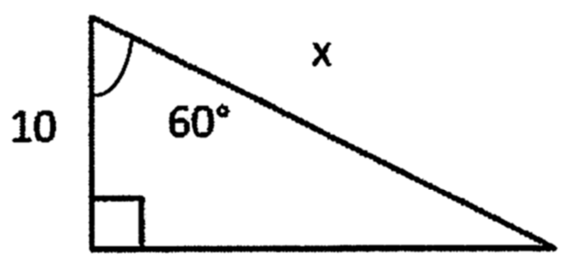
- Topics: trigonometry, algebra, vectors, graphing, geometry

Diagnostic Questions

with samples of correct student responses



What is the value of x?

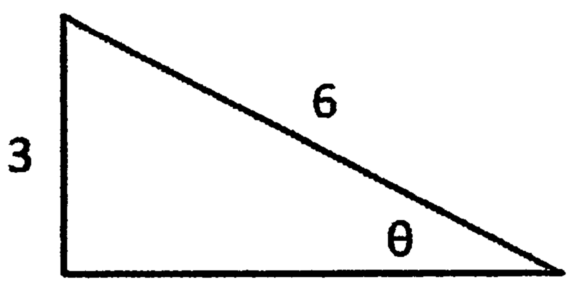


What is the value of x?

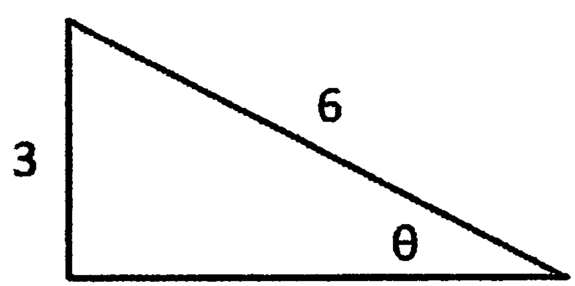
$$\cos 60 = \frac{10}{x}$$

$$x \cos 60 = 10$$

$$x = \frac{10}{\cos 60} = 20$$



What is the value of θ?

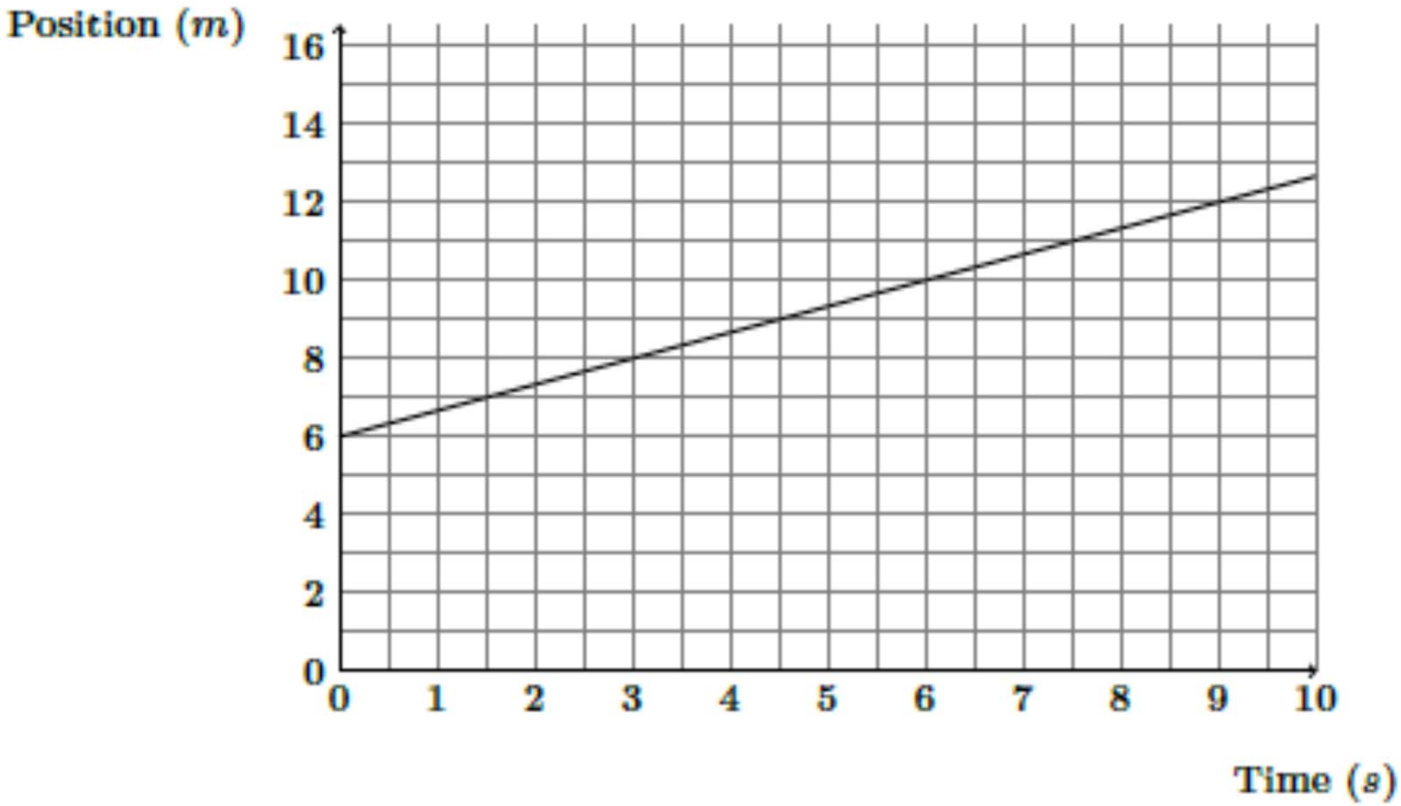


What is the value of θ?

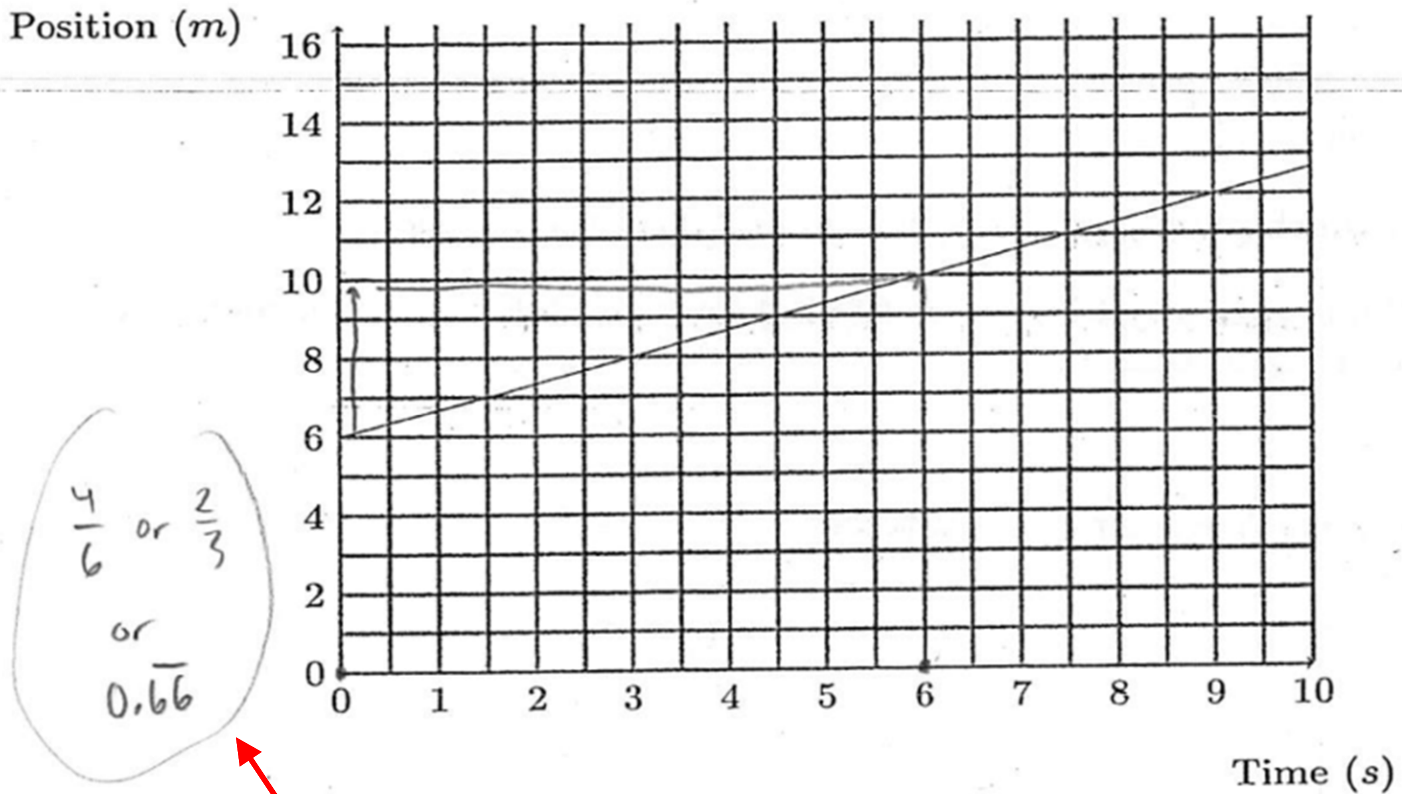
$\sin^{-1}(\theta) = \sin^{-1}\left(3/6\right)$

$\theta = 30°$

What is the slope of the graph below?



[Some physics content]



$\frac{10-6}{6-0} = \frac{4}{6}$ or $\frac{2}{3}$

or

$0.\overline{66}$

Note: Units missing

$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

$$x = \frac{ca}{cb+d}$$

$cy = dx$

$a - y = bx$

$y = a - bx$

$c(a - bx) = dx$

$ca - cbx = dx$

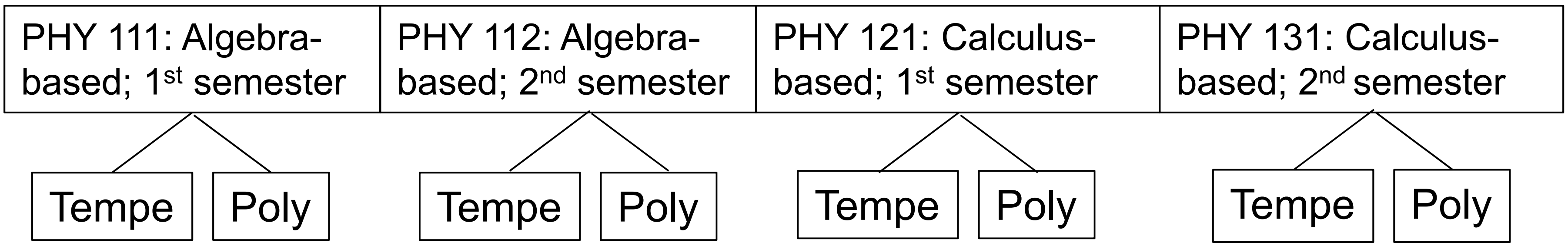
$ca = (cb + d)x$

$x = ?$

Our 4 Sample Populations

PHY 111: Algebra-based; 1st semester	PHY 112: Algebra-based; 2nd semester	PHY 121: Calculus-based; 1st semester	PHY 131: Calculus-based; 2nd semester

Our 8 Sample Populations



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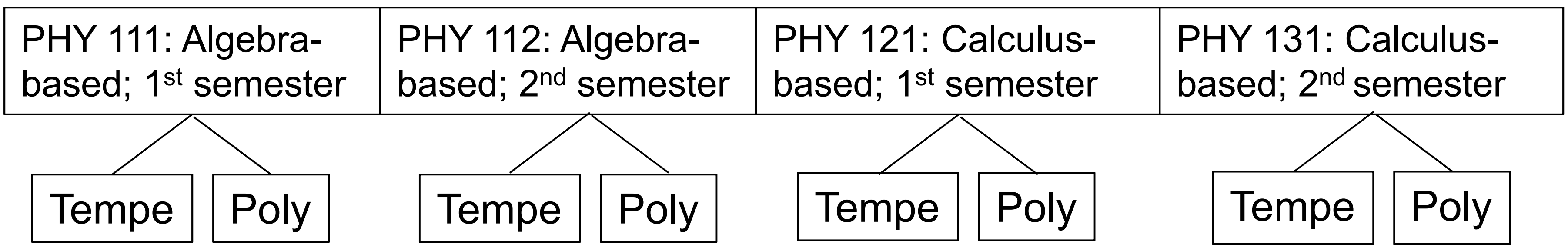
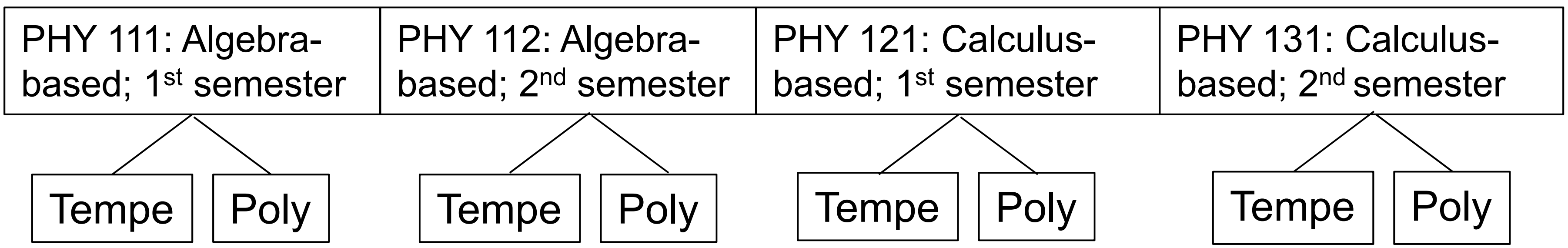
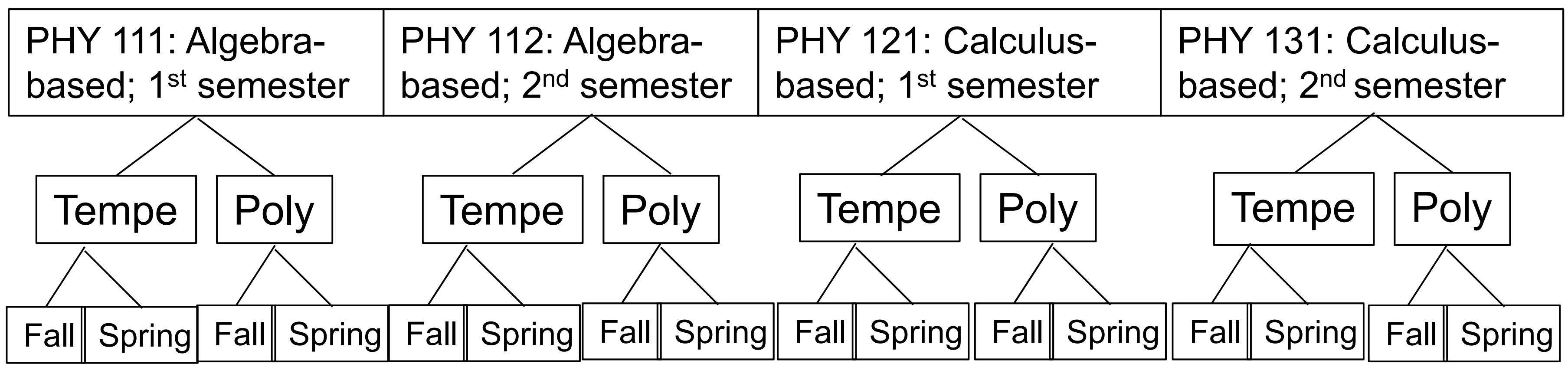


On average, students in the Tempe courses have more extensive background and preparation than those in the corresponding Poly courses.

Our 8 Sample Populations



Our 16 Sample Populations



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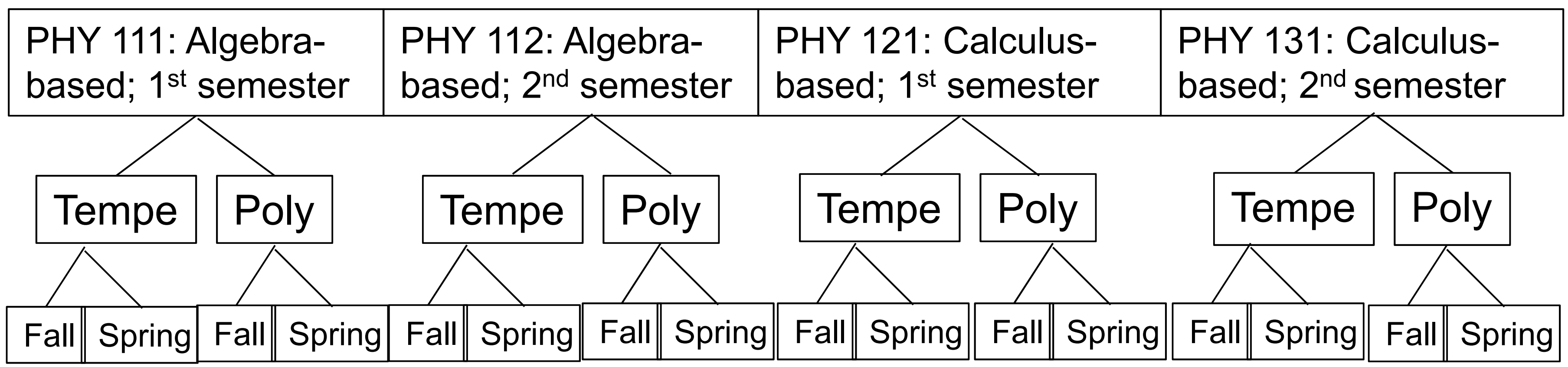
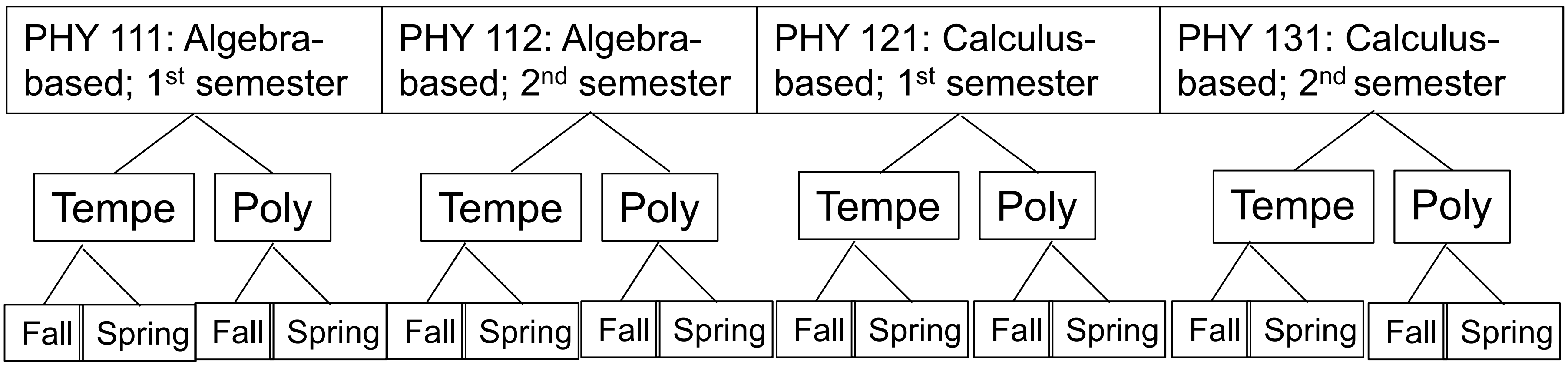


Each of the 16 sample populations has distinct and consistent differences from all of the others!

Our 16 Sample Populations



Our data-collection period (2016-19) included 4 spring semesters and 3 fall semesters, and several of the 16 populations were only sampled once or twice, some with low *N*. So, even with total *N* > 4000, confirmation of consistent patterns is challenging.

Primary Findings

Regardless of course (algebra- or calculus-based), campus (Tempe or Poly), or semester (Spring or Fall):

- Difficulties with basic mathematical operations are widespread; average error rates range from 20-70%;
- Performance on problems using symbols for constants is significantly worse than on problems using numbers;
- During problem-solving interviews, students self-correct approximately 50% of errors following minimal prompts;

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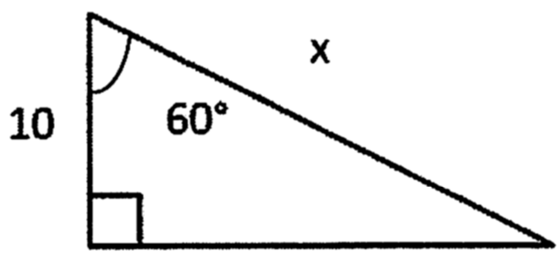

What is the value of x?

$$\cos 60 = \frac{10}{x}$$

$$x \cos 60 = 10$$

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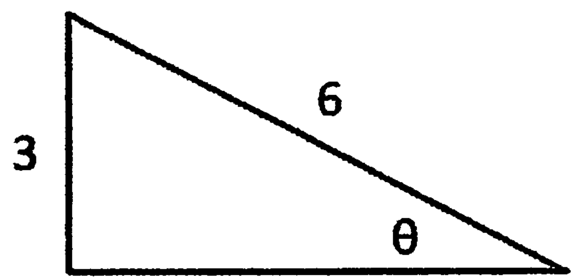
Correct Response Rate, Poly; Fall 2016

Algebra-based course, first semester: 47% (N = 94)

Calculus-based course, first semester: 44% (N = 98)



What is the value of θ?

$$\sin^{-1}(\theta) = \sin^{-1}\left(3/6\right)$$

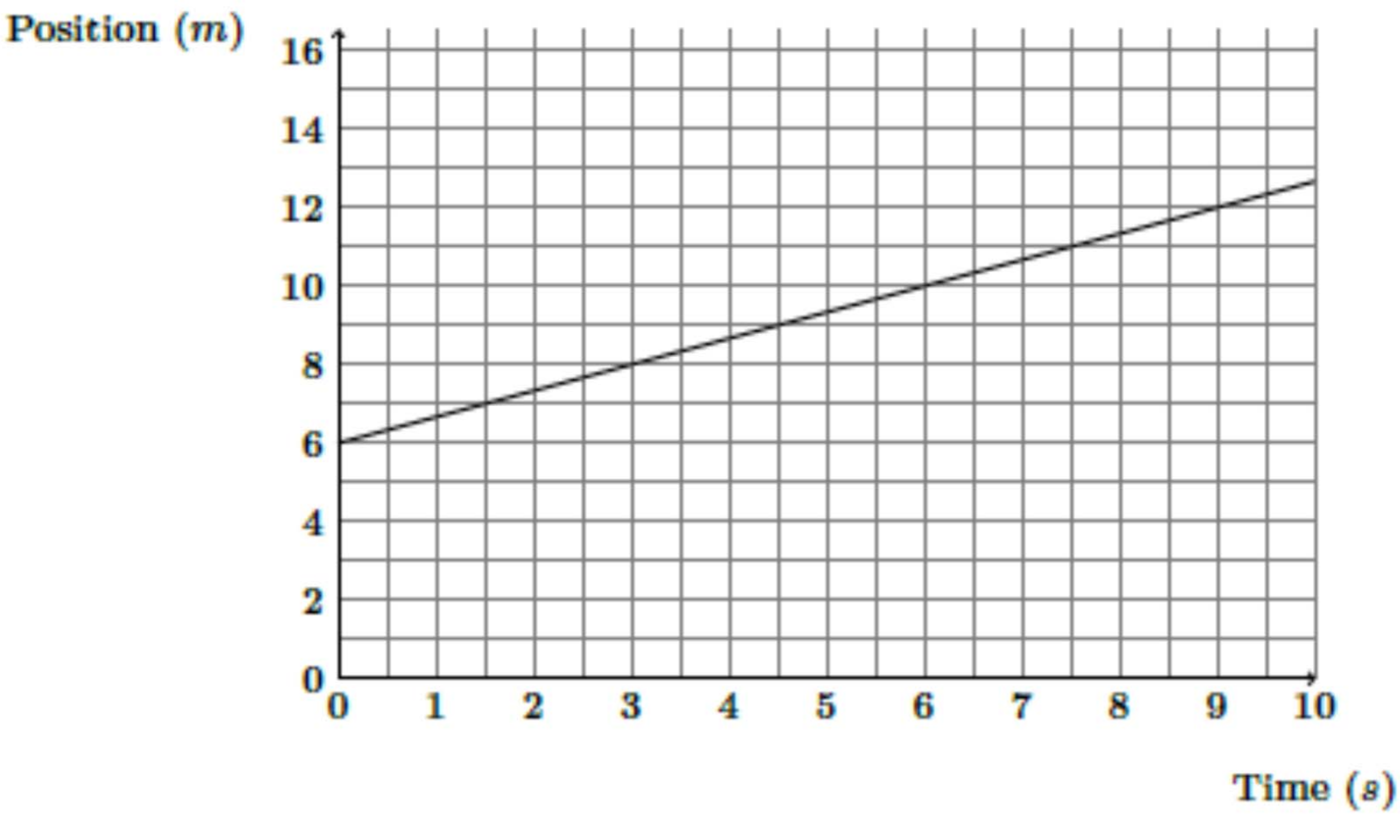
$$\theta = 30^{\circ}$$

Correct Response Rate, Tempe; Fall 2018

Algebra-based course, first semester: 50% (*N* = 308)

Calculus-based course, first semester: 79% (*N* = 330)

What is the slope of the graph below?



Correct Response Rate, Tempe; Fall 2018

Algebra-based course, first semester: 43% (*N* = 305)

Calculus-based course, first semester: 56% (*N* = 329)

$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

Correct Response Rate, Tempe; Spring 2018

Algebra-based course, first semester: 36% (N = 140)

Calculus-based course, first semester: 63% (N = 733)

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Algebra, numeric *vs.* symbolic

Calculus-based course, first semester (Tempe);
Solve for x*: % correct responses:*

$78.4 - y = 8x$ $0.5y = 2x$	***Fall 2018:***	75%	(N = 146)
	Spring 2018:	83%	(N = 897)

$cy = dx$ $a - y = bx$	***Fall 2018:***	47%	(N = 129)
	Spring 2018:	63%	(N = 733)

Primary Findings

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- Difficulties with basic mathematical operations are widespread; average error rates range from 20-70%;
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Student Self-Correction of Errors

Our Interview Findings: Almost half of students' errors on algebra problems were self-corrected by students during interviews, as a consequence of interviewer prompts or unprompted auto-correction.

Prompts Leading to Self-Correction

- *"Explain that step"*
- *"Clarify what you mean*."
- "*What does the problem ask you to do?"*
- *[No specific prompt: Students asked to explain all work]*

Interview Results: $N = 53$

$3x = 2y$
$5x + y = 26$

What are the values of x and y? Show all your steps. For example, $x = 2$, $y = 5$ (These are NOT the correct answers).

Correct:	**83%**
Error, Self-corrected:	**9%**
Error, Uncorrected:	**8%**

Interview Results: N = 53

$x \cos(20°) = y \cos(70°)$
$x \cos(70°) + y \cos(20°) = 10$

What are the values of x and y? Show all your steps. Note: The value for x should NOT include y, and the value for y should NOT include x.

Correct:	**57%**
Error, Self-corrected:	**19%**
Error, Uncorrected:	**25%**

Interview Results: N = 53

$ax = by$
$bx + ay = c$

a, b, and c are constants.
What are the values of *x* and *y* in terms of a, b, and c? Show all your steps. Note: The value for *x* should NOT include *y*, and the value for *y* should NOT include *x*.

Correct:	**55%**
Error, Self-corrected:	**21%**
Error, Uncorrected:	**25%**

Summary: Implications for Instruction

- Difficulties due to **skill-practice deficits** might be addressed by short-term, in- and out-of-class tutorials and assignments, designed to refresh students' previously learned knowledge and skills (e.g., Mikula and Heckler, 2017)
- Difficulties due to **"carelessness"** might be addressed by guiding students to (1) carefully check and re-check key steps in their calculation; (2) slow down, review problem statements, and re-solve when possible

From Torigoe and Gladding (2011):

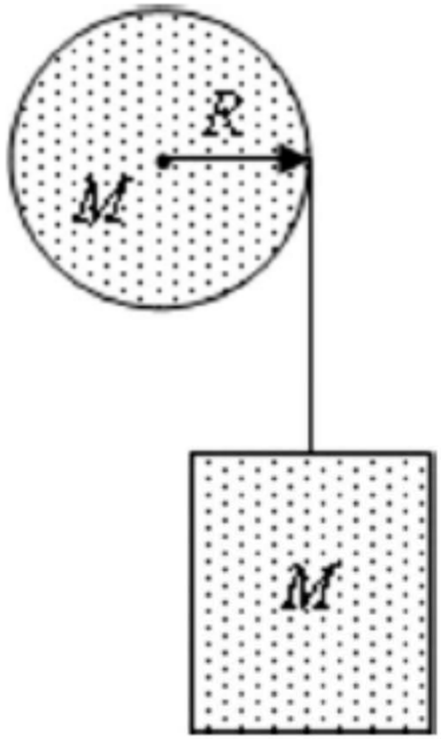


Fig. 7. Diagram for question 10.

Question 10 (numeric). A uniform disk of mass $M=8$ kg and radius $R=0.5$ m has a string wound around its rim. The disk is free to spin about a pin through the center of the disk. A mass $M=8$ kg (same mass as the disk) is connected to the string and is dropped from rest. What is the acceleration a of the block? (See Fig. 7.)

From Torigoe and Gladding (2011):

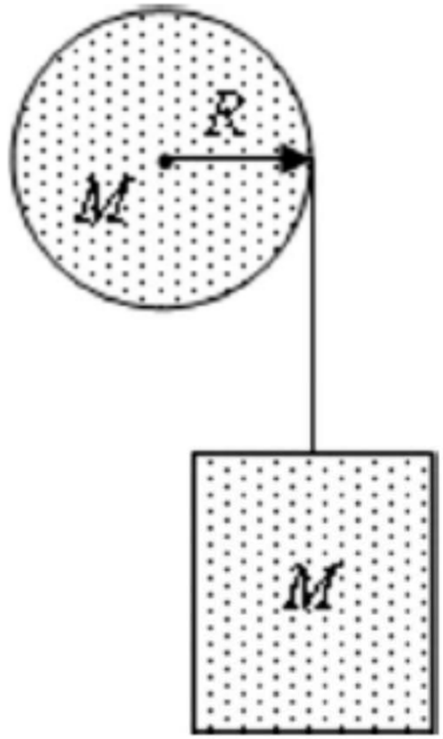
$$Mg - T = Ma$$
$$TR = I\alpha$$
$$[I = ½\, MR^2;\ \alpha = a/R]$$

$$\ldots\rightarrow$$

$$Mg - T = Ma$$
$$TR = [½\, MR^2][a/R]$$
$$a = ?$$

$$78.4 - y = 8x$$
$$0.5y = 2x$$
$$x = ?$$

Our Numeric version



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Why the Difficulties with Symbols?

Some Suggestions Arising from the Interviews

- In elementary math courses, “simplified forms” of equations are emphasized (i.e., few messy symbols and functions).
- Many students get “overloaded” by seeing all the variables, and are unable to carry out procedures that they do successfully with numbers.
- Many students have had *insufficient practice* with algebraic operations to avoid being overwhelmed by standard algebraic manipulations.
  - Students tend to become *careless*